Exploring Asymptotes

Time required
00 minutes

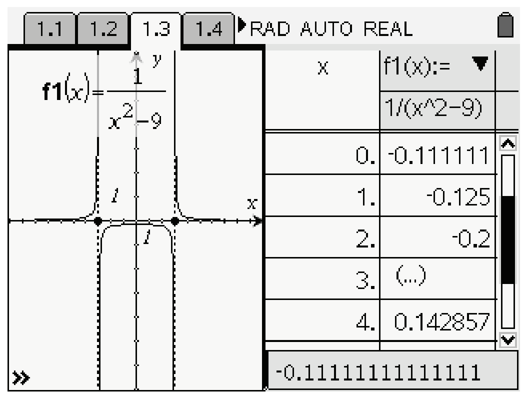
Activity Overview

In this activity, students will explore asymptotes and singularities, paying particular attention to the connection between the algebraic and graphical representations.

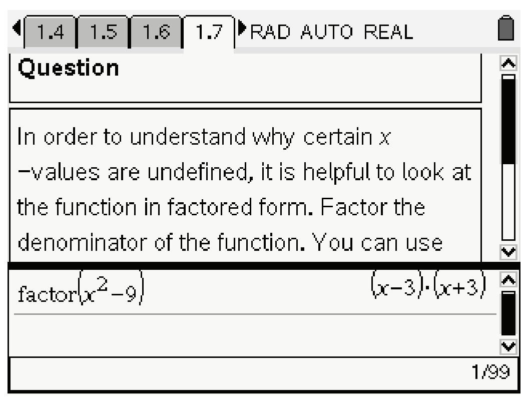
Material

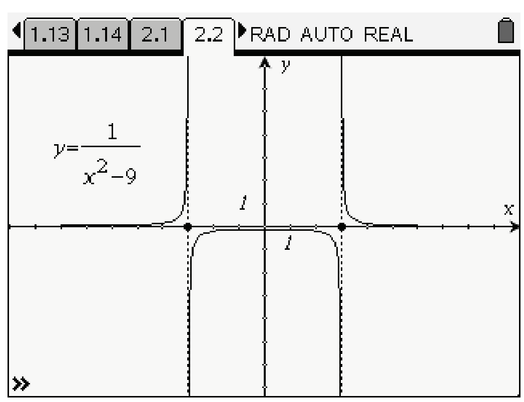
- *Technology*: TI-Nspire handheld, TI-Nspire CAS handheld, or TI-Nspire software
- *Documents*: Asymptotes.tns, Asymptotes_Student.doc

Rational Functions



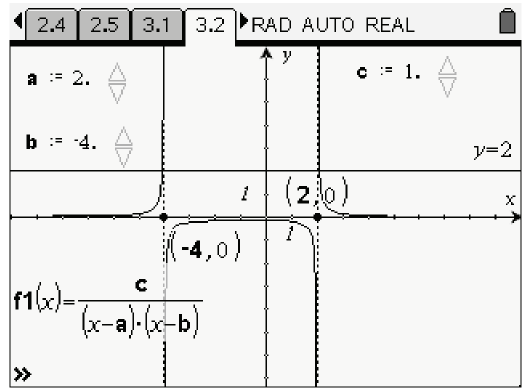
Part 1 involves an exploration of asymptotes involving graphing, exploring a table of values, and algebraic manipulation. Students will use a variety of TI-Nspire tools to identify undefined values of *x*. First, a graphic representation is explored. From the graph, students can get reasonable approximations of undefined values. Ask students how they can be certain of the undefined values. What tools are provided on the graphing screen to help with this?

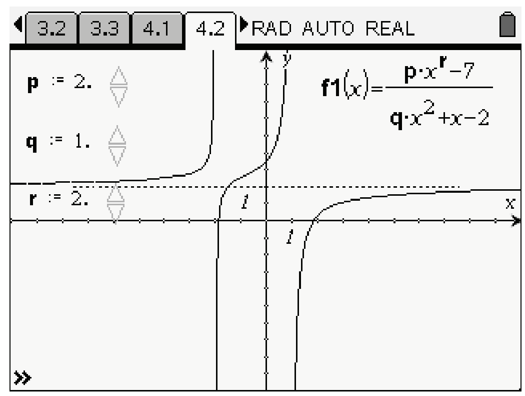




Horizontal Asymptotes

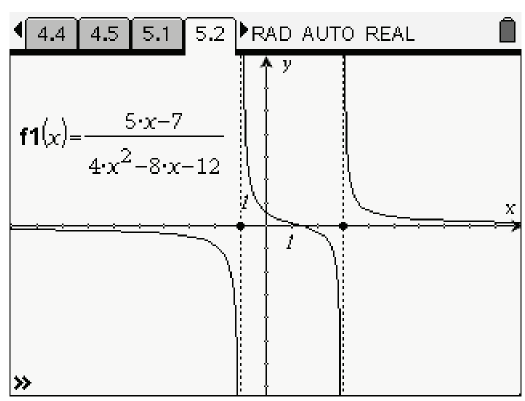
Students explore what was learned in Part 1 to develop an understanding of the patterns involved with asymptotes and rational functions. This part provides a great opportunity for students working in pairs or small groups to develop a deeper understanding of what was learned in Part 1.





Functions and Relations to Vertical & Horizontal Asymptotes

Students apply what was learned in Parts 1 and 2 to finding singularities and asymptotes for the function $f(x) = \dfrac{5x - 7}{4x^2 - 8x - 12}$.



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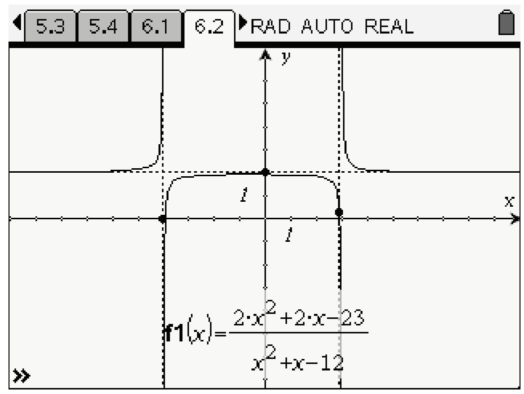
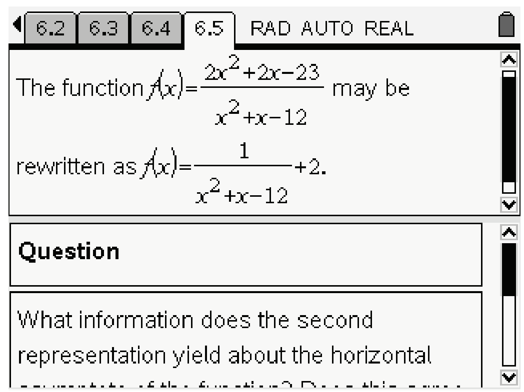
Lead Coefficients and Relations to Asymptotes

Students again apply what has been learned, but the challenge level increases. In this situation, the function is represented in two different ways, one of which helps to illustrate the vertical shift of a simpler graph. You may wish to have students graph $f(x) = \dfrac{1}{x^2 + x - 12}$ and compare it to the second function given. If time allows, consider having students verify algebraically that the two functions given on page 6.5 are equivalent. Also, have students label the asymptotes on the function graph with equations.





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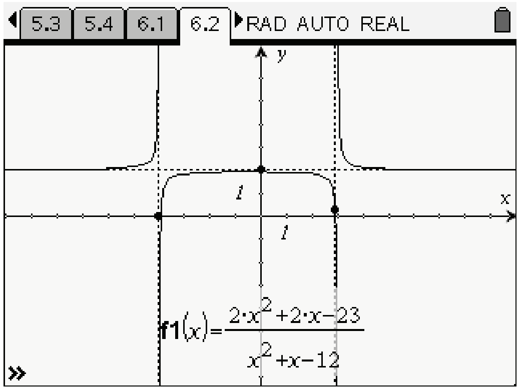
~~Exploring Asymptotes~~ —Student Solutions

1. **a.** *No; certain values of x are skipped*
   b. *The table displays (...) for the undefined x-values.*
   c. *± 3 have no values*
   d. *$(x - 3)(x + 3)$*
   e. *The factors match the skipped x-values. $(x - 3)$ matches with the skipped value of 3, and $(x + 3)$ matches with the skipped value of -3.*
   f. *These values make the denominator equal to zero, causing the function to be undefined at these values of x. Students might expect asymptotes here.*
   g. *Would expect asymptotes at $x = 3$ and $x = -3$.*
   h. *Only one asymptote at $x = -3$.*
   i. *The $x - 3$ cancels for everything except $x = 3$ and so it does not go to infinity.*
2. **a.** *Yes; $y = 0$*
   b. *The denominator gets very small making the fraction very large in magnitude.*
3. *Vertical asymptotes are always at $x = a$ and $x = b$.*
4. **a.** *Horizontal asymptotes are present whenever the degree of the numerator is less than or equal to the degree of the denominator. They may or may not be present when the numerator degree exceeds that of the denominator.*
   b. *$y = p/q$, or at $y =$ (ratio of leading coefficient of numerator to leading coefficient of denominator)*
   c. *$y = 0$*
5. **a.** *$x = -1$ and $x = 3$*
   b. *Yes; $y = 0$*
6. **a.** *$(x - 3)(x + 4)$; singularities at $x = 3$ and $x = -4$*
   b.



   c. *Since $\dfrac{1}{x^2 + x - 12}$ approaches zero as x tends to infinity, the second representation clearly tends to 2.*

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