Lesson Plan:
Exploring the Binomial Theorem

Level

Algebra II

California Content Standards Addressed

18.0 Students use fundamental counting principles to compute combinations and permutations.

19.0 Students use combinations and permutations to compute probabilities.

20.0 Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.

Topics to be covered

Pascal's Triangle
Combinations and Permutations
The Binomial Theorem
The Binomial Probability Theorem

Overview and Purpose

This lesson will introduce students to the binomial theorem through a variety of activities. Students will begin by filling in values of Pascal's triangle. They will then solve a combination problem, and relate the solution back to Pascal's triangle. Probabilities will also be explored through problem solving. The binomial theorem, combinations formula, and the binomial probability function will be explored. Students will also discover how to use the TI-83/TI-84 calculator to find binomial and probability distributions and to create histograms.

Class Time Required

Approximately three or four class periods, depending on students' problem solving and calculator skills. (Does not include time for homework review.)

Objectives

Upon completion of this lesson the participants will be able to:

- Complete Pascal's triangle to any number of rows
- Use Pascal's triangle to determine the number of combinations possible for a given set of parameters, and compute probabilities related to the combinations
- Explain how the binomial theorem, the combinations formula, and the binomial probability function can be used to solve combinations problems
- Understand how to use the TI-83/TI-84 calculator to find binomial and probability distributions

Required Materials

Pascal's triangle template
"Antonio's Pizza Palace" problem
"Do You Feel Lucky?" problem
"sing the TI-83 to analyze binomial distribution"worksheet
TI-83/TI-84 calculator

Historical Background

(to be discussed in context during the lesson)

The binomial theorem states that for positive integers n,

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}, \qquad (1)$$

where $\binom{n}{k}$ are binomial coefficients (Abramowitz and Stegun 1972, p. 10).

The theorem was known for the case $n = 2$ by Euclid around 300 BC, and stated in its modern form by Pascal in a posthumous pamphlet published in 1665. Pascal's pamphlet, together with his correspondence on the subject with Fermat beginning in 1654 (and published in 1679) is the basis for naming the arithmetical triangle in his honor.

Newton (1676) showed that a similar formula (with infinite upper limit) holds for negative integers $-n$,

$$(x+a)^{-n} = \sum_{k=0}^{\infty} \binom{-n}{k} x^k a^{-n-k}, \tag{2}$$

the so-called negative binomial series, which converges for $|x| < a$.

Copied directly from: Eric W. Weisstein. "Binomial Theorem." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/BinomialTheorem.html

Prerequisite Knowledge and Skills

Students should have experience in problem solving, particularly in finding patterns. They should be skilled at interpreting and evaluating with formulas. Some background in the basics of combinations, permutations, and probability is expected. Students should be familiar with the TI-83/TI-84 calculators, and particularly should be proficient at using the lists and statistical functions.

Resources/References

- http://mathforum.org/workshops/usi/pascal/pizza_pascal.html
- http://mathforum.org/workshops/usi/pascal/images/midd.comb1.gif
- TI Activities Exchange: *TI: The Short Way to Expand, Nick Goodbody* http://education.ti.com/educationportal/activityexchange/activity_detail.do?activityid=3489&cid=us
- TI Activities Exchange: *Win or Lose: A Binomial Distribution Investigation* http://education.ti.com/educationportal/activityexchange/activity_detail.do?activityid=3339&cid= us
- TI Activities Exchange: *The Binomial Distribution using the TI-83 Plus, Mary Ann Conners* http://education.ti.com/educationportal/activityexchange/activity_detail.do?activityid=3497&cid=us

Activities/Procedures

- ✓ Teacher will introduce students to Blaise Pascal and the first few rows of his triangle. There will be no explanation of where the numbers come from, their significance, or how to add more rows.
- ✓ Teacher will hand out blank Pascal's triangle. Students will copy the first few rows from the teacher and fill in the rest of the first 12 rows by finding patterns.
- ✓ Students will complete "Antonio's Pizza Palace" problem. Teacher will lead class discussion and summary upon completion. Teacher will introduce the binomial theorem (for n positive integers only) and the combinations formula. Appropriate homework will be assigned for practice.
- ✓ Students will complete "Do You Feel Lucky?" problem. Teacher will lead class discussion and summary upon completion. Teacher will introduce the Binomial Probability Formula. Appropriate homework will be assigned for practice.
- ✓ As a culminating activity the teacher will show students how to use the TI-83/TI-84 calculator to solve the pizza and luck problems and students will use the calculator to further investigate the problems.

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