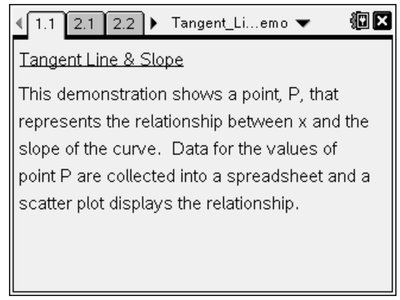


Math Objectives

- Students will make a connection between the slope of the tangent line at a point and the function that represents the slope at all tangent points to a function.

Activity Types

- Teacher Demonstration
- Student Exploration

About the Lesson

- Students will be able to make a connection between the slope of the tangent line at a point and the function that represents the slope at all tangent points to a function.

Directions

- This TI-NSpire demonstration shows a point, *P*, that represents the relationship between *x* and the slope of the curve. Data for the values of point *P* are collected into a spreadsheet and a scatter plot displays the relationship.
- Grab and move point *P* across the screen. Move to page 2.3 to examine the graph of the derivative.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point
- Clear data from spreadsheet column

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
Tangent_Line_Demo_Student.pdf
Tangent_Line_Demo_Student.doc

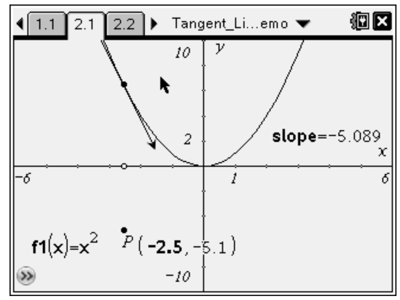
TI-Nspire document
Tangent_Line_Demo.tns

Visit www.mathnspired.com for lesson updates.

Discussion Points and Possible Answers

Move to page 2.1.

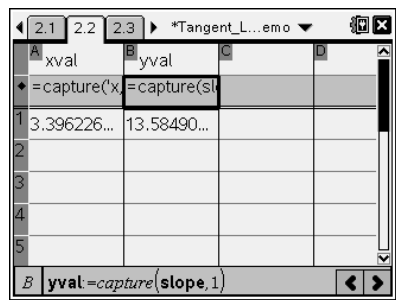
The students explore various functions by changing **f**(x) on the screen. The tangent line and slope are shown. Point P is labeled along with the coordinates. As students move the open circle on the x-axis, the tangent line and point P move.
The example **f**(x) = x^2 is shown at the right.



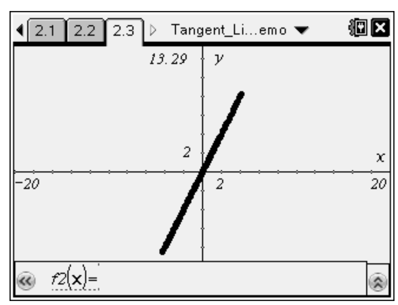
1. Can you predict what function point P is tracing?

   <u>Answer:</u> Linear function

Move to page 2.2.
This page automatically collects the x-value and the slope of the tangent line. The first two columns will fill as the student moves the empty circle along the x-axis.



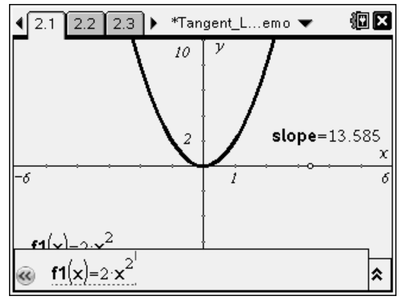
Move to page 2.3.
This graph will display a scatter plot of the points collected on page 2.2. Students can enter in the entry line a function they think the scatter plot represents. When students have the correct function entered, they have found the derivative function for the function from page 2.1.



2. What is the derivative function of **f**(x) = x^2?

   <u>Answer:</u> $\mathbf{f}'(x) = 2x$

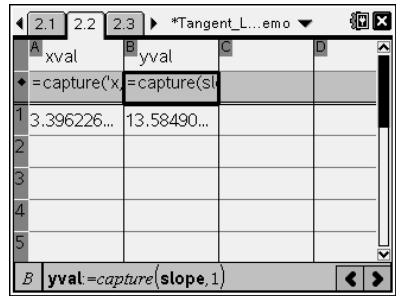
To repeat the process, the following steps will clear the collected data.



Step 1: Go to page 2.1 and change the function by clicking on double arrow in the lower left corner. Click in the entry line and press the up arrow to see **f**1(x) and edit the equation.

Step 2: Go to page 2.2 and highlight the formula cells in columns A and B with the word *capture* and then press (enter) twice.

Step 3: Return to page 2.1 and begin the lesson steps again.



Teacher Tip: Exploration 1 and 2 of the student worksheet asks student to explore variations of x^2 and x^3 to find their derivative functions.

You can return to this lesson file to have students explore non-polynomial functions such as sin(x), ln(x), e^x, and others.

Exploration 1: Now that you have found the derivative function for **f**(x) = x^2, explore some other variations of this function and see if you can find a pattern in their derivatives.

Record the derivative functions and any patterns you saw.

3. **f**(x) = ax^2, where a equals 2, 3, 4, etc., until you see a pattern.

<u>**Answer:**</u> $\mathbf{f}(x) = 2x^2;\ \mathbf{f}'(x) = 4x$

$\mathbf{f}(x) = 3x^2;\ \mathbf{f}'(x) = 6x$

$\mathbf{f}(x) = 4x^2;\ \mathbf{f}'(x) = 8x$

4. **f**(x) = $(x - a)^2$, where a equals 2, 3, 4, etc., until you see a pattern.

<u>**Answer:**</u> $\mathbf{f}(x) = (x-2)^2;\ \mathbf{f}'(x) = 2(x-2)$

$\mathbf{f}(x) = (x-3)^2;\ \mathbf{f}'(x) = 2(x-3)$

$\mathbf{f}(x) = (x-4)^2;\ \mathbf{f}'(x) = 2(x-4)$

5. **f**(x) = $ax^2 + b$; keep *a* constant and change *b.*

Sample answer: $\mathbf{f}(x) = 3x^2 + 1; \mathbf{f}'(x) = 6x$
$\mathbf{f}(x) = 3x^2 + 2; \mathbf{f}'(x) = 6x$
$\mathbf{f}(x) = 3x^2 + 3; \mathbf{f}'(x) = 6x$

Record the derivative functions and any patterns you saw here:

Answer: Patterns: Power Rule and the derivative of a constant is zero.

Exploration 2: Begin by finding the derivative function for **f**(x) = x^3.

6. What is the derivative function of **f**(x) = x^3?

Answer: $\mathbf{f}'(x) = 3x^2$

Now explore some other variations of this function and see if you can find a pattern in their derivatives.

7. **f**(x) = ax^3, *where a* equals 2, 3, 4, etc., until you see a pattern.

Answer: $\mathbf{f}(x) = 2x^3; \mathbf{f}'(x) = 6x^2$
$\mathbf{f}(x) = 3x^3; \mathbf{f}'(x) = 9x^2$
$\mathbf{f}(x) = 4x^3; \mathbf{f}'(x) = 12x^2$

8. **f**(x) = $(x – a)^3$, where *a* equals 2, 3, 4, etc., until you see a pattern.

Answer: $\mathbf{f}(x) = (x-2)^3; \mathbf{f}'(x) = 3(x-2)^2$
$\mathbf{f}(x) = (x-3)^3; \mathbf{f}'(x) = 3(x-3)^2$
$\mathbf{f}(x) = (x-4)^3; \mathbf{f}'(x) = 3(x-4)^2$

9. **f**(x) = $ax^3 + b$; keep *a* constant and change *b.*

Sample answer: $\mathbf{f}(x) = 2x^3 + 1; \mathbf{f}'(x) = 6x^2$
$\mathbf{f}(x) = 2x^3 + 2; \mathbf{f}'(x) = 6x^2$
$\mathbf{f}(x) = 2x^3 + 3; \mathbf{f}'(x) = 6x^2$

Record the derivative functions and any patterns you saw here:

Answer: Patterns: Power Rule and the derivative of a constant is zero.