Dominance Matrices

TI-Nspire Investigation Student 30 min

Are all wins created equal?

Most team sports involve some sort of round-robin type tournament arrangement. As more teams join a competition, the season either needs to get longer or some teams will not get to play each other with the same frequency; in these situations some teams may face a tougher draw than others. The Australian Football League (AFL™) involves 18 teams that compete in the home and away season over 22 rounds. Each team plays every other team in the competition, however this leaves 5 more games, so some teams must play each other twice. At the end of the season the total number of wins and losses is recorded for each team, but are all these wins created equal?

In this investigation you will compare the results of the top four teams based on their form as they played one another throughout the season. The official AFL ladder is available from the AFL website: http://www.afl.com.au/ladder

The top four positions at the end of the 2016 home and away season were as follows:

1. Sydney
2. Geelong Cats
3. Hawthorn
4. GWS Giants

The first[1] encounters for each team have been recorded in the table below:

Team: Sydney	**Round**	**Result**
Sydney vs GWS Giants	3	Sydney
Hawthorn vs Sydney	9	Sydney
Sydney vs Geelong Cats	16	Sydney

Team: Geelong Cats	**Round**	**Result**
Hawthorn vs Geelong Cats	1	Geelong
GWS Giants vs Geelong Cats	2	GWS
Sydney vs Geelong Cats	16	Sydney

Team: Hawthorn	**Round**	**Result**
Hawthorn vs Geelong Cats	1	Geelong
GWS Giants vs Hawthorn	6	GWS
Hawthorn vs Sydney	9	Sydney

| **Team**: GWS Giants	**Round**	**Result**
GWS Giants vs Geelong Cats	2	GWS
Sydney vs GWS Giants	3	Sydney
GWS Giants vs Hawthorn	6	GWS

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[1] **First Encounter:** Some teams played the top teams more than once, only the first encounter has been recorded, regardless of when it occurred.

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TEXAS INSTRUMENTS

Questions

1. Draw a directed network to represent the outcomes of these games.

2. Construct a matrix to represent the network with the first row representing the team that finished on top of the ladder, the second row representing the team that placed second on the ladder and so forth.

Open the TI-nspire file "Dominance".

Use the matrix template to define the matrix from Question 2 (above). Store the matrix in a variable called: **AFL**
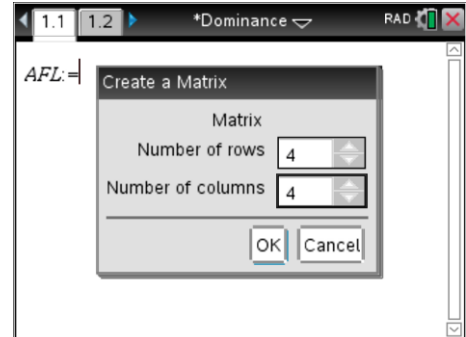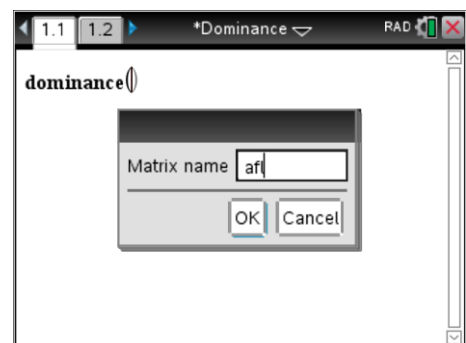
Once the matrix has been entered the dominance program can be used to create the dominance vector.

Press the [**VAR**] key, select the dominance program then press [**ENTER**] to run it.

A prompt will appear, enter the matrix name: AFL

Press [**ENTER**] to see the dominance vector.

Questions

3. Explain how the matrix produced from the dominance program relates to the match results.

4. Write down the matrix result for AFL^2 .

5. In the first encounters of the 2016 season, Sydney defeated GWS Giants who in turn defeated Hawthorn. Sydney also defeated Geelong Cats who in turn defeated Hawthorn. How is this level of dominance displayed in the second order dominance matrix: AFL^2?

6. Run the dominance program again, this time enter AFL^2 when prompted for the matrix name. Order the teams according to this result and interpret the outcome.

7. Suppose the AFL decides to change the ranking system on the ladder to take into account that all wins may not be equal. If the new ranking is based on the following:

$$Ranking = 0.8V_1 + 0.2V_2 \qquad\qquad V_1 = \text{Dominance Vector 1}$$

$$V_2 = \text{Dominance Vector 1}$$

Determine the team rankings based on this model.

TEXAS INSTRUMENTS