Ages 17-19 – CAS your girl friend

A young man is saying good-bye to his girl friend at the platform just outside the last door at the back of a train. When the train signals departure he is kissing her so warmly that he does not hear the signal. We assume that the train has a constant acceleration of a = 0.40 m/s^2 and that he starts running at a constant speed 6.0 s after the train began accelerating. Can he catch the train?

All distances are in meters and time in seconds.

The train starts at $t = 0$ at the position $y = 0$.

The young man runs at the constant speed v m/s.

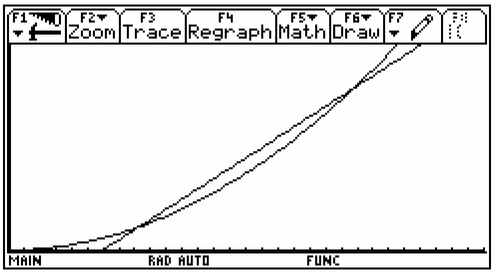
The position of the train door at time t is given by: $y = y1 = \frac{1}{2}at^2 = 0.20 \cdot t^2$.

The position of the young man at time t is given by: $y = y2 = v \cdot (t - 6)$.

We will now analyse this situation in three ways.

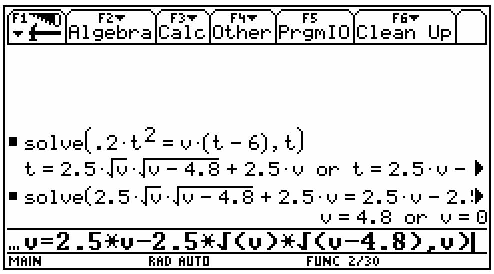
1. CAS the train 1 (using no calculus)

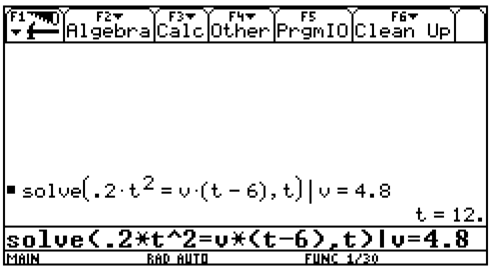
The graphs below show the distances as a function of time if the man runs at v = 6.0 m/s :



We see that in this case he has 2 opportunities to catch the train. If he runs too slowly, he will not be able to catch the train. The minimum speed v_{min} can be found by using the fact that in this case the equation $y1 = y2$ has one and only one solution for t. First solve the equation $y1 = y2$ with any speed v to find the two solutions for t. Then find v such that the two t-values are equal.

Thus the minimum speed in order to catch the train is v_{min} = 4.8 m/s (the solution v = 0 makes no sense). Again, we solve the equation $y1 = y2$ for t, this time with v = 4.8.





We can now conclude that the minimum running speed required for the man to catch the train is 4.8 m/s. At that speed he will catch the train 12 seconds after the train started, after he has been running for 6 seconds and covered a distance of 28.8 m. We can show this in two ways:

He runs for 6 seconds: $4.8 \text{ m/s} \cdot 6 \text{ s} = 28.8 \text{ m}$

The trains travels for 12 seconds: $0.2 \text{ m/s}^2 \cdot (12 \text{ s})^2 = 28.8 \text{ m}$.

If the man runs faster than 4.8 m/s, he will have two opportunities to catch the train.

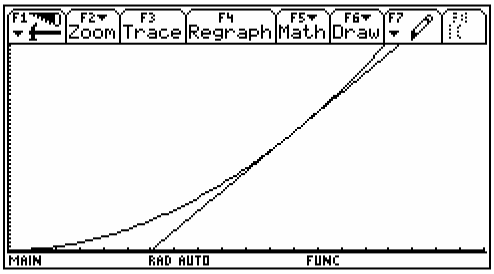
2. CAS the train 2 (using calculus)

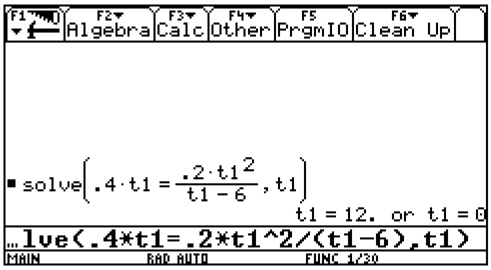
The young man is running at $v = v_{\min}$ and the train has the same speed when he catches it.

Position of train door at time t: $y = \frac{1}{2}at^2 = 0.20 \cdot t^2$.

Speed of train at time t: $v = y'(t) = 0.40 \cdot t$.

The speed of the young man can be written as: $v = \dfrac{0.20 \cdot t_1^2}{t_1 - 6}$, where t_1 is the time when he catches the train.

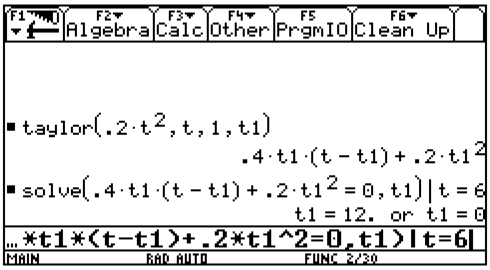




Once again we find that he catches the train $t_1 = 12$ seconds after it started.

3. CAS the train 3 (using calculus)

Below we find the equation of the tangent line at the time t_1 as the Taylor polynomial of order 1.
Then solve for t_1 requiring that $y = 0$ when $t = 6$:



Once again we find that he catches the train $t_1 = 12$ seconds after it started.

<u>Remark</u>: The model is not fully realistic. You might for instance introduce an acceleration period for the young man before he reaches the constant speed.