The Area Function

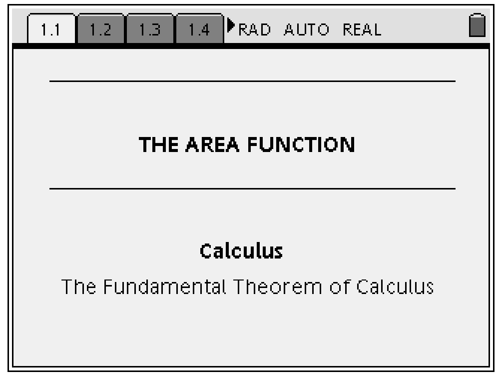
ID: 8298

Name ______________________

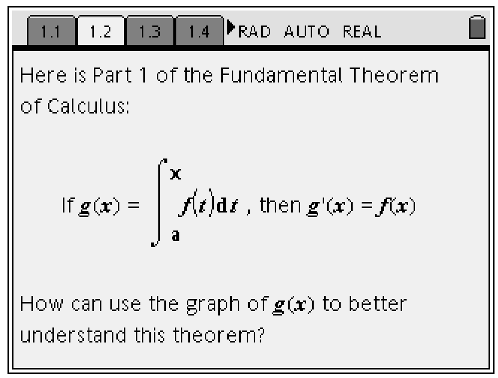
Class ______________________

In this activity, you will explore:

- *the graph of* $g(x)=\int_a^x f(t)\,dt$



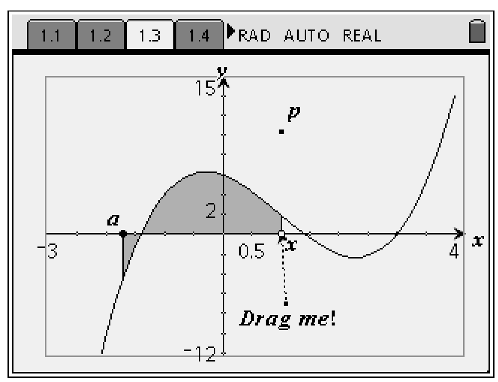
Open the file *CalcAct03_AreaFunction_EN.tns* on your handheld and follow along with your teacher for the first two screens. Use this document as a reference and to record your answers. Move to page 1.2 and wait for further instructions from your teacher.



Part 1 of the Fundamental Theorem of Calculus is shown in the screen at right. One way to demonstrate why this theorem works is to integrate the right side of the equation and then differentiate the result. This will yield $f(x)$, the conclusion of Part 1 of the Fundamental Theorem of Calculus.

In this activity, you will investigate another way to see how this theorem works by looking at the graph of $g(x)$ compared to the graph of $f(t)$.

Problem 1 – Investigating the graph of *g*(*x*)



Move to page 1.3. You will see the screen at right showing the graph of the function $f(x) = x^3 - 3x^2 - 2x + 6$. You will also see three points, *a*, *p*, and *x*, as well as the shaded region representing the area under $f(x)$ from *a* to *x*.

- Explain what point *p* represents.

Drag point *x* slowly along the *x*-axis and observe the movement of point *p*. Describe how point *p* moves in the context of its relationship between the position of points *a*, *x*, and the shaded region.

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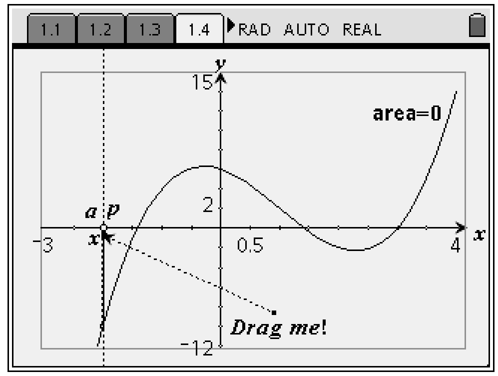
Advance to page 1.4. You will see a screen similar to the one from page 1.3. However, this time points *a*, *x*, and *p* coincide. Also notice that the area is zero.



Drag point *x* slowly along the *x*-axis and observe the trace created by the path of point *p*. Sketch this trace, which we will call $g(x)$, on the diagram provided. Study the trace and answer the following questions.

- For what values of x is $g(x)$ increasing? For what values of x is $g(x)$ increasing?

- What is the relationship between the maximum and/or minimum values of $g(x)$ and the graph of $f(x)$? Explain.

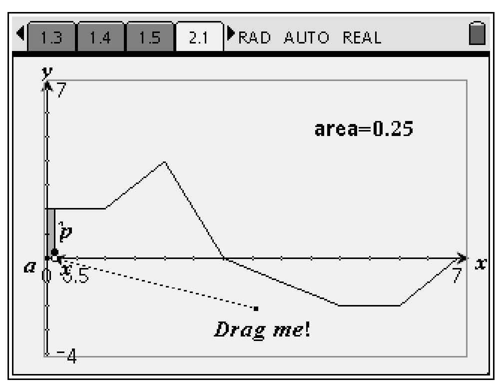
- Explain why $g(x)=\int_a^x f(t)\,dt$ can be thought of as an area function.

- How does the graph of $g(x)$ help demonstrate Part 1 of the Fundamental Theorem of Calculus?

Move to page 1.5 and place the cursor in the formula cell (gray) for Column A. Press [enter] twice to delete the data in this column. Repeat this procedure to delete the data in Column B.

Problem 2 – Investigate another area function

Advance to page 2.1. You will see the screen at right of the function *f*. Let $g(x)=\int_0^x f(t)\,dt$. Before dragging point *x*, answer the following questions.



a) Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(6)$.

b) On what intervals is $g(x)$ increasing?

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c) On what intervals is *g*(*x*) decreasing?

d) Identify the values of *x* for which *g*(*x*) has minimum and maximum value.

e) Make a rough sketch of *g*(*x*) on the diagram provided.

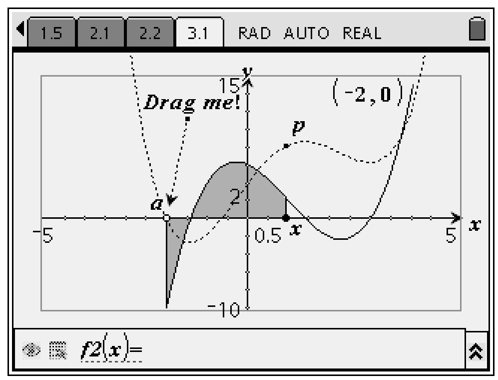
Verify your results by dragging point *x* slowly along the *x*-axis to trace out the graph of *g*(*x*). Then advance to page 2.2 and clear the data as you did in the previous problem.

Problem 3 – Another look at the function from Problem 1

Advance to page 3.1. You will see the screen at right. This screen shows the complete graph of

$g(x) = \int_a^x \left(t^3 - 3t^2 - 2t + 6\right)dt$, with the coordinates of

point *a* shown at the top right corner of the screen. Do not move point *a* yet.



Use the Evaluation theorem, with a lower limit of integration *a* = –2, to determine the formula for *g*(x). Graph this formula in *f2*(*x*). Did your result match the graph of *g*(*x*) already shown?

Now drag point *a* along the *x*-axis in either direction. What effect does moving point *a* have on the graph of *g*(*x*)?

Explain why Part 1 of the Fundamental Theorem of Calculus holds for any value of *a*.

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