

# Building Concepts: Dividing a Fraction by a Whole Number

TEACHER NOTES

## Lesson Overview

This TI-Nspire™ lesson uses a unit square to explore division of a unit fraction and a fraction in general by a whole number. The concept of dividing a quantity by a whole number,  $n$ , can be thought of as separating the quantity into  $n$  parts and selecting one of those parts (i.e.,  $\frac{1}{2}$  divided by 2 is separating  $\frac{1}{2}$  into two equal parts, each  $\frac{1}{4}$ , and selecting one of them).



Students can use the relationship between division and multiplication to solve problems involving dividing fractions by whole numbers.

## Learning Goals

Students should understand and be able to explain each of the following:

1. Dividing a unit fraction by a whole number partitions the unit fraction into the number of pieces determined by the whole number;
2. To divide any fraction by a whole number, divide the unit fraction by the whole number and multiply the result by the numerator of the fraction (i.e., to divide  $\frac{2}{3}$  by 4, consider  $\frac{1}{3}$  divided into 4 parts so each part is  $\frac{1}{12}$ ; thus,  $\frac{2}{3}$  divided by 4 would be 2 times  $\frac{1}{12}$  or  $\frac{2}{12}$ , which could be written as  $\frac{1}{6}$ );
3. The relationship between multiplication and division justifies  $\left(\frac{1}{3}\right) \div 4 = \frac{1}{12}$  because  $\left(\frac{1}{12}\right) \times 4 = \frac{1}{3}$ .

## Prerequisite Knowledge

*Dividing a Fraction by a Whole Number* is the eleventh lesson in a series of lessons that explore fractions. Students should have experience with the concepts of equivalence and the relation of fractions to unit squares. These can be found in *What is a Fraction?*, *Equivalent Fractions*, *Creating Equivalent Fractions*, and *Fractions and Unit Squares*. Prior to working on this lesson students should understand:

- the concept of related multiplication and division facts.
- that a fraction  $\frac{1}{b}$  is one part of a unit square that has been partitioned into  $b$  equal parts.
- that the product of two numbers is not affected by the order in which they are multiplied.

## Vocabulary

- **non unit fraction:** a fraction that has a number other than 1 in the numerator
- **quotient:** the answer after you divide one number by another

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## Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

## Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Dividing a Fraction by a Whole Number\_Student.pdf
- Dividing a Fraction by a Whole Number\_Student.doc
- Dividing a Fraction by a Whole Number.tns
- Dividing a Fraction by a Whole Number\_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to <http://education.ti.com/go/buildingconcepts>.

## Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:



**Class Discussion:** Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.



**Student Activity Sheet:** The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.



**Bulls-eye Question:** Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.



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## Mathematical Background

This TI-Nspire™ lesson uses a unit square to explore division of a unit fraction and a fraction in general by a whole number. The concept of dividing a quantity by a whole number,  $n$ , can be thought of as separating the quantity into  $n$  parts and selecting one of those parts (i.e.,  $\frac{1}{2}$  divided by 2 is separating  $\frac{1}{2}$  into two equal parts, each  $\frac{1}{4}$ , and selecting one of them). To divide  $\frac{a}{b}$  by a whole number  $c$ , students can consider dividing the unit fraction  $\frac{1}{b}$  into  $c$  parts, selecting one of them and then multiplying that answer by  $a$ . For example,  $\frac{5}{6}$  divided by 2 could be thought of in this way:  $\frac{1}{6}$  divided by 2 gives  $\frac{1}{12}$  and 5 copies of  $\frac{1}{12}$  is  $\frac{5}{12}$ .

A second approach to dividing a fraction by a whole number is to consider the related multiplication problem, that is:  $\frac{a}{b}$  divided by  $c$  equals  $d$  implies that  $\frac{a}{b}$  is equal to the product of  $c$  and  $d$

$\left( \left( \frac{a}{b} \right) \div c = d \Rightarrow \frac{a}{b} = c \times d \right)$ . In thinking about this approach, students might want to refer back to earlier work on multiplication of fractions by a whole number (the focus of the lesson *Multiplying Fractions by a Whole Number*).

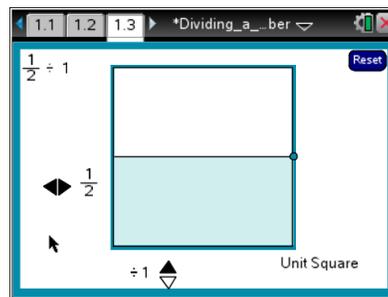
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## Part 1, Page 1.3

Focus: Students will use unit squares to investigate dividing a fraction by a whole number.

In this activity a unit square is used to show the division of a fraction by a whole number. On page 1.3 the arrows to the left of the unit square set the denominator of a unit fraction, shown in the unit square. Moving the dot sets the numerator. The arrows underneath the unit square set the whole number by which the fraction is divided. The unit square is then partitioned into equal parts according to that whole number.



### TI-Nspire Technology Tips

Students may find it easier to use the **tab** key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

Help students to recognize that the unit square measures one unit vertically and one unit horizontally. Point out that these vertical and horizontal sides are almost like having perpendicular number lines. Thus, the  $\frac{1}{2}$  on page 1.3 marks  $\frac{1}{2}$  on the vertical side of the unit square;  $\frac{1}{3}$  would mark  $\frac{1}{3}$  of that side and so on. Dividing the fraction by 2 on the bottom is represented by the division of the unit square into two congruent parts indicated by the dashed lines, and the result is shown by the shaded portion of the unit square.

**Teacher Tip:** Have students begin with a unit fraction and observe the result of dividing by whole numbers beginning with 1. The mathematical relationship should become clear and they can generalize what they observe. Once they understand that relationship, students can make the transition to non-unit fractions.

Be sure students understand how the interaction with the unit square supports the mathematics. Asking them how the representation of the unit square is connected to the process of dividing by a whole number can lead to a productive discussion about their interpretation both of the unit square and of the mathematical question.



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## Class Discussion

Have students...

- *On page 1.3 of the activity use the bottom arrow to change the divisor to 2. Explain how the display on the screen represents the problem  $\frac{1}{2}$  divided by 2.*
- *Make a conjecture about the answer to  $\frac{1}{3}$  divided by 2.*
- *Change the fraction using the arrows on the left of the unit square and check your conjecture.*
- *What do you think the answer for  $\frac{1}{3}$  divided by 3 will be? Check your answer using the unit square.*
- ✓ *Make a conjecture about  $\frac{1}{3}$  divided by 4. Check your answer using the unit square.*

(Question #1 on the Student Activity sheet.)

Look for/Listen for...

Answer: The entire unit square was divided into 4 parts. Each  $\frac{1}{2}$  region was divided into 2 parts.

One of the parts is shaded, so  $\frac{1}{2}$  divided by 2 is  $\frac{1}{4}$ .

Possible answer: It will probably be  $\frac{1}{6}$  because each of the  $\frac{1}{3}$  parts will be cut into two parts, so there will be 6 parts all together. One of the 6 parts will be shaded.

Possible answers: I was correct or I was not right.

Possible answer: My conjecture was that it would be 1. But it was  $\frac{1}{9}$  because all of the copies of  $\frac{1}{3}$  were divided into 3 parts, making 9 parts all together.

Possible answer: My conjecture was  $\frac{1}{12}$ , and I was right.

**Teacher Tip:** Have students illustrate their conjectures by completing the unit square on their worksheets (Question 1) before working together on the interactive unit square.



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## Class Discussion (continued)

Use some of the following examples to practice dividing fractions by whole numbers.  
(You do not have to do them all.)

- $\frac{1}{5}$  divided by 3, 4, 5, 6, 8
- $\frac{1}{6}$  divided by 3, 4, 5, 6, 8

Use the unit square to divide some of the following

- $\frac{2}{3}$  divided by 2, 3, 4, 6, 8
- $\frac{5}{6}$  divided by 2, 3, 4, 5, 6, 8
- $\frac{5}{12}$  divided by 2, 3, 4, 5, 6, 8, 10, 12

- **After you have done some of the problems, make a conjecture about how to divide a fraction by a whole number. Try dividing by a few other fractions to test your conjecture.**

Possible answer: My conjecture was that you used the same procedure as dividing by the unit fraction and just kept the same numerator. Some of the fractions could be reduced, though. The examples supported my conjecture.

**Based on your thinking in the problems above, decide which of the following statements seem to be true and be ready to explain your reasoning.**

**To divide a fraction by a whole number,**

- **you multiply the denominator of the fraction by the whole number and use the same numerator.**

Answer: True

- **you multiply the numerator of the fraction by the whole number and use the same denominator.**

Answer: False

- **if the whole number and the numerator of the fraction have a common factor, you can reduce the result.**

Answer: True



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## Class Discussion (continued)

- *if the whole number and the denominator of the fraction have a common factor, you can reduce the result.*

Answer: False

- *the answer will always be less than the original fraction.*

Answer: True

**Answer each of the following and explain your thinking in each case.**

- $2 \times \text{what number} = \frac{1}{2}$  ?

Answer:  $\frac{1}{4}$ ; you need 2 copies of something to make  $\frac{1}{2}$  on the number line; 2 copies of  $\frac{1}{4}$  is  $\frac{1}{2}$ .

- $4 \times \text{what number} = \frac{2}{3}$  ?

Answer:  $\frac{1}{6}$ ; you need to find a fraction that will leave a 3 in the denominator after a 2 has been reduced.  $\frac{1}{6}$  works.

- $10 \times \text{what number} = \frac{15}{2}$  ?

Answer:  $\frac{3}{4}$ ; you need a fraction that has a multiple of 2 in the denominator so it has to be an even number 4, 6, 8, ...; the fraction has to have a 3 in the numerator in order to make 15. The first fraction that works is  $\frac{3}{4}$ .

- $6 \times \text{what number} = 1$  ?

Answer:  $\frac{1}{6}$ ; 6 copies of  $\frac{1}{6}$  is  $\frac{6}{6}$ , or 1.

**Recall that multiplication and division are related:  $20 \times 4 = 80$  can be written as  $80 \div 4 = 20$ .**

- **What other division statement could you make from  $20 \times 4 = 80$  ?**

Answer:  $80 \div 20 = 4$

- ✓ **Write  $4 \times \frac{1}{10} = \frac{2}{5}$  as a division problem.**

(Question #2 on the Student Activity sheet.)

Answer:  $\frac{2}{5} \div 4 = \frac{1}{10}$  or  $\frac{2}{5} \div \frac{1}{10} = 4$



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## Class Discussion (continued)

*Rewrite the multiplication sentences as division statements where the divisor is a whole number.*

- $2 \times \frac{1}{4} = \frac{1}{2}$

Answer:  $\frac{1}{2} \div 2 = \frac{1}{4}$

- $4 \times \frac{1}{6} = \frac{2}{3}$

Answer:  $\frac{2}{3} \div 4 = \frac{1}{6}$

- $10 \times \frac{3}{4} = \frac{15}{2}$

Answer:  $\frac{15}{2} \div 10 = \frac{3}{4}$

- $6 \times \frac{1}{6} = \frac{6}{6} = 1$

Answer:  $\frac{6}{6} \div 6 = \frac{1}{6}$

- ***Remember your conjecture about dividing a fraction by a whole number: you multiply the denominator of the fraction by the whole number and use the same numerator. Create a division problem and its related multiplication problem to see if your conjecture is true.***

Possible answer: Using the rule on

$\frac{4}{3} \div 8 = \frac{4}{24} = \frac{1}{6}$  you would multiply  $8 \times 3$  and

keep the numerator 4 to get  $\frac{4}{24}$ . Considering

the corresponding multiplication problem, you would have to find a fraction that multiplies a whole number to produce the original fraction.

For example, you can multiply 8 by some

fraction to get  $\frac{4}{3}$ . In other words,  $8 \times \frac{a}{b} = \frac{4}{3}$ .

You have to reduce 8 by 2 to get 4 in the numerator, so the denominator,  $b$ , needs a

multiple of 2 and 3. So a logical answer is  $\frac{1}{6}$ .



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## Class Discussion (continued)

The unit square allows you to only divide a fraction less than or equal to one by a whole number. What do you think each of the following would be? Explain your reasoning. (Hint: you might want to think about the previous problems.)

- $\frac{7}{4} \div 2$

Answer:  $\frac{7}{8}$

- $\frac{10}{3} \div 5$

Answer:  $\frac{10}{15}$  or  $\frac{2}{3}$

- $\frac{10}{3} \div 3$

Answer:  $\frac{10}{9}$

In each case, you use the same rule or way of thinking described in the previous problem.



Suppose that you know  $\frac{1}{6} \div 4 = \frac{1}{24}$ . How

could you use that information, without using the unit square, to answer the following?

Use the unit square to check your answers.

- $\frac{5}{6} \div 4$

Answer:  $\frac{5}{6}$  is the same as 5 copies of  $\frac{1}{6}$  and so

the answer is  $\frac{5}{24}$ .

- $\frac{3}{6} \div 4$

Answer:  $\frac{3}{6}$  is the same as 3 copies of  $\frac{1}{6}$  and so

the answer is  $\frac{3}{24} = \frac{1}{8}$ .

- $\frac{1}{12} \div 4$

Answer:  $\frac{1}{12} = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)$  so

$$\frac{1}{12} \div 4 = \left(\frac{1}{2}\right)\left(\frac{1}{6} \div 4\right) = \left(\frac{1}{2}\right)\left(\frac{1}{24}\right) = \left(\frac{1}{48}\right)$$

Answer: If I am dividing by a number twice as small, the answer should be twice as big or

$$2\left(\frac{1}{24}\right) = \frac{1}{12}.$$



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## Class Discussion (continued)

Answer each and explain your thinking in each case.

- ✓ A board is  $\frac{4}{9}$  yard long. If you divide the length of the board into 3 equal parts, how long will each part be?

(Question #3 on the Student Activity sheet.)

Answer:  $\frac{4}{27}$  yard.

**Teacher Tip:** Have students discuss how they showed the solution on the unit square in the student worksheet (Question 3). Ask them to explain why the denominator in the quotient is not 9.

- How much chocolate will each person get if 6 people share  $\frac{1}{2}$  pound of chocolate equally?

Answer:  $\frac{1}{12}$  pound

- You have  $\frac{3}{4}$  pounds of meat to share equally with 3 people. How much will each person get?

Answer:  $\frac{1}{4}$  pound

- Sonja says the problem below is a division problem. Traci says it is a multiplication problem. Use a diagram and what you know about multiplication and division of fractions to decide what kind of a problem it is and why.

How many  $\frac{1}{3}$ -cup servings are in 4 cups of cereal?

Answer: Although all division problems (except when 0 would be a divisor) can be rewritten as multiplication, Sonja is correct if you interpret it as written. It is a division problem, but it is a whole number divided by a fraction. You have 4 and are looking for how many  $\frac{1}{3}$ s are in 4. There are 12  $\frac{1}{3}$ s in 4, so the answer is 12 one-third cup servings.



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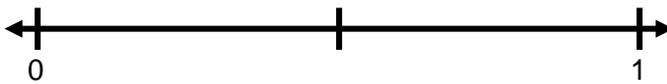
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## Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Which is the smallest and which is the largest of the following? Estimate where the value for the smallest and largest would be on the number line.

a.  $\frac{1}{4} \div 2$       b.  $\frac{1}{4} \div 3$       c.  $\frac{3}{4} \div 3$       d.  $\frac{3}{8} \div 2$       e.  $\frac{3}{8} \div 8$



**Answer: e is the smallest and c is the largest.**

2. Which of the following problems have the same solution?

a.  $\frac{3}{8} \div 2$       b.  $\frac{3}{16} \div 2$       c.  $\frac{3}{16} \times \frac{1}{2}$       d.  $\frac{3}{2} \div 8$

**Answer: a and d are both  $\frac{3}{16}$  and b and c are both  $\frac{3}{32}$ .**

3.  $\frac{3}{4}$  of a package of pens is to be shared among six people.

- a. How much of the whole package does each person get?

**Answer: Each person will get  $\frac{3}{24}$  or  $\frac{1}{8}$  of the package.**

- b. If each package has 16 pens, how many pens will each person get?

**Answer: 2 pens**

4. Fill the boxes with numbers to make a true statement.

a.  $\frac{7}{\square} \div 2 = \frac{\square}{\square}$       Possible answer:  $\frac{7}{3} \div 2 = \frac{7}{6}$

b.  $\frac{\square}{10} \div \square = \frac{1}{5}$       Possible answer:  $\frac{12}{10} \div 6 = \frac{1}{5}$



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5. Beti claims that if you divide a fraction between 0 and 1 by a whole number, the answer will always be smaller than the original fraction. Which is the best reason to support or to disprove her statement?
- It is not true because when you use a whole number, the answers always get larger than the one you started with.
  - It is true because when you divide, the result is always smaller than what you start with.
  - It is true because the corresponding multiplication problem would multiply the answer by two, which would double the size.
  - It is true because when you divide by a whole number you are basically increasing the size of the denominator of the original fraction and so you have more parts and each part is smaller.
  - It is not true because it depends on whether the numerator and the whole number have any common factors.

***Answer: d) It is true because when you divide by a whole number you are basically increasing the size of the denominator of the original fraction and so you have more parts and each part is smaller.***



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## Student Activity solutions

### Vocabulary

**non unit fraction:** a fraction that has a number other than 1 in the numerator

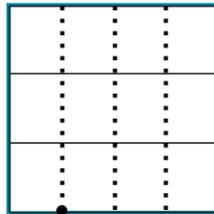
**quotient:** the answer after you divide one number by another

In this activity, you will use a unit square and the relationship between multiplication and division to divide a fraction by a whole number.

1. Make a conjecture about  $\frac{1}{3}$  divided by 4. Write your answer. Complete the unit square to show your reasoning. Check your answer using the interactive unit square.



*Possible answer: My conjecture was  $\frac{1}{12}$ , and I was right.*



2. Recall that multiplication and division are related:

$20 \times 4 = 80$  can be written as  $80 \div 4 = 20$ . Write  $4 \times \frac{1}{10} = \frac{2}{5}$

as a division problem.

*Answer:  $\frac{2}{5} \div 4 = \frac{1}{10}$  or  $\frac{2}{5} \div \frac{1}{10} = 4$*



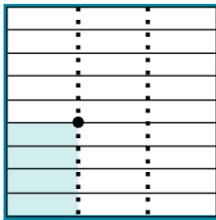
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3. A board is  $\frac{4}{9}$  yard long. If you divide the length of the board into 3 equal parts, how long will each part be? Complete the unit square to show your reasoning.



**Answer:**  $\frac{4}{27}$  yard



4.  Suppose that you know the following:  $\frac{1}{8} \div 3 = \frac{1}{24}$ . How could you use that information to answer  $\frac{6}{8} \div 6$ ?

**Answer:**  $\frac{6}{8}$  is the same as 6 copies of  $\frac{1}{8}$  and so the answer is  $\frac{6}{48}$  or  $\frac{1}{8}$ .