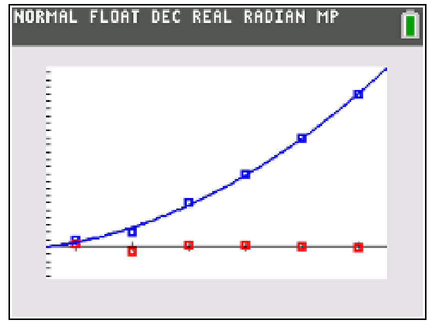
Stacking Bricks

Student Activity

Name ____________________

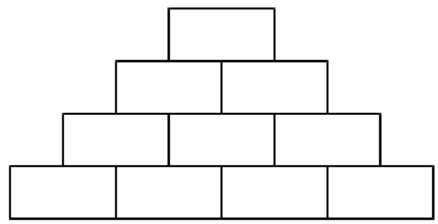
Class ____________________



This activity presents a real-world situation—stacking bricks in a pile—that can be modeled by a polynomial function. Students create a small table to show how the number of bricks relates to the number of rows, then calculate the first, second, and third differences of the data to determine what degree of polynomial model to use. Next, they use the handheld's statistical calculation functions to perform the correct regression. Finally, they evaluate the model using a variety of methods: by graphing the model and the data together, by examining the value of R^2, analyzing Residual plots, and by discussing the model's applicability to the real-world situation.

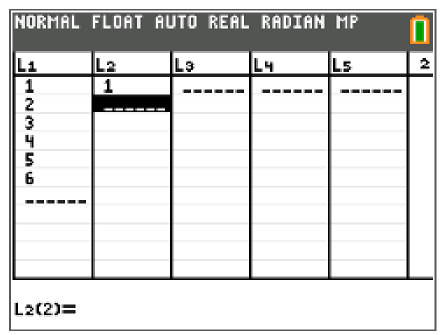
Problem 1 – A Flat Triangular Stack

In this problem, you are stacking bricks according to the pattern shown at the right. Each row contains one more brick than the row above it.



How many bricks will be in the stack when it is 50 rows high?

You can solve the problem easily by creating a polynomial model to describe the number of bricks in the stack, $f(x)$, given a number of rows x.



Look for a pattern using a small table. Press [stat] [enter] and enter the numbers in **L1** as shown.

L1 = the number of rows in the stack
L2 = the number of bricks in the stack

Complete list **L2**.

Which polynomial model should you use—linear, quadratic, cubic, or quartic?

To decide, calculate the successive differences.

Enter the first differences in **L3** (by hand or by using the Delta List command on the handheld by moving your cursor to the top of **L3** and pressing **2nd stat**, **ops**, **7: Delta List(L2)**, then **enter**), the second differences in **L4**, and the third differences in **L5**. Record your lists at the right.

L1	L2	L3	L4	L5
rows	bricks	1st Diff	2nd Diff	3rd Diff
1	1			
2				
3				
4				
5				
6				

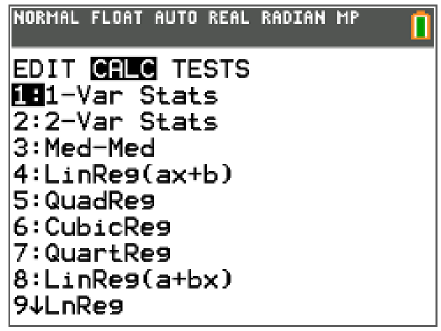
If the first differences are constant or close to constant, a first degree (linear) model is a good fit for the data. If the second differences are constant or close to constant, a second degree (quadratic) model is a good choice, and so on.

1. Which set of differences is constant?

2. What degree polynomial is the best fit for this data?

Return to the HOME screen. Use the regression command to create the model for the data.

Press [stat], arrow to the Calc menu, and choose appropriate **Reg** command. Enter **L1, L2, Y1** after the command.

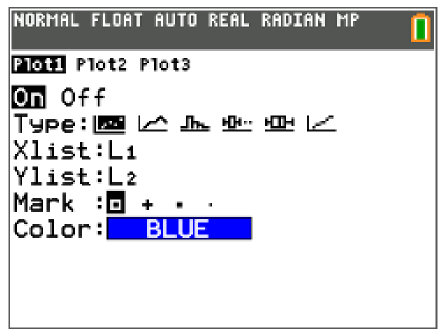


3. Record the equation of the model here:

Check your model graphically by plotting the points with the model. Press [2nd] [stat plot], select **Plot1**, and match the settings shown at the right.

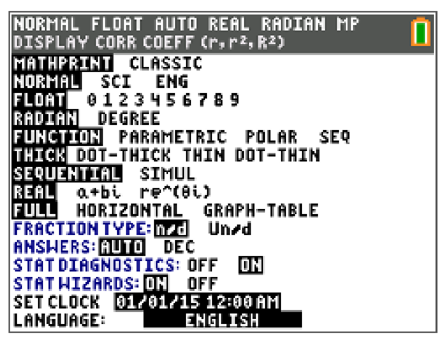
Press [zoom] and select **ZoomStat**.

If the model is correct, its graph will pass through all the data points.



Now check your model by calculating the coefficient of determination, R^2. The closer the R^2 value is to 1, the better the model fits the data.

Press [mode], select **ON** next to **STATDIAGNOSTICS**, and press [2nd] [quit].



Press [enter] to run the regression again.

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4. What is the R^2 value for your model? What does that mean?

5. When you checked your model, did the function go through all the points?

6. If your model is correct, use it to calculate the number of bricks in a stack 50 rows high. (Remember that **Y1(X)** is the number of bricks and **X** is the number of rows.)

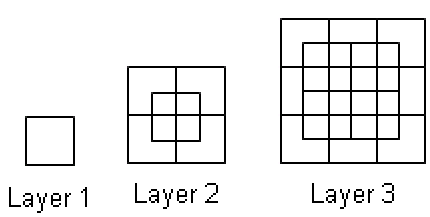
7. Discuss the shortcomings of the model for this situation. For what numbers of rows is it valid? For what numbers of rows does it not make sense?

8. Write a domain for this model.

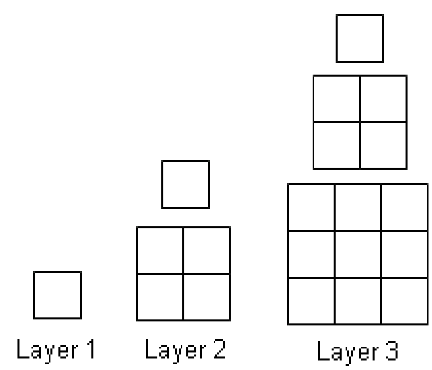
Problem 2 – A Pyramidal Stack

In this problem, you are stacking bricks in pyramids.

The diagram below shows the stacks from above.



To see the pattern more clearly, the layers of the pyramids are shown separately below.



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9. Use the method from Problem 1 to find the number of bricks in a pyramid with 50 layers. Calculate the successive differences and record the values in the table. What do you notice about the common differences?

L_1	L_2	L_3	L_4	L_5
layers	**bricks**	**1st Diff**	**2nd Diff**	**3rd Diff**
1	1			
2				
3				
4				--
5			--	--
6		--	--	--

10. Choose and perform a polynomial regression. Record it here.

11. Look at the R^2 value for the regression. What is it? What does this mean?

12. Check your model. Graph total bricks vs. number of layers as a scatter and graph your model into $\mathbf{Y_1}$. Does the model go through all the points?

13. If your model is correct, use it to calculate the number of bricks in a pyramid 50 layers high.

14. Discuss the shortcomings of your model for this situation. For what numbers of layers is it valid? For what numbers of layers does it not make sense?

15. Write a domain for this model.

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Problem 3 – Extension, Beyond R^2

In this final problem, students will analyze the residuals created by the results in problem 2. A **residual** value is the difference between the actual value (given data) and the predicted value (value found by using the regression model). Students will examine the residual plot from problem 2's model and discuss what they notice.

16. Press **y =**, and make sure your regression from Problem 2 is currently in $\mathbf{Y_1}$. Press **2nd y =** (stat plot) and make sure that your data is still in Plot 1. Go to Plot 2 and make sure that your X-List is L_1. Go to your Y-List and press **2nd stat** (list), go to the bottom and press enter for **RESID**. Press **zoom**, **9: ZoomStat** to see the residual plot. Discuss with another student what you recognize about the residual plot. Explain what each of the values represent.

17. Using the residual plot, how can you tell if the model found in Problem 2 is a good fit?