EXPLORATIONS ™

Activity 6

Objectives

- Graph quadratic inequalities
- Graph linear-quadratic and quadratic systems of inequalities

Inequalities, They Are Not Just Linear Anymore!

Introduction

What do satellite dishes, car headlights, and camera lenses have in common? All of these commonly used devices, as well as many others, have foundations in quadratic relationships.

In this activity, you will look at quadratic relationships and will explore the process of graphing a quadratic inequality and systems of quadratic inequalities. You will be able to solve these inequalities algebraically and graph them on a coordinate plane.

Problem

How does the graph of $y > x^2$ differ from the graph of $y = x^2$?

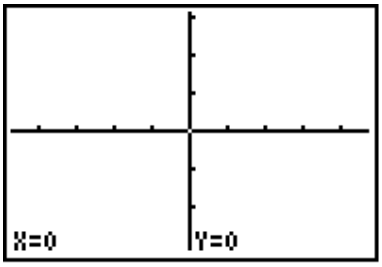
Exploration

The first step in solving the problem is to graph $y = x^2$.

1. Without using your graphing handheld, draw a sketch of the graph of $y = x^2$ in the grid.

What is the *y*-intercept? _____

Verify your graph on your graphing handheld.



2. Locate the point (1, 1) and mark this point on your graph. Substitute the coordinates of this point into the inequality, $y > x^2$ and solve algebraically. Is the inequality statement true or false?

The coordinate points (2, 4) and (⁻3, 9) are also on the parabola of $y = x^2$. What happens when you substitute them into $y > x^2$? Are these points part of the solution set? Record your results in the table below.

Ordered Pair	$y > x^2$	Solution Set?
(1, 1)	1 > 1	No
(2, 4)		
(⁻3, 9)		

What can you conclude about the points that lie on the parabola? Are they part of the solution set for $y > x^2$?

What change could you make to the equation so that the coordinate points in the table *are* part of the solution set for $y > x^2$?

3. The parabola that you graphed in step **1** divides the coordinate plane into two parts. In which of these parts do you think the solution set that satisfies the inequality $y > x^2$ lies?

4. Locate the point (1, 4). Mark this point on your graph. Substitute these coordinates into $y > x^2$. Is the statement true? Are these coordinates part of the solution set for $y > x^2$?

5. Repeat this process with the points (0, 5) and (-2, 6). Are they part of the solution set? Record your results in the table below.

Ordered Pair	$y > x^2$	Solution Set?
(1, 4)	4 > 1	Yes
(0, 5)		
(-2, 6)		

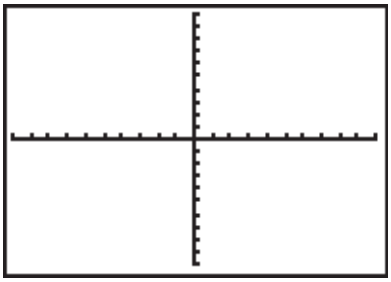
How many of these points are located inside the graph of $y = x^2$?

6. Locate the point (3, -2) and mark it on your graph. Substitute the coordinates into $y > x^2$. Are these coordinates part of the solution set? Add this ordered pair to the table above. Choose another point that lies outside the graph and substitute the coordinates into $y > x^2$. Add the results to the table.

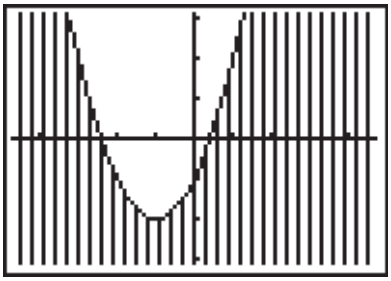
7. Based on your table, what conclusion can you draw about the solution set of $y > x^2$?

8. Sketch the graph to show the solution set of $y > x^2$. How is this graph different from the graph you drew in step **1**?

Verify your answer by using the Inequality Graphing application to graph $y > x^2$.



9. Which quadratic inequality is represented by this graph?



Note: *The* **ZDecimal** *viewing window has been used.*

A. $y \geq (x+1)^2 + 2$ **B.** $y > (x+2)^2 + 2$

C. $y \leq (x+1)^2 - 2$ **D.** $y \geq (x+2)^2 - 2$

Extension

Just as it is possible to graph a system of linear inequalities, it is also possible to graph systems that involve quadratic inequalities. First, you will investigate a linear-quadratic system of inequalities, and then a quadratic system of inequalities.

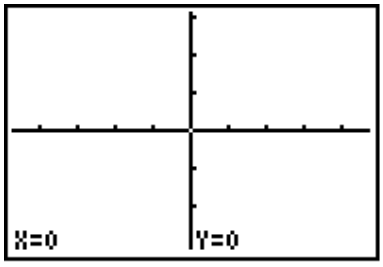
Let's look first at this system of inequalities: $y > -1$; $y \leq -x^2 + 2$.

1. Describe the graph you would expect to see for $y > -1$.

2. Describe the graph you would expect to see for $y \leq -x^2 + 2$.

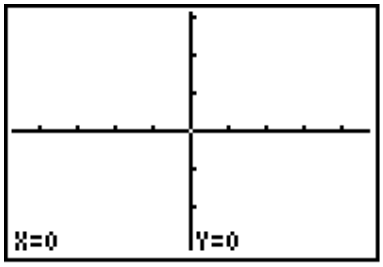
3. Will these two inequalities intersect in one or more points? Explain your thinking.

4. Use the Inequality Graphing App to check your predictions. Sketch the graph of the system.

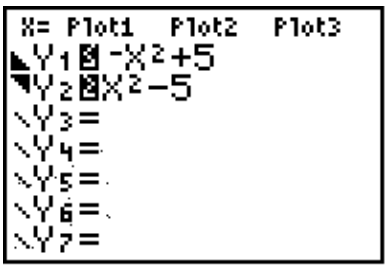


A quadratic system of inequalities is made up of more than one quadratic inequality.

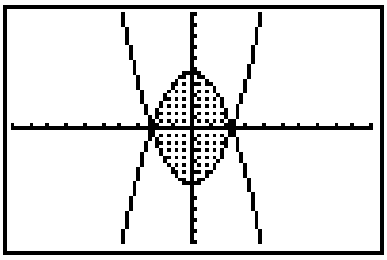
5. What do you think the graph of a system of quadratic inequalities, $y \leq -x^2 + 5$ and $y \geq x^2 - 5$, would look like? Sketch the graph.



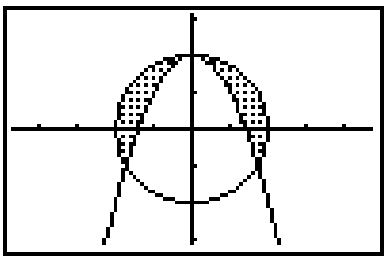
6. Use the Inequality Graphing application to graph the system of inequalities. Enter the inequalities in the **Y=** editor. Then graph the system.



7. Set the **Shading** to **Ineq Intersection**. Press ZOOM **6** to select **6:ZStandard** to set the viewing window.



Quadratic systems can also include other conic relationships, such as circles, ellipses, and hyperbolas. The graph on the right contains both parabolic and circular components.



Let us figure out what inequalities represent each of the components (parabolic and circular) in the graph.

The inequality that created the parabolic component is: _____.

The inequality that created the circular component is: _____.

Generating the graph of the quadratic system of inequalities, $x^2 + y^2 < 4$; $y > -x^2 + 2$, with a graphing handheld is tricky because circles are not functions. However, the Inequality Graphing App can be used only when the graphing handheld is operating in **Func** mode, its default setting.

8. The first step to graphing this system of inequalities is to solve the equation $x^2 + y^2 = 4$ for y.

Note: *You obtain two roots when taking the square root during this process. One root is positive, the other negative.*

$y =$ _____

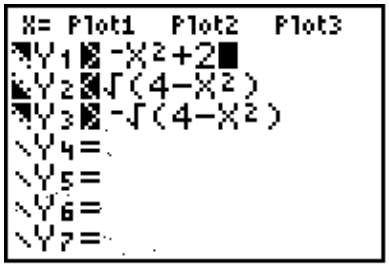
$y =$ _____

9. Replace the equal symbols with inequality symbols that will produce the desired solution.

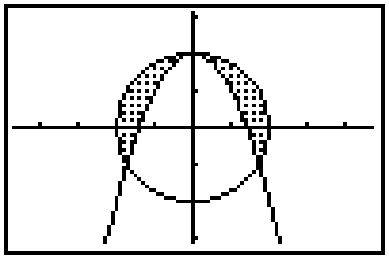
$y >$ _____

$y <$ _____

10. Enter the three inequalities in the **Y=** editor and graph the system.



11. Press ZOOM **4** to select **4: ZDecimal**. Set the **Shading** to **Ineq Intersection**. The graph on your graphing handheld should look like the one shown.



Student Worksheet

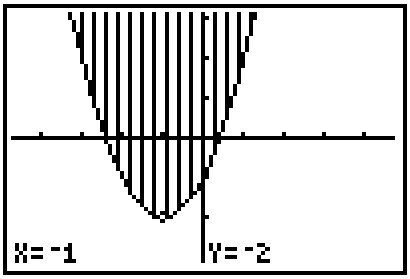
Name _____

Date _____

For items **1–4**, write the quadratic inequality most likely used to generate each graph. Use the Inequality Graphing application to check your answers.

Note: *The* **ZDecimal** *viewing window was used to create each graph.*

1.



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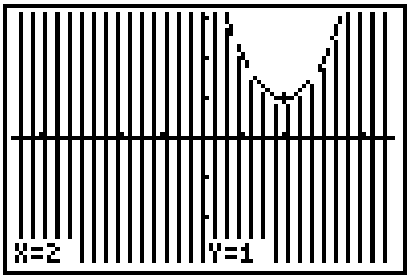
2.



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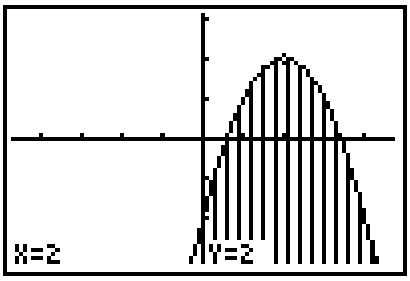
3.



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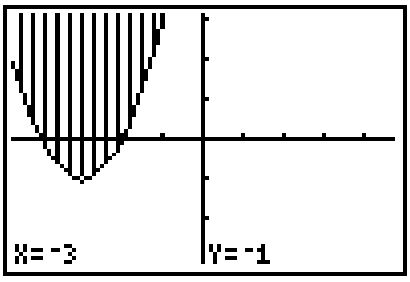
4.



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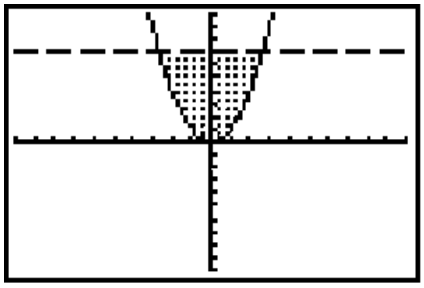
For items **5–8**, determine the system of linear-quadratic inequalities most likely used to generate each graph. Use the Inequality Graphing application to check your answers.

Note: *The* **ZStandard** *viewing window was used to create each graph.*

5.



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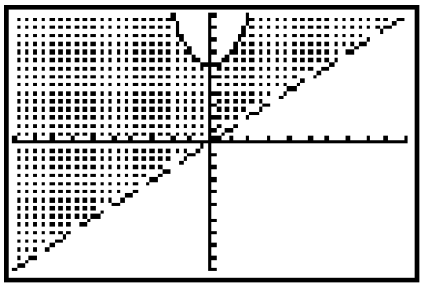
6.



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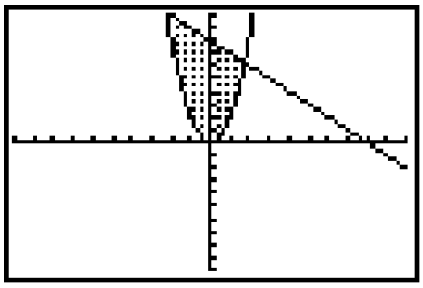
7.



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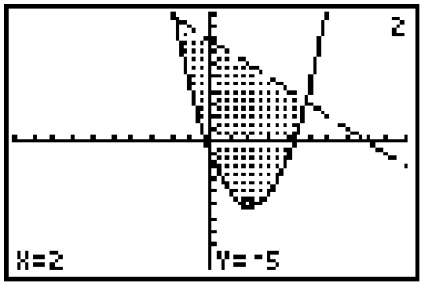
8.



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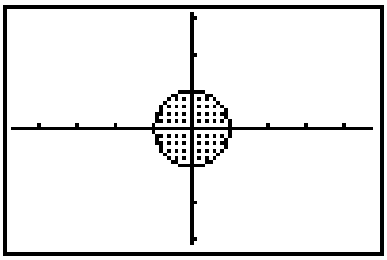
For items **9–12**, choose the system of inequalities used to generate each graph. Use the Inequality Graphing application to check your answers.

Note: *The* **ZInteger** *viewing window was used to create graphs* **9** *and* **10**. *The* **ZDecimal** *viewing window was used for graphs* **11** *and* **12**.

9.



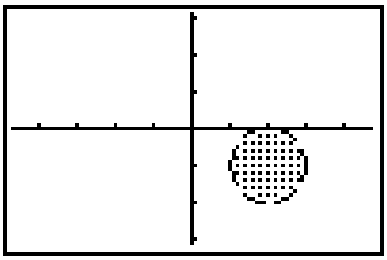
A. $x^2 + y^2 < 100$

B. $x^2 + y^2 \leq 100$

C. $x^2 + y^2 > 100$

D. $x^2 + y^2 \geq 100$

10.



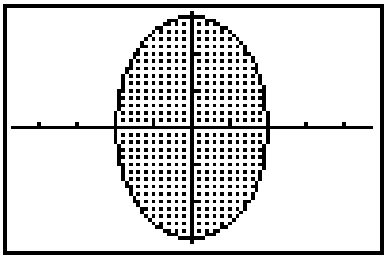
A. $(x - 20)^2 + (y + 10)^2 < 100$

B. $(x - 20)^2 + (y + 10)^2 \leq 100$

C. $(x - 20)^2 + (y + 10)^2 > 100$

D. $(x - 20)^2 + (y + 10)^2 \geq 100$

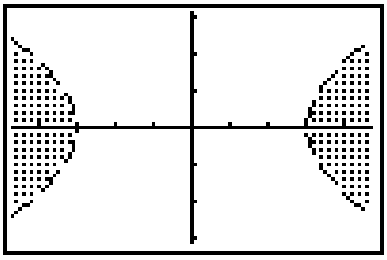
11.



A. $\frac{x^2}{4} + \frac{y^2}{9} > 1$ **B.** $\frac{x^2}{4} + \frac{y^2}{9} \geq 1$

C. $\frac{x^2}{4} + \frac{y^2}{9} < 1$ **D.** $\frac{x^2}{4} + \frac{y^2}{9} \leq 1$

12.



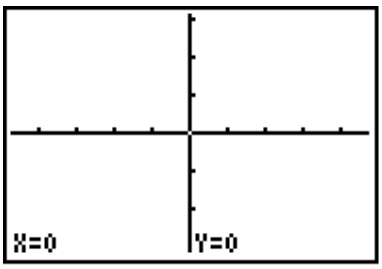
A. $\frac{x^2}{9} - \frac{y^2}{4} > 1$ **B.** $\frac{x^2}{9} - \frac{y^2}{4} \geq 1$

C. $\frac{x^2}{9} - \frac{y^2}{4} < 1$ **D.** $\frac{x^2}{9} - \frac{y^2}{4} \leq 1$

13. Graph the system $\frac{x^2}{9} + \frac{y^2}{4} \leq 1$;

$$9x^2 + 4y^2 < 36$$

Use the Inequality Graphing App to check your answer.



Teacher Notes

Activity 6

Inequalities, They Are Not Just Linear Anymore!

Objectives

- Graph quadratic inequalities
- Graph linear-quadratic and quadratic systems of inequalities

Materials

- TI-84 Plus/TI-83 Plus
- Inequality Graphing application

Teaching Time

- 90 minutes

Prerequisite Skills

- Graphing two-variable linear inequalities in the coordinate plane
- Fundamentals of conic sections
- Solving equations for a variable in terms of another variable

Management

Students might benefit from working in pairs when doing the Student Worksheet. Encourage students to identify the coordinate points of the vertex as well as the slope of the parabolic curve in each graph.

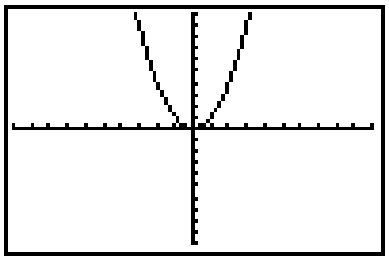
Notes about Exploration

Students might benefit from a brief review of the basic graphical distinctions between linear and quadratic functions before beginning the Exploration items.

The Exploration section guides students through the process of graphing quadratic inequalities. The Extension section addresses the graphing of systems of linear-quadratic and quadratic inequalities. Teachers may choose to present this lesson one section at a time, depending on the course curriculum.

Answers to the Exploration Questions

1.



y-intercept = 0

2. The statement, 1 > 1, is false. The ordered pair is not part of the solution set.

Ordered Pair	$y > x^2$	Solution Set?
(1, 1)	1 > 1	No
(2, 4)	4 > 4	No
(⁻3, 9)	9 > 9	No

The points that lie on the parabola are not part of the solution set for $y > x^2$.

Change the equation sign to $\geq$.

3. The solution set lies inside the curve.

4. Yes, 4 > 1 is a true statement, so (1, 4) is part of the solution set.

5.

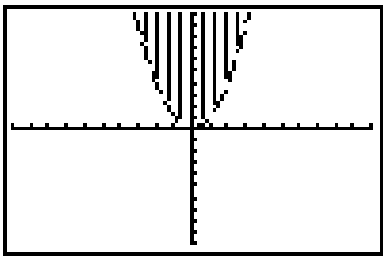
Ordered Pair	$y > x^2$	Solution Set?
(1, 4)	4 > 1	Yes
(0, 5)	5 > 0	Yes
(⁻2, 6)	6 > 4	Yes
(3, ⁻2)	⁻2 > 9	No
Answers will vary.		

6. The ordered pair (3, ⁻2) is not part of the solution set. ⁻2 > 9 is a false statement.

Answers will vary, but ordered pairs should not be part of the solution set.

7. All the points that lie inside the parabola satisfy the given inequality and are part of the solution set. Points that lie outside the parabola are not part of the solution set.

8. Students should note that the curve is dashed rather than solid and that the area inside the parabola is shaded.



9. **C**

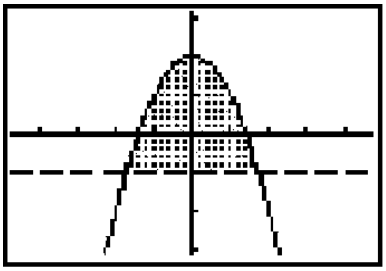
Extension

1. Answers will vary. Students should recognize that there will be a dashed horizontal line at $y = -1$, with the area above shaded.

2. Answers will vary. Students should note that the parabola is a solid curve with a vertex at (0, 2). The area inside the parabola is shaded.

3. Answers will vary. Students should realize that the inequalities will share a common region.

4.



5. Students' sketches will vary, but should consist of two parabolas since they are graphing two quadratic inequalities. Students should make the parabolas solid curves and should shade appropriate regions.

7. Inequality that generated the parabolic component: $y \geq -x^2 + 2$

Inequality that generated the conic component: $x^2 + y^2 \leq 4$

8. $y = \sqrt{4 - x^2}$, $y = -\sqrt{4 - x^2}$

9. $y < \sqrt{4 - x^2}$, $y > -\sqrt{4 - x^2}$

Answers to the Student Worksheet

1. $y \geq (x + 1)^2 - 2$

2. $y < (x - 2)^2 + 1$

3. $y \leq -(x - 2)^2 + 2$

4. $y \geq (x + 3)^2 - 1$

5. $y < 7; < 7;\ y \geq x^2$

6. $y > x;\ y \leq x^2 + 6$

7. $y \leq {}^{-}x + 8;\ y \geq 2x^2$

8. $y < {}^{-}x + 8;\ y \geq (x-2)^2 - 5$

9. B

10. A

11. D

12. A

13. $y \leq \sqrt{\frac{36-4x^2}{9}};\ y \geq {}^{-}\sqrt{\frac{36-4x^2}{9}};\ y < \sqrt{\frac{36-9x^2}{4}};\ y > {}^{-}\sqrt{\frac{36-9x^2}{4}}$