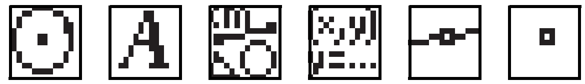
EXPLORATIONS™

Activity 6

Objective

- To investigate the relationships between the equation of a circle, the length of its radius, and the coordinates of its center

Cabri® Jr. Tools



Circles in the Plane

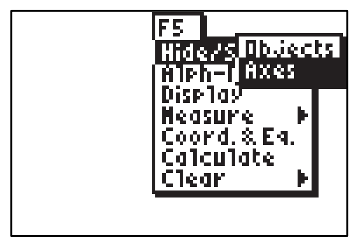
Introduction

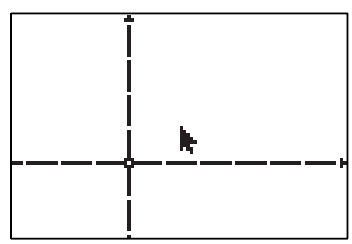
This activity uses the Cabri Jr. application to explore circles in the plane.

Construction

Show the coordinate axes and create a circle.

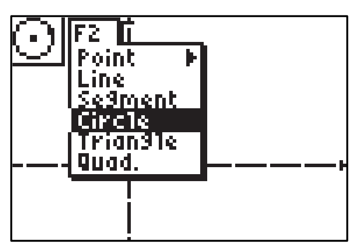
1. Open the **Display Tools Menu** and highlight **Hide/Show**. Press ▶ to view the **Hide/Show Menu**. Highlight **Axes**. Press ENTER to display the coordinate axes.



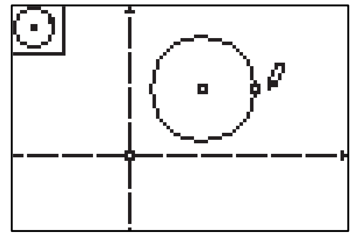


2. Open the **Drawing Tools Menu**, and then highlight **Circle**. Press ENTER.

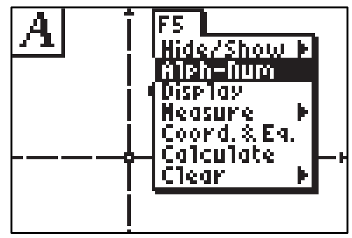
Note: *The tool icon at the top left of the screen indicates that the* **Circle** *tool is active.*



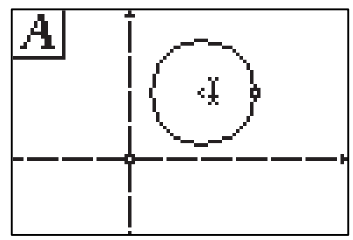
3. Move the cursor to the first quadrant and press [ENTER] to anchor the center of the circle. Move the cursor and press [ENTER] again to anchor the radius point of the circle.



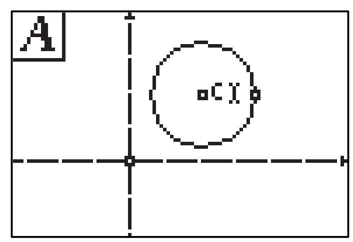
4. Open the **Display Tools Menu** and highlight **Alpha-Num**. Press [ENTER].



5. Move the cursor to the center of the circle. The point blinks when the cursor is close enough to the point to select it.

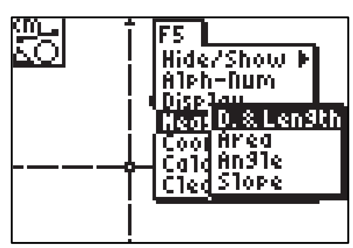


6. Press [ENTER] to create a label for this point. Label the point *C* (**C** is located above [PRGM]), and then press [ENTER] to complete the label.



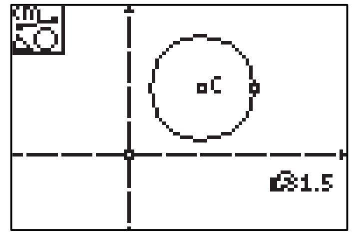
 *Measure the radius of the circle, and display the equation of the circle and the coordinates of the center.*

7. Open the **Display Tools Menu**, and then highlight **Measure**. Press [▸] to view the **Measure Menu**. Highlight **D. & Length**. Press [ENTER].

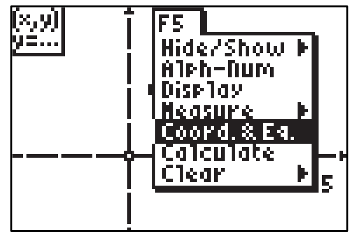


8. Move the cursor to point *C* and press [ENTER]. Move the cursor to the radius point of the circle and press [ENTER]. Press [ENTER] a final time to anchor the measurement

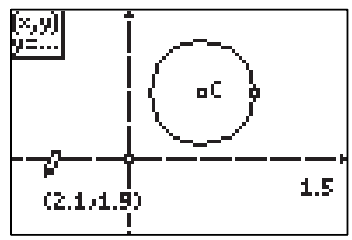
Note: *Be careful to grab the point itself, not the label* C.



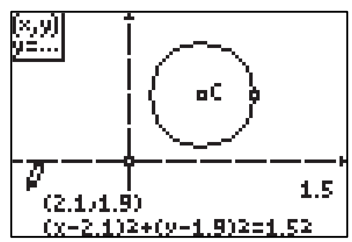
9. Open the **Display Tools Menu**, and then highlight **Coord. & Eq**. Press [ENTER].



10. Move the cursor to point *C* and press [ENTER]. Move the cursor to a desired location and press [ENTER] to anchor the coordinates.



11. Move the cursor to the circle and press [ENTER]. Move the cursor to a desired location and press [ENTER] to anchor the equation. Press [CLEAR] to exit the **Coordinate and Equations** tool.



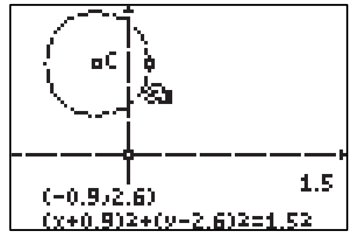
Drag the circle, the center, and the radius point.

12. Move the cursor to the side of the circle and press [ALPHA] to grab it.

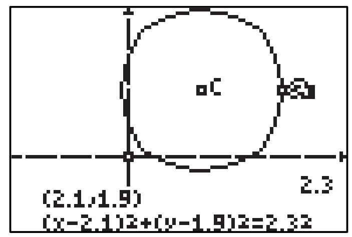
Note: *The pointer changes to an outlined arrow when it is near an available object.*

Note: *To drag the circle around the screen, grab the circle itself, not its radius point.*

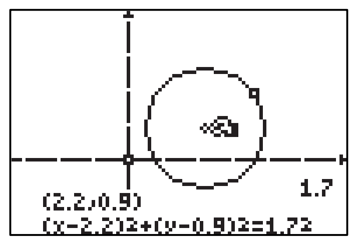
Use the cursor keys to drag the circle and observe any changes in the coordinates and equation.



13. Press [CLEAR] to release the circle. Move the cursor to the radius point and press [ALPHA] to grab it. Use the cursor keys to change the radius, and observe the results.

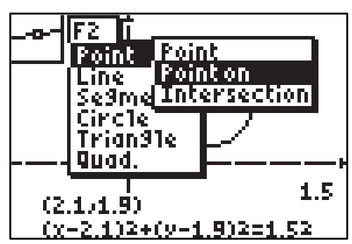


14. Press [CLEAR] to release the radius point. Move the cursor to the center point *C* and press [ALPHA] to grab it. Use the cursor keys to move the center, and observe the results. Try locations in all four quadrants, on the *x*- and *y*-axis, and at the origin.

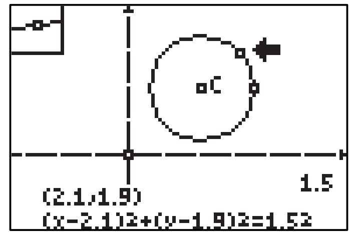


Construct a point on the circle, and measure its distance to the center.

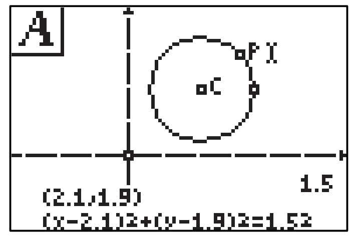
15. Open the **Drawing Tools Menu**, and then highlight **Point**. Press [▶] to view the **Point Menu**. Highlight **Point on**, and then press [ENTER].



16. Move the cursor to the circle and press ENTER to construct a point on the circle.

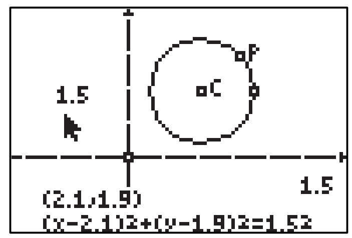


17. Use the **Alpha-Numeric** tool to label this point *P* (**P** is above 8).

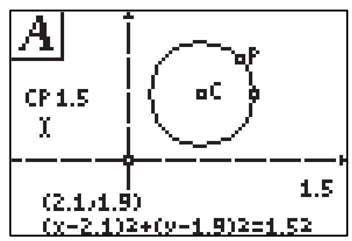


18. Use the **Distance & Length** tool to measure the distance from point *C* to point *P*. When finished, press CLEAR to exit the tool.

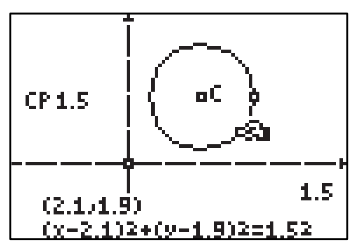
Note: *There is no need to connect these two points with a segment if you press* ENTER *to select each of the two points.*



19. If desired, you can label the measurement using the **Alpha-Numeric** tool. Move the cursor to the left of the desired measurement, press ENTER to create the label, and then press ENTER again to complete the label. When finished, press CLEAR to exit the tool.

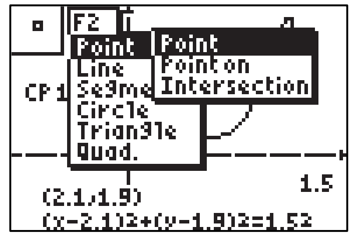


20. Move the cursor to point *P* and press ALPHA to grab it. Drag point *P* around the circle, and explain why the distance from point *C* to point *P* remains constant.

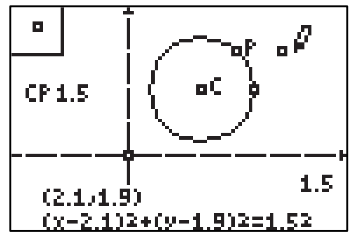



Construct a point not on the circle, and measure its distance to the center.

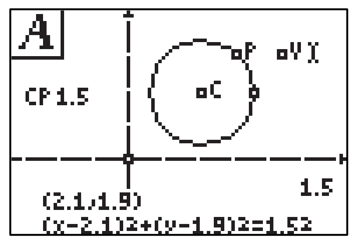
21. Open the **Drawing Tools Menu**, and then highlight **Point**. Press [▸] to view the **Point Menu**. Highlight **Point**, and press [ENTER].



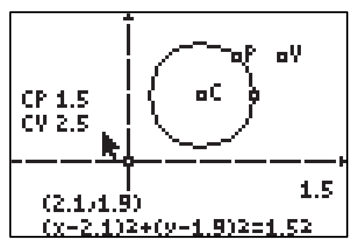
22. Move the cursor to a location *not* on the circle and press [ENTER] to create a point.



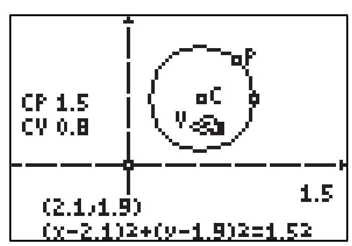
23. Use the **Alpha-Numeric** tool to label this point *V* (**V** is above [6]).



24. Use the **Distance & Length** tool to measure the distance from point *C* to point *V*. When finished, press [CLEAR] to exit the tool.



25. Move the cursor to point *V* and press [ALPHA] to grab it. Drag point *V* around the screen so it is both inside and outside the circle. Explain any relationships you observe.



Data Collection and Analysis

Name ______________________

Date ______________________

Questions and Conjectures

1. Explain the relationships among the coordinates of the center of the circle, the length of the radius, and the equation of the circle.

2. Explain why the distance from point *C* to point *P* remains constant as point *P* is dragged around the circle.

3. Describe the relationship between the distance from point *C* to point *V* and the radius of the circle when point *V* is inside, on, and outside the circle.

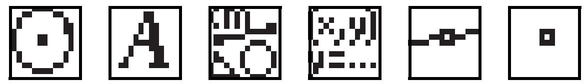
4. Research the Distance Formula and explain the connection that this mathematical principle has with the circles and points investigated in this activity.

Teacher Notes

Objective

- To investigate the relationships between the equation of a circle, the length of its radius, and the coordinates of its center

Cabri® Jr. Tools



Activity 6

Circles in the Plane

Construction Notes

When dragging the circle, the first inclination students have is to drag the center of the circle to move it around in the plane. This causes the radius of the circle to change since the radius point does not move when the center is dragged. Different results occur when the circle itself is dragged, when the center is dragged, and when the radius point is dragged.

The **Distance & Length** tool can be used to select one object, such as a segment, or it can be used to measure a point-to-point distance by pressing [ENTER] on each of the two points. There is no need to connect the desired points with a segment.

Answers to Questions and Conjectures

1. Explain the relationships among the coordinates of the center of the circle, the length of the radius, and the equation of the circle.

As the circle is dragged around the screen, the coordinates of point *C* change accordingly and the radius of the circle remains constant. The equation of a circle is of the form $(x - a)^2 + (y - b)^2 = r^2$. The coordinates of point *C* are represented in the equation of the circle as *a* and *b*. Change the length of the radius of the circle to change the value of r^2 in the equation.

Place the circle in the plane so that its center point *C* is at the origin or on the *x*- or *y*-axis to help students understand the special cases of the equation of a circle.

2. Explain why the distance from point *C* to point *P* remains constant as point *P* is dragged around the circle.

As point *P* is dragged around the circle, the distance from point *C* to point *P* remains constant since all points on the circle are equidistant from the center. When point *P* is on the circle, the distance from *C* to *P* is equal to *r*, representing the points on the circle since the equation of the circle is $(x - a)^2 + (y - b)^2 = r^2$.

Dragging point *P* around the circle provides a more convincing visualization of this equality. The distance from *C* to *P* remains constant, as expected. This visualization also suggests the common locus definition of a circle as the set of points equidistant from a given point. Students should see this happen and then be asked to give their own definition of a circle.

3. Describe the relationship between the distance from point *C* to point *V* and the radius of the circle when point *V* is inside, on, and outside the circle.

When point *V* is inside the circle, the distance from point *C* to point *V* is less than the radius of the circle. All of the points in the interior of the circle are represented by the inequality $(x - a)^2 + (y - b)^2 < r^2$ since the distance from *C* to *V* is less than *r*. When *V* is outside the circle, the distance from *C* to *V* is greater than the radius of the circle. All of the points exterior to the circle are represented by the inequality $(x - a)^2 + (y - b)^2 > r^2$ since the distance from *C* to *V* is greater than *r*.

4. Research the Distance Formula and explain the connection that this mathematical principle has with the circles and points investigated in this activity.

If students have seen the Distance Formula before, they might see the connection between the equation of the circle and this formula. If they have not studied the Distance Formula, then draw their attention to the similarity between the equation of the circle and the computed relationship $(x_1 - x_2)^2 + (y_1 - y_2)^2$. When point *P* is on the circle, its coordinates (x_1, y_1) dynamically represent all of the points on the circle. The coordinates of point *C*, (x_2, y_2), represent the center of the circle as they do in the equation of the circle. Using the Pythagorean Theorem, the expression $(x_1 - x_2)^2 + (y_1 - y_2)^2$ represents the square of the distance from *C* to *P*, or the square of the radius of the circle. This is also a good time to repeat the locus definition of a circle: the set of points equidistant from a single point. This is the heart of the Distance Formula and the derivation of the general equation of a circle.