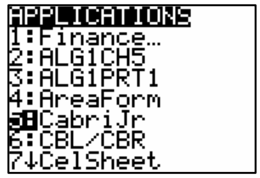
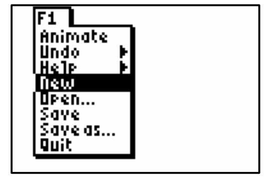
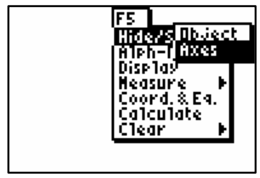
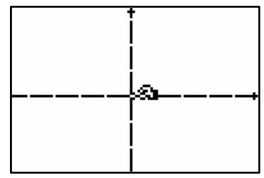
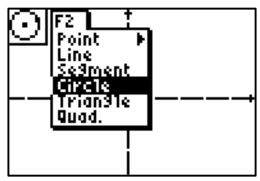
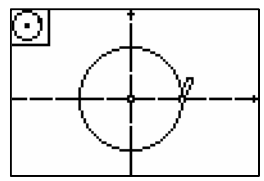
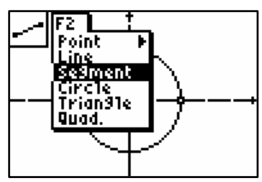
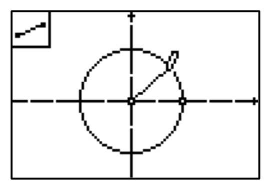
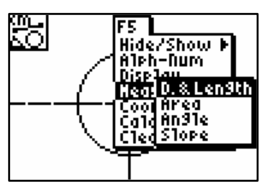
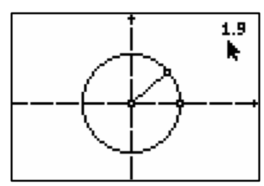
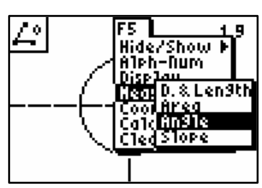
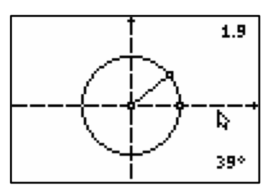
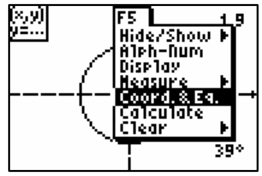
Investigating the Sine and Cosine Functions – Part 1

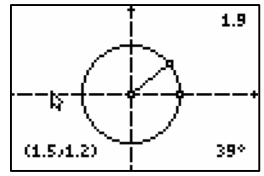
**Before beginning the activity review the basic features of Cabri Jr. (creating circles & segments, grabbing & moving, and measuring lengths & angle.) along with the method for finding angles in quadrants III and IV using reference angles and coterminal angles. Note that data can be collected as a class rather than individually also.*

Set-Up

• Press [APPS]. Move down to **5: Cabri Jr** and press [ENTER]. • Press [Y=] for the F1 menu and select **New**.	 
• Press [GRAPH] for F5 and select **Hide/Show > Axes**. • Move to the origin and press [ALPHA] to drag the origin to the center of your screen. • Press [ENTER] when you have reached the spot where you want to drop the origin.	 
• Press [WINDOW] for F2 and select **Circle**. • Draw a circle with its center at the origin and a radius of your choice. (Press [ENTER] to mark the radius on the x-axis).	 
• Press F2 again and select **Segment** to draw a segment from the center of the circle to a point on the circle in quadrant I.	 
• Press F5 and choose **Measure > D & Length** to measure the radius of the circle. • Drag and drop this length off to the upper right hand side of your screen so it is out of your way.	 
• Measure the angle formed by the x-axes and the segment you drew in quadrant I by pressing F5 and choosing **Measure > Angle**. • Drop this length off to the lower right hand side of your screen so it is out of the way.	 

- Press F5 again and select **Coord & Eq**. Move to the point where the terminal side of the angle touches the circle and press ENTER to label the coordinates.
- Again, drag and drop the coordinates off to the bottom left of your screen so they are out of the way as shown.



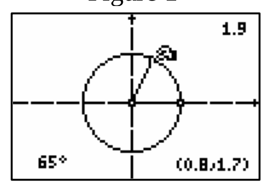


Collecting Data

What is the radius of your circle? <u>*Answers will vary depending on size of circle*</u>

- Move to the point whose coordinates you found in the last step above.
- Press ALPHA when the point is highlighted to select and drag the point. (See figure 1.)
- You are going to collect data by dragging this point around your circle. As you move the point record the measure of the **angle** in the 1st column below, the **x-coordinate** in column 2 (We will use this column later), and the **y-coordinate** in column 3.
- You are going to choose two points from each quadrant along with each of the four **quadrantal angles**.
- Remember that as you move into quadrants III and IV, Cabri Jr.will give the angle as a measure between 0° and 180°. It is your job to use your knowledge of the coordinate axes and reference angles to convert this angle to an appropriate measure between 180° and 360°.

Figure 1



**Make sure that students choose 2 different angles from each quadrant. The x and y-coordinates will vary depending on the size of the circle drawn.*

Angle Measure	X-Coordinate	Y-Coordinate
0°	*Length of radius*	*0*
90°	*0*	*Length of radius*
180°	*- Length of radius*	*0*
270°	*0*	*- Length of radius*
360°	*Length of radius*	*0*

Investigating the Relationship

You are now going to examine the relationship between the **y-coordinate** and the **angle measure** around the circle.

Press [STAT] > **Edit**.

Enter the **angle measure**s from above into [L1] and the **y-coordinates** into [L2].

We are now going to define [L3] as **the y-coordinate divided by the radius** of the circle. To do this move to [L3] so that the name of the list is highlighted and press [ENTER]. Press [L2] [÷] the value you found to be the radius of your circle (figure 2). Then press [ENTER].

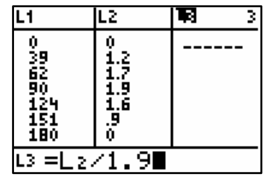


Figure 2

Press [2nd] [Y=]. Turn plot one on by pressing [1] and then pressing [ENTER] when On is highlighted as shown in figure 3.

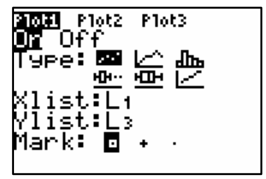


Figure 3

Define your x-list to be [L1] and your y-list to be [L3].

Press [WINDOW] and set your window to fit your data as shown in figure 4.

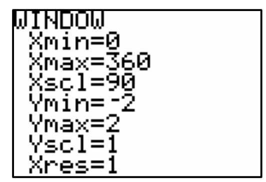
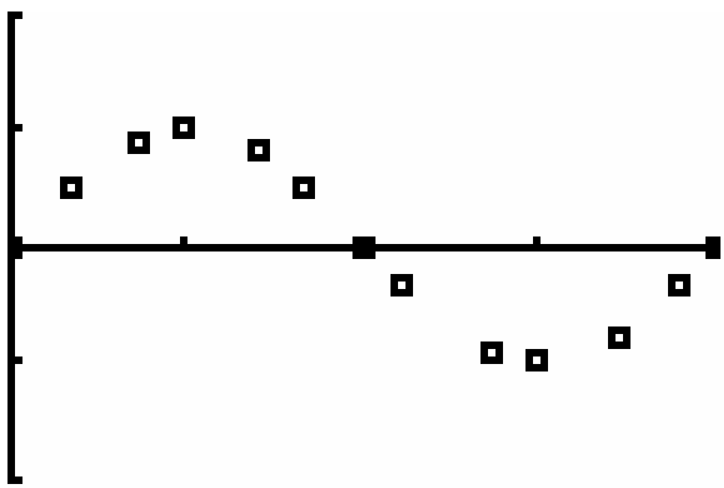


Figure 4

Now press [GRAPH] and sketch the scatter plot that you see on the axes below. What is special about the result?



**Remind students how to label the axes and scale appropriately for trig graphs.*

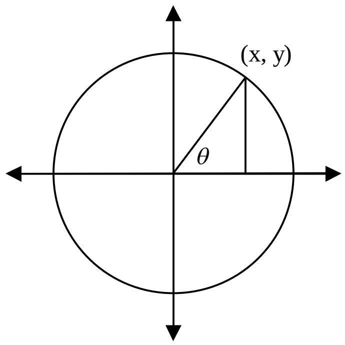
What trig function is represented above? *The sine function*

Questions

1. Based on you results, define sin θ for any point along the circle.

   sin θ = *y-coordinate/radius*

2. Label the opposite, adjacent, and hypotenuse for the following triangle and use the diagram to explain your answer in question #1.



Since sin θ = opposite/hypotenuse and the length of the opposite side = y and the radius = the hypotenuse of the triangle, then it makes sense that sin θ = y/r

3. According to your answer to #1 and the graph you found, in which quadrant(s) is the sine function positive?

   The sine function is positive in quadrants I and II because that is where the y-values are positive.

4. As you recall, a **quadrantal angle** is an angle whose terminal side lies on the x or y - axis. Using your conjecture in question #1, give the sine for the following **quadrantal angles**:

sin 0° =	*0*
sin 90° =	*1*
sin 180° =	*0*
sin 270° =	*-1*
sin 360° =	*0*

5. What is the equation for the unit circle? What are the radius and center of the unit circle?

   Equation: $x^2 + y^2 = 0$ Center: *(0, 0)* Radius: *1*

6. If you are given a point on the unit circle, what do you know about the sine of the angle whose terminal side passes through that point?

   sin θ = *y-coordinate*

Investigating the Relationship

You are now going to examine the relationship between the **x-coordinate** and the **angle measure** around the circle. Make a prediction for what the result will represent?

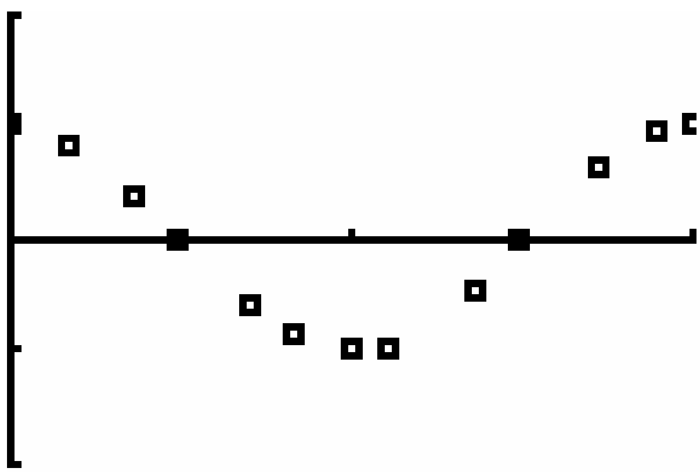
Answers will vary.

Leaving the angle measures in [L1], enter the **x-coordinates** into [L2] as you did with the y-coordinates previously in Part I.

This time we are going to define [L3] as the **x-coordinates divided by the radius** of the circle. Remember to move to [L3] so that the name of the list is highlighted and press [ENTER]. Press [L2] [÷] the value you found to be the radius of your circle. Then press [ENTER].

Make sure that Plot One is turned on and define your x-list to be [L1] and your y-list to be [L3] as you did previously.

Using the same window as in part one, press [GRAPH] and sketch the scatter plot that you see on the axes below.



Was your prediction correct?

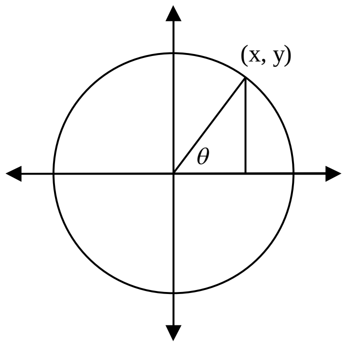
Yes, if the prediction was the cosine function. Otherwise student should state that this is the cosine function now.

Questions

1. Based on you results, define cos θ for any point along the circle.

   cos θ = *x-coordinate/radius*

2. Label the opposite, adjacent, and hypotenuse for the following triangle and use the diagram to explain your answer in question #1.



Since cos θ = adjacent/hypotenuse and the length of the adjacent side = x and the radius = the hypotenuse of the triangle, then it makes sense that cos θ = x/r

3. According to your answer to #1 and the graph you found, in which quadrant(s) is the cosine function positive? *The cosine function is positive in quadrants I and IV because that is where the x-values are positive.*

4. Find the cosine for the following **quadrantal angles**:

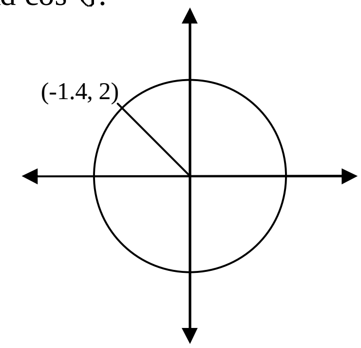
cos 0° =	*1*
cos 90° =	*0*
cos 180° =	*-1*
cos 270° =	*0*
cos 360° =	*1*

5. If you are given a point on the unit circle, what do you know about the cosine of the angle whose terminal side passes through that point?

   cos θ = *x-coordinate*

What Have You Learned?
**This section can be assigned as a follow-up to the activity or as a homework assignment.*

1. Given the following circle with a radius of 3 and the angle, θ, whose terminal side passes through the point (-1.4, 2) as shown, find the sin θ and cos θ.



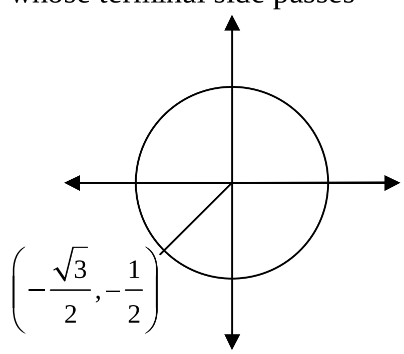
sin θ = *2/3*

cos θ = *-1.4/3*

2. If sin θ < 0 and cos θ > 0, in which quadrant could the terminal side of θ lie?

Quadrant IV only

3. Find the sin θ and cos θ for the angle on the unit circle whose terminal side passes through the point $\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$?



$\sin\theta = -\frac{1}{2}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

What is the measure of θ? *210°*

4. Evaluate the following:

a) sin 180° = *0*

b) cos (-90°) = *0*

c) sin (-270°) = *1*

d) cos 360° = *1*

e) sin (-180°) + cos (90°) = *0 + 0 = 0*