Area Accumulation

Student Activity

Name _____

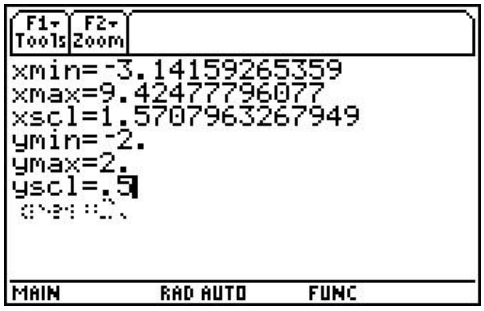
Class _____

Part 1 – Explore Area Accumulation

In this activity, you will explore the integral numerically and graphically using area accumulation.

Let $F(x) = \int_a^x f(x)\,dx$ and let $f(x) = \sin(x)$.

Press ◆ [Y=]. Select **F1:Tools > 8:Clear Functions**. In **y1**, type the expression **sin(*x*)**. Press ◆ [WINDOW] and change the window settings to match those on the right. Note that the window limits for *x* are $-\pi \le x \le 3\pi$ and **xscl** is $\frac{\pi}{2}$. Then, press ◆ [GRAPH] to display the sine function.



Select **F5:Math > 7:** $\int f(x)\,dx$ and enter 0 for the lower limit and π for the upper limit to find $F(\pi)$ for $a = 0$. Then select **F4:Regraph** to clear the previous integral. Repeat the procedure to find each of the following integrals.

1. What is $F\left(\frac{\pi}{2}\right)$ for $a = 0$?

2. What is $F\left(\frac{3\pi}{2}\right)$ for $a = \frac{\pi}{2}$?

3. What is $F(2\pi)$ for $a = \pi$?

4. What is $F\left(\frac{\pi}{2}\right)$ for $a = -\frac{\pi}{2}$?

Press [HOME]. To find $F(2\pi)$ for $a = \frac{\pi}{2}$, enter **∫(sin(*x*),x,π/2,2π)** and press [ENTER]. Let $f(x) = \sin(x)$ and $a = \frac{\pi}{2}$, and complete the following table.

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
***F*(*x*)**							

Use your observations to answer each question.

5. What function fits the values in the table?

6. For a fixed value of *a*, when is $F(x)$ increasing? When is $F(x)$ decreasing? Explain.

7. For $\int_a^x f(x)dx$, is the integral *always*, *sometimes*, or *never* negative when $x < a$? Explain.

8. Describe two different sets of circumstances that would result in a negative integral: one set when $a < x$ and one set when $x < a$.

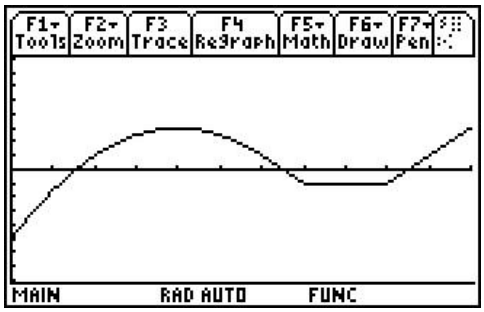
9. Describe two different sets of circumstances that would result in a positive integral: one set when $a < x$ and one set when $x < a$.

10. $\int_{\pi}^{\pi/2} \sin(x)dx =$ _____

Part 2 – Curve Sketching and Kinematic Calculus

In this section, you will explore the kinematic relationship between an acceleration graph and its corresponding velocity graph.

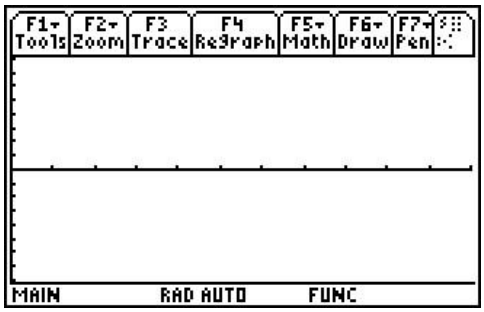
The graph at the right shows the acceleration of an object in m/s^2.



1. What is the definite integral of acceleration with respect to time?

2. What is the definite integral of velocity with respect to time?

3. Sketch the corresponding kinematic velocity graph (in m/s) at the right.



4. Use your velocity graph to describe what is occurring physically to the object.