## Activity Overview

In this activity, students will use a temperature probe to generate a cooling curve and develop an appropriate regression equation to model collected data.

Topic: Cooling \& Heating Curves

- Exponential Regression
- Data Analysis
- Data Collection with Probeware


## Teacher Preparation and Notes

- This activity was designed for use with TI-Nspire technology, both CAS and non-CAS versions in conjunction with Vernier Easy Link temperature probes. Please note that an Easy Link adapter will be necessary if the probes to be used do not have a mini USB link.
- Problem 1 involves generating data using ice cubes and water that is slightly warm. Make certain that the cups or containers used are large enough to accommodate adding ice and stirring of contents. It will be necessary for students to work in pairs or small groups so that one may start the data sampling while another student drops ice into the water and stirs. Problem 2 is a related problem involving heating of water. If time allows, the teacher may choose to use hot plates and have students generate unique heating curves. Data is provided in the .tns file for this problem should time not allow for generation of data.
- To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "12146" in the quick search box.


## Associated Materials

- HowCoolltls_Student.doc
- HowCoolltts.tns
- HowCoolttls_Soln.tns


## Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- Cooling Rates (TI-Nspire technology) - 8546
- Half-Life (TI- Nspire technology) - 9288
- Chill Out (TI-Nspire technology) - 11323


## Problem 1 - Cooling Curve

Students are to drop 6 standard sized ice cubes into approximately 1 cup of tepid or lukewarm water.

It is recommended that the teacher test this to ahead of time to ensure that the ice cubes do not melt too quickly as temperature changes are to be recorded over a 2 minute period. The ice should last to the very end of the 2 minute period.

When on page 1.4, students can connect the probe to the TI-Nspire. (This will place the spreadsheet in the .tns file in the same place for all students.)

Selecting Lists \& Spreadsheet from the Auto Launch window will enable students to store the data gathered from the probe into the first two columns of the spreadsheet. The first column stores the times and the second column stores the temperature.

To change the unit from ${ }^{\circ} \mathrm{C}$ to ${ }^{\circ} \mathrm{F}$, press MENU > Sensors > Change Units.

On page 1.6, students are to create a scatter plot of the data by selecting dc01.time for the $x$-axis and dc01.temp1 for the $y$-axis. They are to add the Exponential Regression to the scatter plot. They should see that the equation does not fit the data very well.


Much exploration is done in the rest of the activity to obtain an equation that best models the given data. Students use regression models as well as sliders to adjust values of $a$, $b$, and $c$ in the development of an exponential equation of the form $f(x)=a b^{x}+c$.

Several questions are asked throughout the activity to guide students in the development of an equation to best model the generated data.
Discuss with students how the apparent horizontal asymptote relates to the exponential equation form $f(x)=a b^{x}+c$. Ask the students what limitations the TI-Nspire has in regard to the exponential regression performed. This will be helpful in providing clarification for the need to adjust the data in obtaining the second regression equation. The regression equation obtained with the TI-Nspire is of the form $f(x)=a b^{x}$. A vertical shift is not taken into account.

For the second regression equation, students will generate a second set of temperature data in the spreadsheet. In the formula bar of Column C, students are to subtract a value from dc01.temp1.

Students can then use page the Calculator application on page 1.12 to calculate and compare the exponential regressions of the original data and the adjusted data.


On page 1.15, students will graph the original data (NOT the adjusted data) as a scatter plot (MENU > Graph Type > Scatter Plot).

They will also graph 3 equations: (1) the original regression equation, (2) the adjusted data regression equation, and (3) the adjusted data regression equation + the value they subtracted in the spreadsheet.

In conclusion to the first part of the activity, students will use sliders to find their own regression equations. Discuss with students which equation best fits the data and why.

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## Problem 2 - Heating Curve

Students may either generate their own data or use the data provided in the TI-Nspire document. If students generate their own data, hot plates and glass cups will be needed. Again, students explore regression equations and apply the use of sliders to find an equation to best fit the given data.

This problem provides students with an opportunity to practice and apply what was learned in Problem 1.


## Solutions - Student Worksheet

1. exponential
2. Result depends upon student data (should be close to $\left.f(x)=20(0.99)^{x}\right)$.
3. no
4. adjust for the vertical shift, or apparent horizontal asymptote/test values for $a, b$, and $c$ in $f(x)=a b^{x}+c$
5. answer should be yes, for the solution data, this line appears to be near $y=12$
6. for the solution data, the equation is roughly $f(x)=36 \cdot 0.92^{x}+11.999$
7. Answers will vary, but the values of $r$ should be very close
8. The adjusted regression equation provides a much better fit visually.
9. test values for $a, b$, and $c$ in $f(x)=a b^{x}+c$
10. Students should that $f(x)=21 \cdot 0.94^{x}+12$ or an equation close to this one works well. If the first few seconds of data are disregarded, this equation provides a very good fit. All 3 regression equations are relatively close, but the $2^{\text {nd }}$ and $3^{\text {rd }}$ regression equations are closest and provide the best fitting models. The $2^{\text {nd }}$ and $3^{\text {rd }}$ equations only show a very significant difference in the value of a.
11. Our impatience seems to lead us to feel that drinks that are too hot cool slowly in reaching a drinkable temperature. The most dramatic temperature change occurs at the start of the experiment, when temperature is highest.
12. $f(x)=8.93716 \cdot 1.23764^{x}$
13. This equation seems to fit the data reasonably well
14. It appears that this graph may have a horizontal asymptote, possibly around $y=9$.
15. $f(x)=1.958 \cdot 1.43727^{x}+9$
16. For the first regression, $r \approx 0.99$, and for the second regression, $r \approx 0.94$, indicating that the first regression provides a better fit. This will vary depending upon the apparent horizontal asymptote chosen by the student for obtaining the second regression equation.
17. This answer may vary depending upon the apparent horizontal asymptote chosen. The work shown on the solution file indicates the best fit with the first regression equation.
18. $f(x)=8.6 \cdot 1.3^{x}+0$
