Medians in a Triangle – ID: 9690

By Judy Hicks

Time required
50 minutes

Activity Overview

In this activity, students will study medians and some of their properties. A median of a triangle connects a vertex of the triangle with the midpoint of the opposite side.

Concepts

- *Midpoints*
- *Medians*
- *Centroid*
- *Slopes and equations of lines*

Teacher Preparation

This activity is designed to be used in a high-school geometry classroom.

- *This activity assumes a basic working knowledge of the TI-Nspire device, such as drawing shapes and finding lengths and areas of segments or figures.*
- *The screenshots on pages 2–5 (top) demonstrate expected student results. Refer to the screenshots on pages 5 (bottom) and 6 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "9690" in the quick search box.***

Classroom Management

- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *The student worksheet GeoAct32_MedianTriangle_worksheet_EN helps to guide students through the activity and provides a place to record their answers.*
- *The TI-Nspire solution document GeoAct32_MedianTriangle_Soln_EN.tns shows the expected results of working through the activity.*

TI-Nspire™ Applications

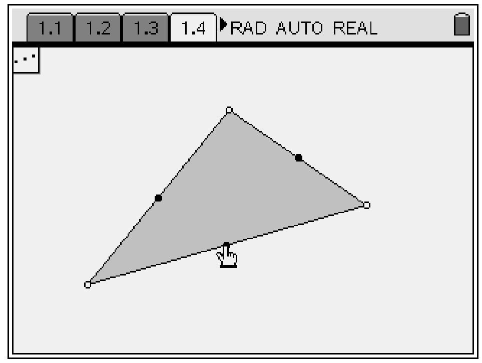
Graphs & Geometry, Lists & Spreadsheet, Notes, Data & Statistics

This activity allows students to interactively discover, confirm, and explore the theorems relating to medians of triangles. You may need to help students use the handheld's different tools; instructions are provided in this document at point of first use.

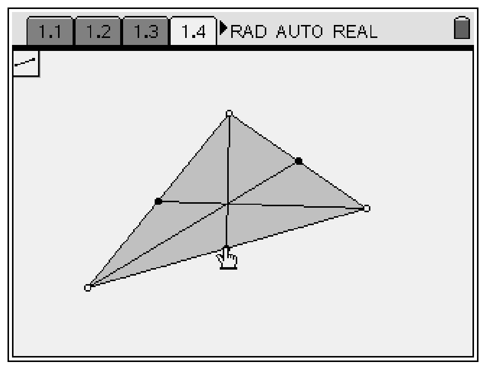
Problem 1 – Medians and concurrency

On page 1.3, students will construct the three medians of the triangle.

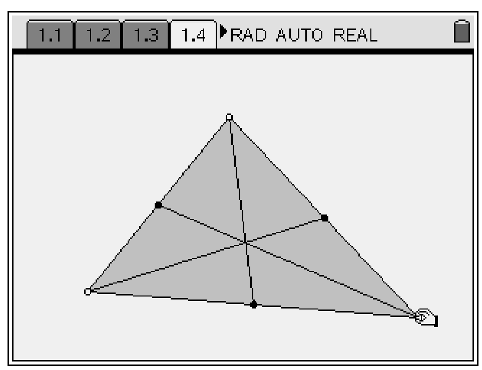
To do so, they first need to display the midpoints of each side using the **Midpoint** tool from the Construction menu.



Then, they can use the **Segment** tool from the Points & Lines menu to join each vertex with the midpoint of the opposite side.

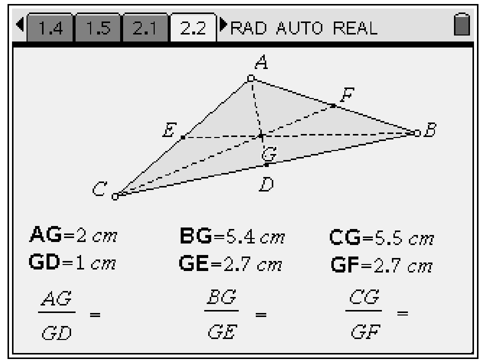


Now students should drag the vertices around the screen to change the size of the triangle. Encourage them to notice that the third median intersects the other two at their point of intersection; that is, the three medians are concurrent at a point called the *centroid*, which they will explore in the next problem.

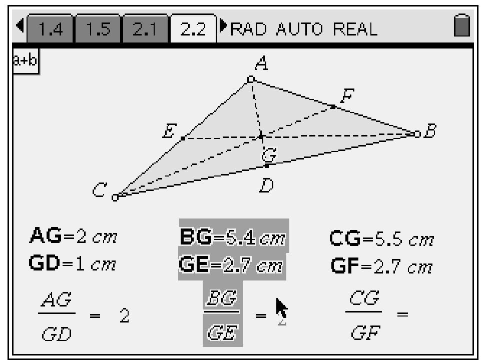


Problem 2 – Parts of a median

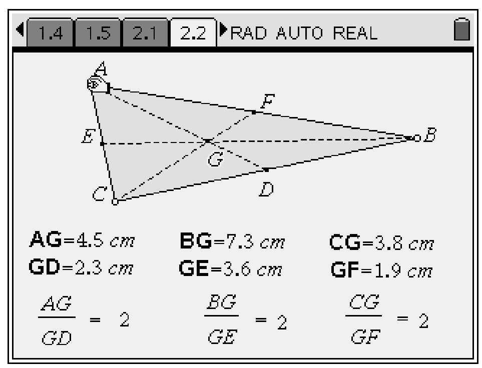
On page 2.2, students are given △*ABC*, along with all three of its medians. The centroid (point of concurrency of the medians) has been labeled point *G*, and the measurements from each vertex and midpoint to point *G* have been displayed.



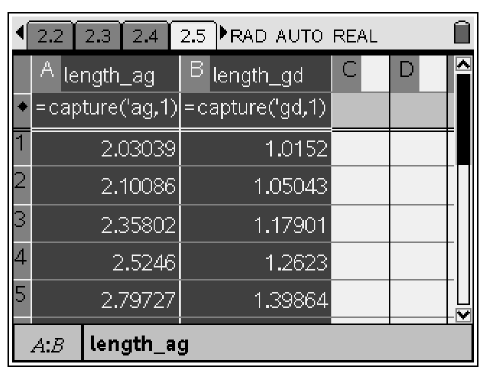
Using the **Calculate** tool from the Actions menu, students are asked to calculate the value of each ratio displayed at the bottom of the screen. (These are the ratios of the "parts" of each median.) To use the **Calculate** tool, click first on the expression to be evaluated and then on the value to use for each variable, in the order you are prompted.



After calculating the ratios, students should again drag vertices to explore the relationship among the parts of the median. Students should find that the centroid divides each median into two pieces, with one piece twice as long as the other piece.

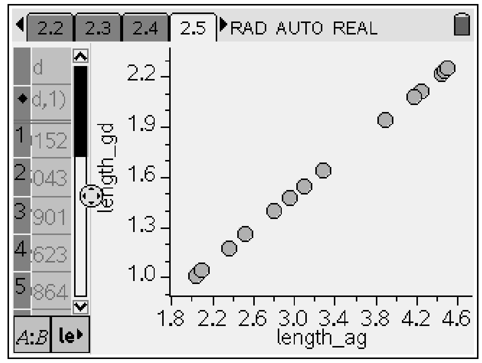


While students were dragging vertices on page 2.2, the lengths of segments *AG* and *GD* were collected into the spreadsheet on page 2.5, via an automatic data capture.



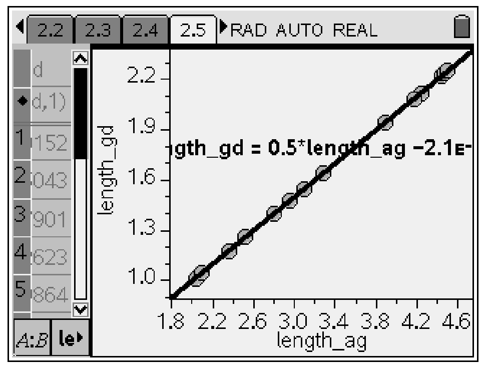
Direct students to position the cursor in Column A and press up on the NavPad until the column is selected. Holding down the **SHIFT** key (⇧), students can then press right on the NavPad to extend the column to include Column B.

Students should now select **MENU > Data > Quick Graph**, which brings up a *Data & Statistics* application (in a split screen) in which the data from Columns A and B have been displayed.



At this point, you might wish to have students press ctrl + ⌂ to access the **Tools** menu, from which they can select **Page Layout > Custom Split** and arrow left to increase the size of the scatter plot.

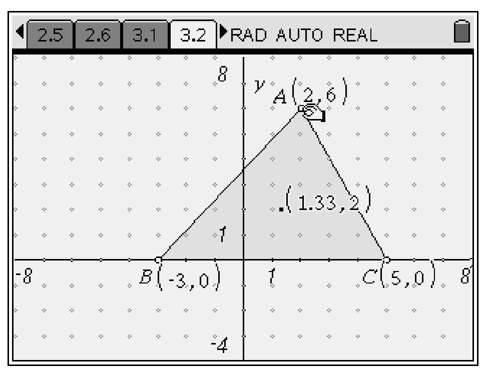
Instruct students to choose **MENU > Actions > Add Movable Line.** They should then drag and rotate the line until it is an appropriate model for the data.



Notice that the equation of this line is displayed. Because the equation of the line is given in slope-intercept form, have students determine the slope of the line what the slope represents. (The slope of the line should be 0.5. The slope represents the ratio between the lengths of the "parts" of the median created by the centroid.)

Problem 3 – Coordinates of the centroid

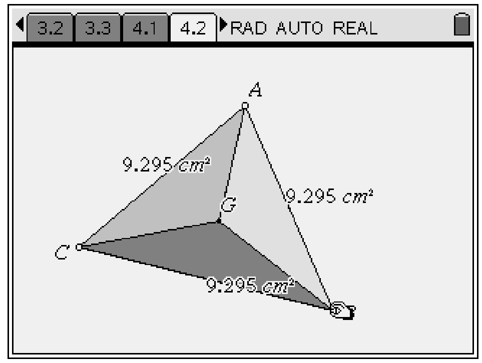
In Problem 3, a triangle and its centroid is shown on a coordinate grid. With two vertices on the *x*-axis, prompt students to look at the *x*-coordinates of the vertices and try to determine the relationship between the *x*-coordinate of the centroid. Then have them repeat for the *y*-coordinates.



After speculating, students should drag vertices to verify the relationship. (The *x*-coordinate of the centroid is the mean of the *x*-coordinates of the vertices; the *y*-coordinate of the centroid is the mean of the *y*-coordinates of the vertices.)

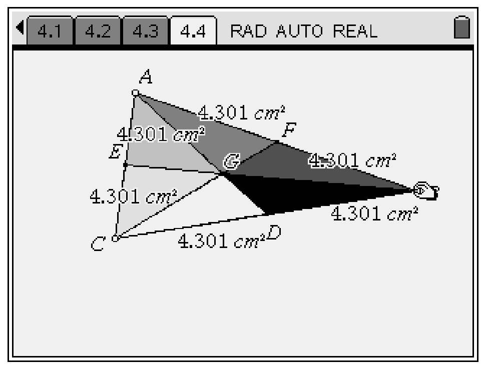
Problem 4 – Area relationships

On page 4.2, students are given a triangle with centroid *G* drawn and they are asked to draw the 3 triangles *AGC*, *AGB*, and *BGC*. After measuring the areas of these three triangles, students should find that the three areas are equal; thus it makes sense that the centroid is the "center of gravity" of the triangle.



In the diagram to the right, the **Attributes** tool (from the Actions menu) was used to make the triangles appear with different shades.

On page 4.4, students take the exploration from page 4.2 one step further, looking at the areas of the 6 small triangles created by the three medians. It should come as a surprise to students that these 6 triangles *also* have equal areas, despite the fact that they are not generally congruent!



Again, the triangles in the figure to the right have been shaded to ease visibility.

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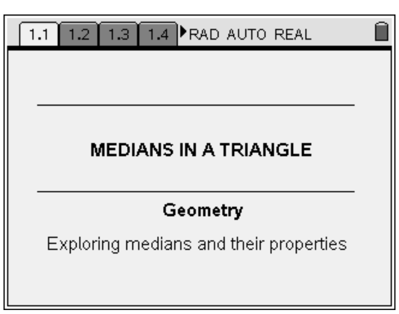
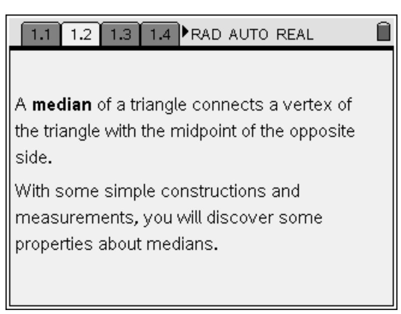
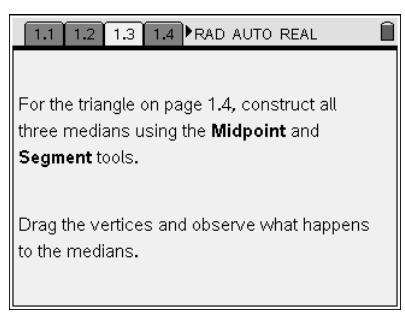
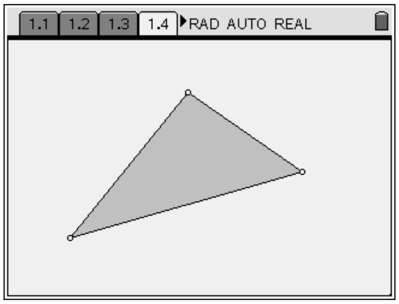
(Student)TI-Nspire File: *GeoAct32_MedianTriangle_EN.tns*









1.2 1.3 1.4 1.5 RAD AUTO REAL

Question

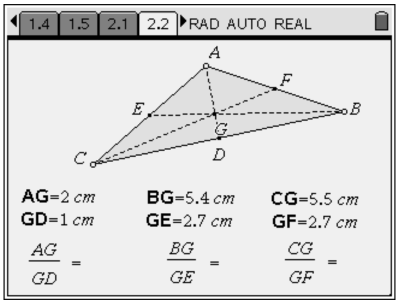
What do you notice about the third median with respect to the other two?

Answer

1.3 1.4 1.5 2.1 RAD AUTO REAL

On page 2.2, triangle *ABC* is shown with all medians constructed. The point of concurrency of the medians, called the centroid of triangle *ABC*, has been labeled point *G*. The lengths of some segments are given, and certain ratios are displayed.

Calculate each ratio. Then drag the vertices and observe what happens.



1.5 2.1 2.2 2.3 RAD AUTO REAL

Question

How does the centroid of a triangle divide each median?

Answer

2.1 2.2 2.3 2.4 RAD AUTO REAL

The spreadsheet on the next page captured the lengths *AG* and *GD* as you dragged the vertices on page 2.2.

Select Columns A and B and choose **MENU > Data > Quick Graph.**

Then choose **MENU > Actions > Add Movable Line** to display a fit line. Drag the line as needed to model the data points.

2.2 2.3 2.4 2.5 RAD AUTO REAL

	A length_ag	B length_gd	C	D
◆	=capture('ag,1)	=capture('gd,1)		
1	2.03039	1.0152		
2				
3				
4				
5				

A1 =2.03039405042

2.3 2.4 2.5 2.6 RAD AUTO REAL

Question

What is the slope of the fit line? What is the significance of this slope?

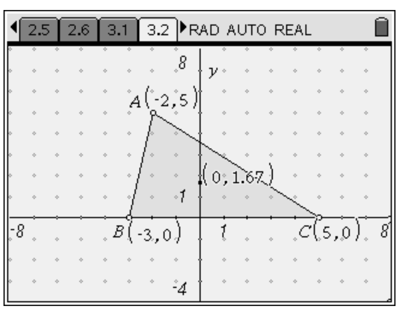
Answer

2.4 2.5 2.6 3.1 RAD AUTO REAL

On page 3.2, a triangle and its centroid is shown on a coordinate grid.

Look at the *x*−coordinates of the vertices and try to find how they are related to the *x*−coordinate of the centroid. Do the same for the *y*−coordinates of the vertices and the *y*−coordinate of the centroid.

Drag vertices and test your conjecture.



2.6 3.1 3.2 3.3 RAD AUTO REAL

Question

What is the relationship between the coordinates of the vertices of a triangle and the coordinates of the centroid?

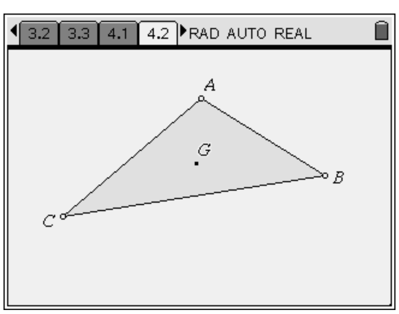
Answer

3.1 3.2 3.3 4.1 RAD AUTO REAL

The centroid of a triangle is also its center of gravity (or mass), provided the triangle has a uniform thickness and density.

On page 4.2, verify this by drawing triangles *AGC*, *AGB*, and *BGC*, and measuring their areas. Then change the size of the triangle.

What do you notice?



3.3 4.1 4.2 4.3 RAD AUTO REAL

Take the exploration from the previous page one step further...

On the next page, draw and measure the areas of each of the 6 small triangles formed by the 3 medians. Then drag the vertices.

What do you observe?