**The Pirate Problem** **Teacher Notes**
**A TI-Nspire Activity**

## Overview

This activity is an adaptation from an article that appeared in the September 2003 *Mathematics Teacher*, Dynamic Visualization and Proof: A New Approach to a Classic Problem by Daniel Scher.

## Materials Needed:

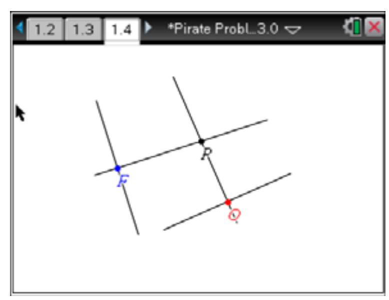
TI-Nspire handhelds

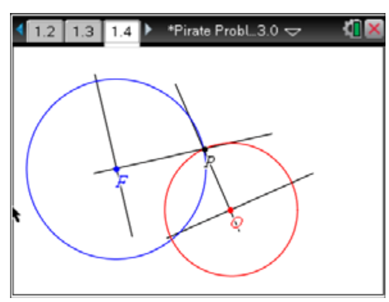
## Objectives:

- Students will be able to make geometric constructions using dynamic geometry software
- Students will apply geometric methods to solve problems with constraints.
- Students will apply geometric properties of circles, triangles, perpendiculars, and congruence theorems to a context.
- Students will be able use inductive reasoning and formal proof to support their conclusions.

This activity would be suitable for high school formal geometry students who have studied congruence theorems for triangles and have knowledge of lines, perpendiculars, and circles. With minor adaptations, this activity can be modified for middle school students who are motivated by the setting of the story.

This lesson uses guided inquiry methods to search for an explanation of the observations made when the students animate the diagrams they construct. Open ended questions are sequenced to help students use critical thinking skills to solve the mystery.




Page 1.2 is meant for students to attempt to construct the treasure map based on their prior knowledge and their geometric thinking. If the level of your students does not lend itself to this style of inquiry-based learning, then moving p. 1.4 in front of the question on p. 1.3 may be advisable. Solution to constructing the treasure map:

1. menu >Points&Lines>Line. Draw line FP and line OP.
2. menu >Construction>Perpendicular. Draw a perpendicular to line FP at F and a perpendicular to line OP at O.
3. menu >Shapes>Circle. Draw a circle with center at F that goes through P. Draw a circle with center at O that goes through P.
4. menu >Points&Lines>Intersection Point(s). Select the circle (center F) and the perpendicular through F. Label that point S1 by pressing menu >Actions>Text. Select the intersection point by making it blink and press enter. Type S1 and press enter.
5. Do the same for the intersection of circle (Center O) and the perpendicular through O. Label that point S2 by pressing menu >Actions>Text. Select the intersection point by making it blink and press enter.
6. menu >Points&Lines>Segment. Draw line segment $S_1S_2$.
7. menu >Construction>Midpoint. Label the midpoint T by pressing

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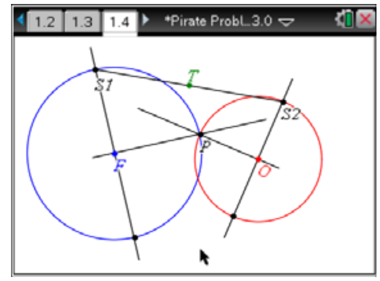
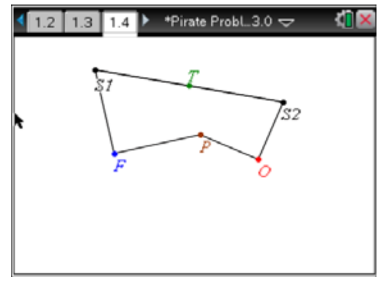
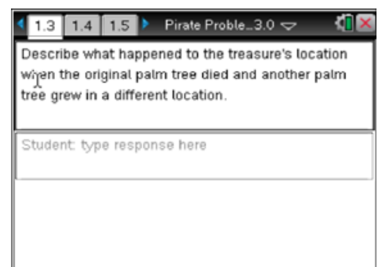
menu >Tools>Text. Select the point by making it blink and press enter. Type T and press enter. Congratulations! You have found the treasure.



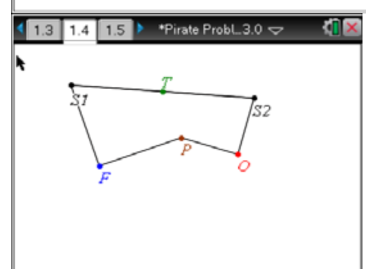

1. To make the diagram less cluttered, press menu>Actions>Hide/Show. Press enter. To hide the perpendiculars and the circles, make each one blink and press enter. When you are done, press esc.
2. menu>Shapes>Polygon. Press enter. Draw the polygon $FS_1S_2OPF$. Press enter. Press esc. In this way, students can move the palm tree and better see what happens to the treasure even when a different palm tree in a different location is used to step off the paces from the rocks.
3. Now students may select the palm tree, P, and press ctrl to lock on to point P and move it to new positions.




Page 1.3 This page gives students the opportunity to record the observations they made when moving the palm tree.

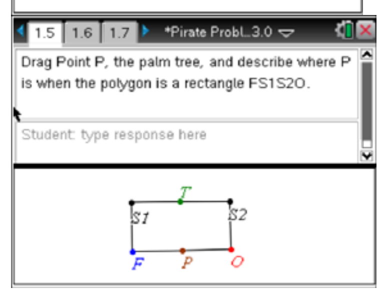



Page 1.4: For students having difficulty constructing their own "treasure map," page 1.4 is provided.

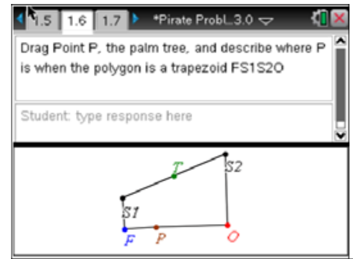



Page 1.5

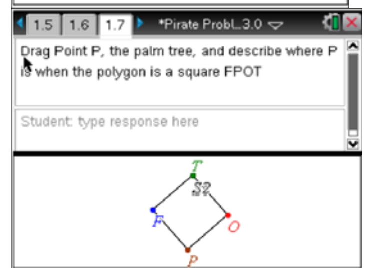
When the palm tree is on the midpoint of line segment $\overline{FO}$, then polygon FS1TS2OP is a rectangle.



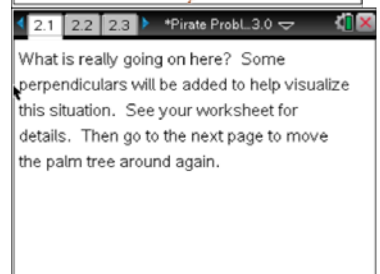
Page 1.6
When the palm tree lies on line segment $\overline{FO}$, then the polygon is a trapezoid.



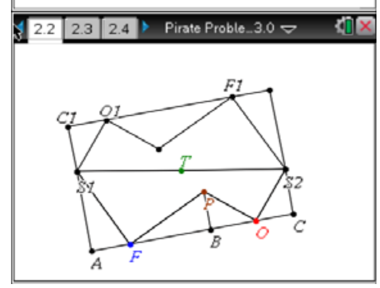

Page 1.7
When the palm tree is on the perpendicular bisector of segment $\overline{FO}$ so that S1 and S2 meet at T.




We need to investigate what is really going on here. The diagram on page 2.2 shows perpendiculars drawn from S1, S2, and P to segment FO. Then the polygon AS1FPOS2C is rotated 180° about T.

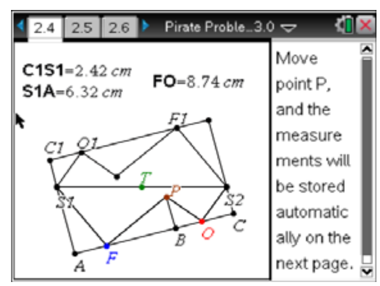



Then students move the palm tree, P, and record their observations on page 2.3.

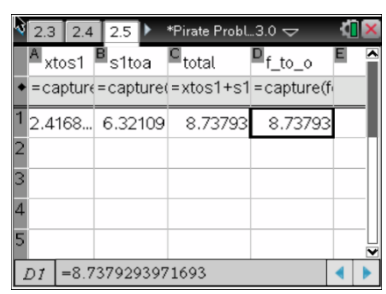



Possible answers: Students may observe that the rectangle stays a rectangle. They may observe that the height of the rectangle stays the same. They may repeat the observation that the treasure stays at the same place. Some might observe that the treasure is always the middle point of the rectangle.

Page 2.4 is set up to capture the measurements of the segments that make up the height of the rectangle and the measurement of the distance between the rocks. As the student moves point P, the spreadsheet is populated with data.




Students should be able to see that the height of the rectangle is the distance between the falcon and owl rocks. Thus their observations from the previous page should be confirmed inductively





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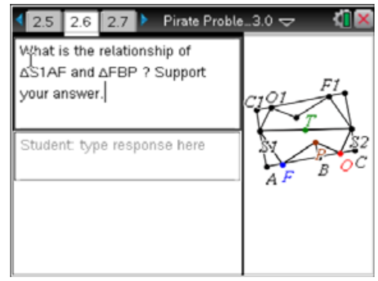
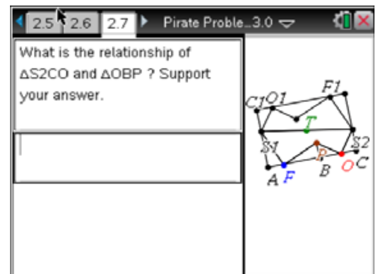
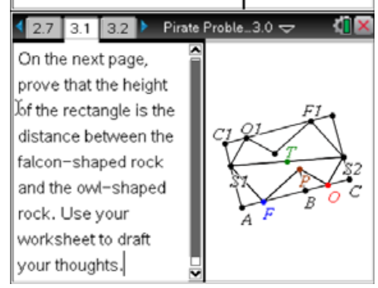
ΔS1AF and ΔFBP are congruent. We know that ∠AFS1 and ∠PFB are complementary (∠ AFS1 + ∠S1FP + ∠PFB = 180 and ∠S1FP = 90). We also know that ∠AFS1 and ∠AS1F are complementary since they are acute angles of a right triangle. That means ∠AS1F and ∠PFB are congruent, because the two pairs of angles share the same complement. Likewise, ∠S1FA is congruent to ∠FPB, because they are complements to the same angle. These triangles have equal hypotenuse lengths by construction. Thus ΔS1AF and ΔFBP are congruent using ASA congruence.



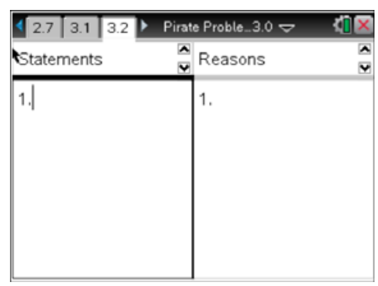

ΔS2CO and ΔOBP are congruent using the same reasoning as the previous page with the respective triangles




The proof could be similar to the following:




| | |
|---|---|
| ΔS1AF and ΔFBP are congruent. | ASA Postulate |
| Δ S2CO and ΔOBP are congruent | ASA Postulate |
| Δ S2CO and Δ S1C1O1 are congruent | Properties of rotation by construction |
| Δ S1C1O1 and ΔOBP are congruent | Transitive Property of Equality |
| C1S1 and BO are congruent | Corresponding parts of congruent triangles |
| S1A and FB are congruent | Corresponding parts of congruent triangles |
| Therefore, S1C1 + S1A = FB + BO | Substitution |
| C1S1 + S1A = C1A | Segment Addition Postulate |
| FB + BO = FO | Segment Addition Postulate |
| Therefore, C1A = FO | Substitution |




Since the height of the rectangle does not change, the midpoint of the midline of the rectangle stays the same. This means that the directions that the pirate wrote on the parchment in the 1800's will still lead to the treasure no matter where you find a palm tree on the island. As long as you follow the directions, you will find the treasure.

Addendum:
To construct the diagram with the perpendiculars on pages 2.2 and 2.4:

1. Press $\boxed{\text{menu}}$>Points&Lines>Line. Press $\boxed{\text{enter}}$. Select point F. Press $\boxed{\text{enter}}$. Select point O. Press $\boxed{\text{enter}}$.

2. Press [menu] >Construction >Perpendiculuar. press [enter]. Select point S1 and line FO to drop a perpendicular to line FO through point S1
3. Do the same to drop a perpendicular to line FO through point S2 and a perpendicular to line FO through point P.
4. [menu]>Points&Lines>Intersection Point(s). Place point A, B, and C at the respective intersections of the perpendicular line and line FO.
5. Label the points by right clicking on the points and select Label.
6. Press [menu] >Action>Hide/Show. Press [enter]. Hide the perpendiculars
7. [menu]>Shapes>Polygon. Press [enter]. Draw the polygon $AS_1FPOS_2C$. Press [enter]. Press [esc].
8. Press [menu] **>**Action>Text. . Type 180.
9. Press [menu] >Transformation>Rotation. press [enter]. Select the polygon that you constructed in step 15, select point T, and select the 180$^o$ and press [enter].