The Higher Order Derivative

ID: 9323

Time Required
45 minutes

Activity Overview

In this activity, students will calculate the second derivative of a function, inspect a graph and give the intervals for concave up and concave down and find the point of inflection.

Topic: Formal Differentiation

- *Use the rules for differentiation to compute the higher order derivatives of differentiable functions.*
- *Interpret the second derivative of a function as the rate of change of the slope of its graph.*
- *Graph a function and identify the intervals in which $f''(x) > 0$, $f''(x) < 0$, and $f''(x) = 0$.*

Teacher Preparation and Notes

- *This investigation uses nested derivatives. The students should be familiar with keystrokes for the **Derivative** command. The student should also be able to graph functions and use the command **Show Tangent**.*
- *The calculator should be set in radian mode before the trigonometric derivatives are taken.*
- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. The student worksheet is intended to guide students through the main ideas of the activity and provide a place for them to record their observations.*
- *Before starting this activity, students should go to the home screen and select **F6:Clean Up > 2:NewProb**, then press* ENTER*. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9323" in the quick search box.***

Associated Materials

- *CalcWeek09_HigherOrder_Worksheet_TI89.doc*

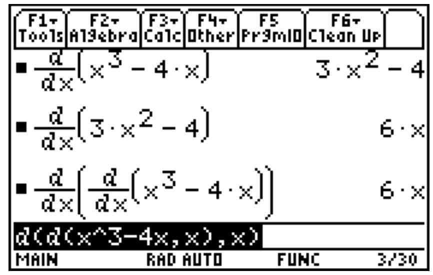
Suggested Related Activities

- *Higher Order Derivatives (TI-Nspire technology) — 9325*

Problem 1 – The Second Derivative of functions

In this section, be sure that students can write the higher order derivatives properly in either the $\frac{d^2y}{dx^2}$ form or the $f''(x)$ form and that numbers are used for the fourth derivative on up.

Students are to use the **Derivative** command (**F3:Calc>1:Derivative**) to find the derivative of $y = x^3 - 4x$ and then find the derivative of that.

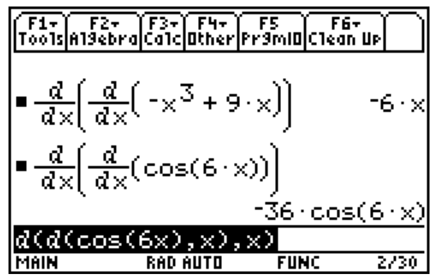


Then students can nest the **Derivative** command to find the second derivative. They should see that the calculator is taking the derivative of the function and then taking the derivative of the result, just like they did above.

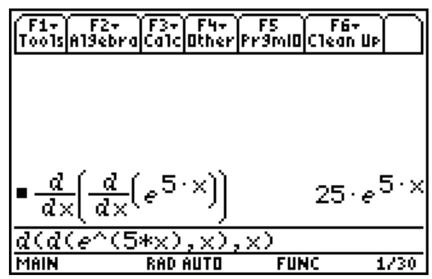
Students will use the calculator to find the second derivative of the following functions.

$g(x) = -x^3 + 9x \rightarrow g'(x) = -3x^2 + 9 \rightarrow g''(x) = -6x$

$h(x) = \cos(6x) \rightarrow h'(x) = -6\sin(6x) \rightarrow h''(x) = -36\cos(6x)$

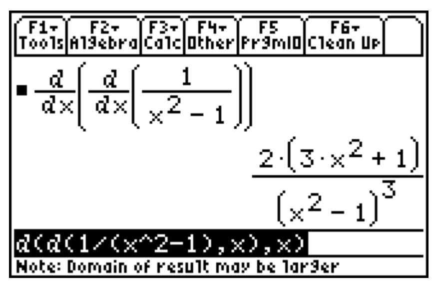


$j(x) = e^{5x} \rightarrow j'(x) = 5e^{5x} \rightarrow j''(x) = 25e^{5x}$



$k(x) = \dfrac{1}{(x^2 - 1)} = (x^2 - 1)^{-1} \rightarrow k'(x) = -1(x^2 - 1)^{-2}\,(2x)$

$\rightarrow k''(x) = \dfrac{d}{dx}\left(\dfrac{-2x}{(x^2-1)^2}\right) = \dfrac{2(3x^2+1)}{(x^2-1)^3}$

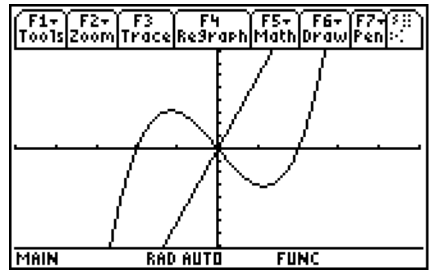


Problem 2 – Concavity

Remind students that concave up is $f''(x) > 0$ and concave down is $f''(x) < 0$.

Students are to graph the function $f(x) = x^3 - 4x$. If they were to draw a line segment between two points in the section to the left of the y-axis, the line segment will be below the curve. If they draw a line segment between two points in the section to the right, the line segment will be above the curve.

Students are to graph the function $f(x) = x^3 - 4x$ and its second derivative, $f''(x) = 6x$. The student screen should look like the one at the right.

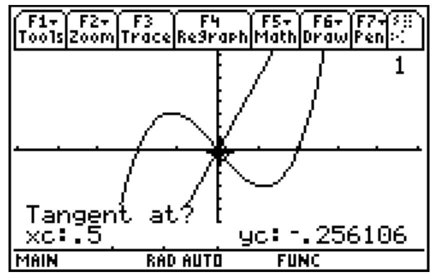


Note that the second derivative is positive when $x > 0$ and the function is concave up there.

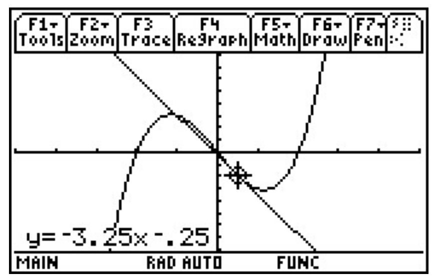
Note that the second derivative is negative for $x < 0$ and the function is concave down there.

At $x = 0$, the function changes concavity and that point is called the point of inflection.

Students are to use the **Tangent** command to find the equation of the tangent line at a specific *x*-value.



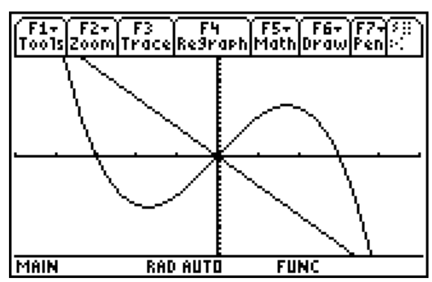
	Equation of Tangent
$x = 0.5$	$y = -3.25x - 0.25$
$x = 1$	$y = -x - 2$
$x = 2$	$y = 8x - 16$
$x = -2$	$y = 8x + 16$
$x = -1$	$y = -x + 2$
$x = -0.5$	$y = -3.25x + 0.25$



Students should see that as x got larger, the slope increased. Thus the points are indeed in the concave section of the function.

The values of the slope decrease as x moves to the right. Thus these points are in the section where the function is concave down.

Students are to graph $g(x) = -x^3 + 9x$ and its second derivative. They should see the graph at the right and determine the following:



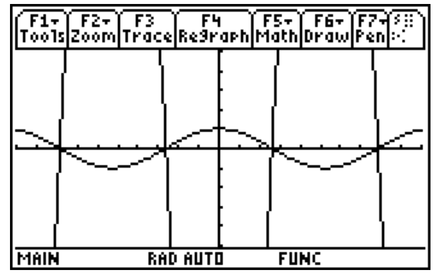
It is concave up if $x < 0$.

It is concave down if $x > 0$.

It has a point of inflection at $x = 0$.

Problem 3 – Concavity for other functions

Students are to graph both $h(x) = \cos(6x)$ and $h''(x) = -36\cos(6x)$.

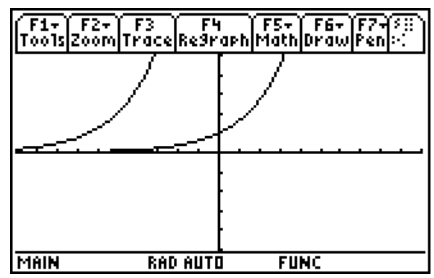


Because the second derivative is so large and the function is so small, encourage the students to change the window so they can see what is happening. The $\cos(6x)$ is periodic with a period of $\frac{\pi}{3}$.

The function is concave down in the intervals $\left[\frac{-\pi}{12}, \frac{\pi}{12}\right]$ and $\left[\frac{\pi}{4}, \frac{5\pi}{12}\right]$, however there are more such intervals. The function is concave up in the interval $\left[\frac{\pi}{12}, \frac{\pi}{4}\right]$ and other intervals.

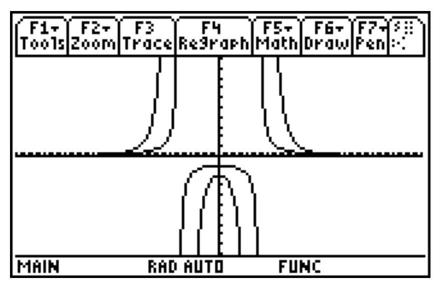
There are multiple points of inflection at $\frac{\pi}{12} \pm \frac{n \cdot \pi}{6}$. Remind students that when we have periodic functions, there will be multiple intervals for concavity.

Students will graph both $j(x) = e^{5x}$ and $j''(x) = 25e^{5x}$.



They should see that the second derivative is always positive and thus the function is always concave up. There is no point of inflection since there is no change in concavity.

Students are to graph both $k(x) = \frac{1}{x^2 - 1}$ and $k''(x) = \frac{2\left(3x^2 + 1\right)}{\left(x^2 - 1\right)^3}$.



Remind students that the first graph was the function and the second was the second derivative.

Students should see that the function is concave up in $(-\infty, -1) \cup (1, \infty)$ and concave down in $(-1, 1)$.

There is no point of inflection because the function is not defined at $x = -1$ or at $x = 1$. The function does change concavity at the asymptotes but since the function is not defined there, there is no point of inflection.