The Mean Value Theorem

by – Vicki Carter

Activity overview

Students are presented with a several examples of functions to discover the hypotheses and conclusion of the Mean Value theorem. They will explore the concept of continuity and differentiability as related to the Mean Value Theorem.

Concepts

Parallel Lines
Secant Lines and Tangent Lines
Slopes of Tangent Lines
Continuity
Derivatives and Differentiability

Teacher preparation

This investigation could be used as an introduction to The Mean Value Theorem in calculus. Students should be familiar with the derivative as the slope of a tangent line.
Download the MVT.tns file.

Classroom management tips

This activity is intended to be ***student-centered*** *with the teacher acting as a facilitator while students work cooperatively. Students will answer the questions posed on the Q&A Notes pages.*

- *As all questions are posed in the .tns file, the intent of this activity is for the teacher to collect the document from the students at the conclusion of the activity. As an alternative, you may wish to have the class record their answers on a separate sheet of paper or simply use the questions posed to engage the students in a class discussion.*

TI-Nspire Applications

Graphs & Geometry, Lists & Spreadsheet, Notes, Notes with Q&A templates, Calculator with CAS

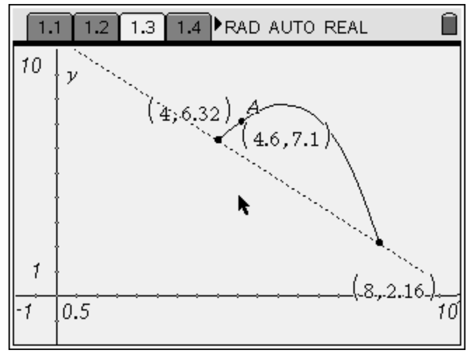
by: Vicki Carter
Grade level: 9-12
Subject: Calculus
Time required: 45 minutes
Materials: MVT.tns

Step-by-step directions

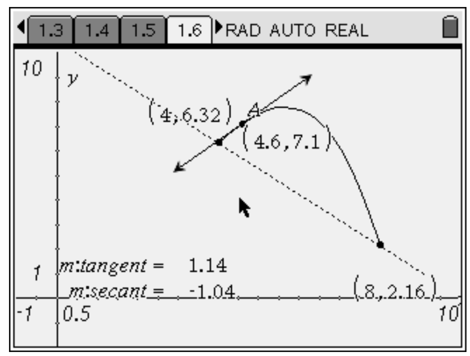
Investigating The Mean Value Theorem graphically and numerically

<u>**Problem 1**</u>

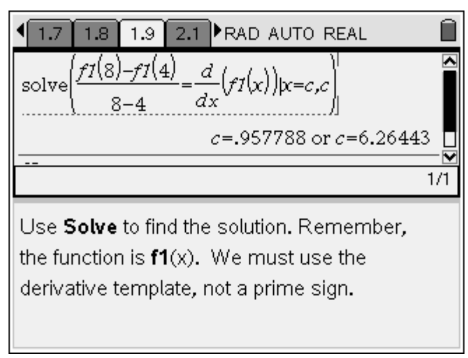
Step 1: On page 1.3, students should grab point A and approximate the position of A so that a tangent drawn to point A is parallel to the dotted secant line. The student will have additional visual references on page 1.6



Step 2: On page 1.6, students are instructed to again drag point A so that the slope of the tangent line is approximately 1.14 which is the slope of the secant line on [4, 8].

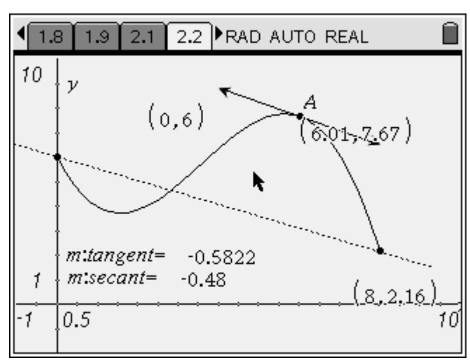


Step 3: Students may need some assistance in writing the equation to solve in order to find the value of *c* that satisfies the Mean Value Theorem. The calculation is shown to the right.

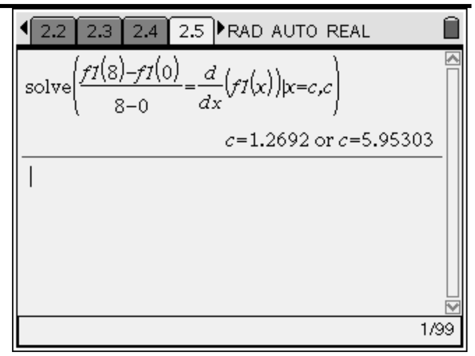


<u>**Problem 2**</u>

Step 4: On page 2,2, students should be able to drag point A so that there are two values of *c* for which the slope of the tangent is approximately -0.5822.

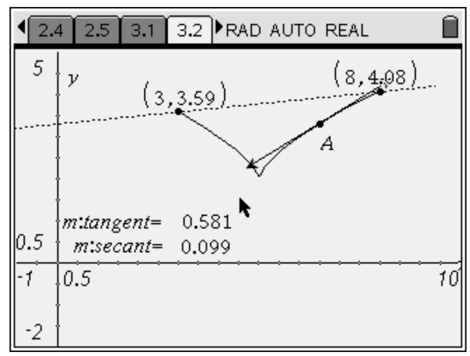


Step 5: Students may still need some assistance in writing the equation to solve in order to find the value of *c* that satisfies the Mean Value Theorem. The calculation is shown to the right.

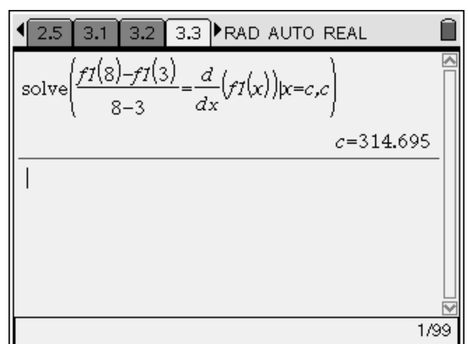


Problem 3

Step 6: On page 3.2, the students investigate a function that is not differentiable at $x = 5$. In the interval [3,8], the students will not be able to find a *c* to satisfy the Mean Value Theorem.

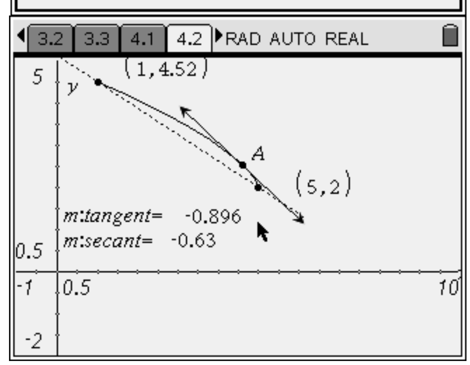


Step 7: You might consider having them insert a Calculator application to solve for *c*. The value found is not in the interval [3, 8].

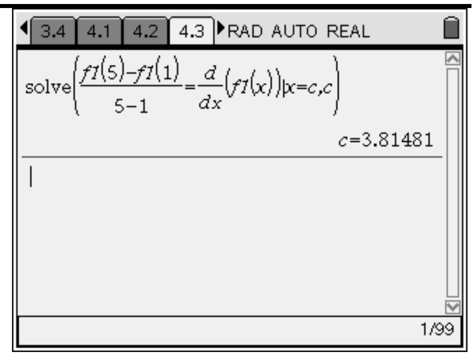


Problem 4

Step 8: On page 4.2, the students are investigating the same function but now we consider the interval [1, 5]. Remind the students that the function is not differentiable at the endpoint $x = 5$.

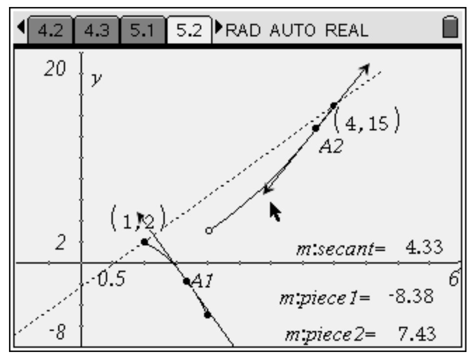


Step 9: Again consider having the students insert a calculator page to solve for the value of *c*.

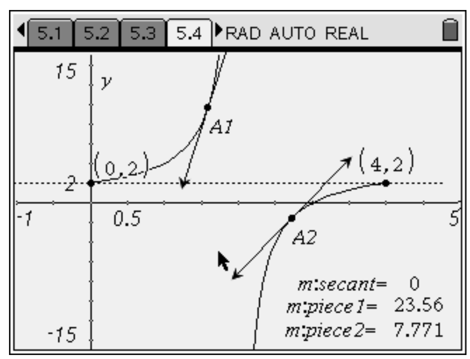


Problem 5

Step 10: On page 5.2 with the first piece-wise defined function, the students should be able to find a *c* in the piece defined from (2,4] where the slope of the tangent is approximately equal to the slope of the secant from *x* = 1 to *x* = 4.



Ste11: On page 5.4 with the second piece-wise defined function, the students will not be able to find a *c* where the slope of the tangent is 0 (the slope of the secant).



Assessment and evaluation

The teacher could collect the document from the students at the conclusion of the activity to check for understanding. As an alternative, you may wish to have the class record their answers on a separate sheet of paper or simply use the questions posed to engage the students in a class discussion.

Student TI-Nspire Document
MVT.tns

1.1 1.2 1.3 1.4 RAD AUTO REAL

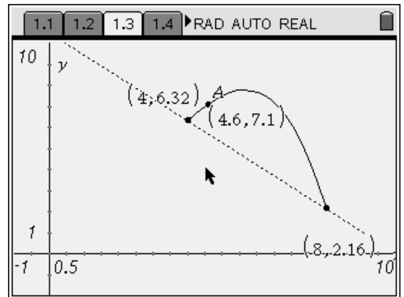
The Mean Value Theorem

AP Calculus

by: Vicki Carter

1.1 1.2 1.3 1.4 RAD AUTO REAL

On page 1.3, the graph of $f1(x)=-.12x^3+1.3x^2-3.2x+6$ is shown on the interval $4\le x\le 8$. The secant line through the endpoints has been drawn. There is a value $x=c$ between 4 and 8 at which the tangent to the graph is parallel to the secant line. Drag point A to approximate the value of c.



1.1 1.2 1.3 1.4 RAD AUTO REAL

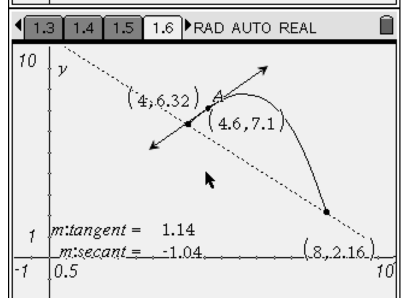
Question

What is your best estimate of the value of c for a tangent parallel to the secant?

Answer

1.2 1.3 1.4 1.5 RAD AUTO REAL

On page 1.6, the slopes of both the secant line and the tangent line are shown. Drag point A until the tangent is parallel to the secant.



1.4 1.5 1.6 1.7 RAD AUTO REAL

Question

What is your new value of c? How does this value compare to the one you found on page 1.3?

Answer

1.5 1.6 1.7 1.8 RAD AUTO REAL

Use the calculator page (page 1.9) to find the value of c where

$$\frac{f(8)-f(4)}{8-4}=f'(c)$$

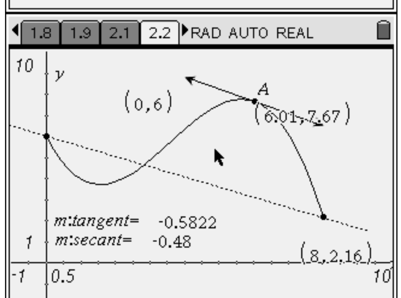
1.8 1.9 2.1 2.2 RAD AUTO REAL

0/99

Use **Solve** to find the solution. Remember, the function is **f1**(x). We must use the derivative template, not a prime sign.

1.8 1.9 2.1 2.2 RAD AUTO REAL

On page 2.2, the graph $f1(x)=-.12x^3+1.3x^2-3.2x+6$ is shown on the interval $0\le x\le 8$. The secant line through the endpoints has been drawn. Investigate the tangent line. Is it possible to have more than one value of c where the tangent is parallel to the secant line?



1.9 2.1 2.2 2.3 RAD AUTO REAL

Question

Approximately what are the two values of c?

Answer

2.1 2.2 2.3 2.4 RAD AUTO REAL

Use the calculator page (page 2.5) to find the value of c where

$$\frac{f(8)-f(0)}{8-0}=f'(c)$$

2.2 2.3 2.4 2.5 RAD AUTO REAL

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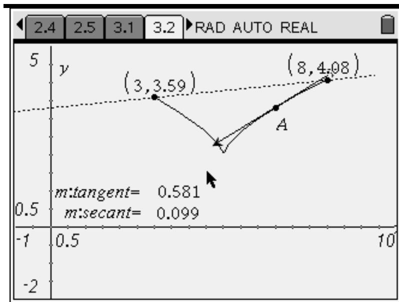
2.3 2.4 2.5 3.1 RAD AUTO REAL

On page 3.2, $f1(x)=2+(x-5)^{\frac{2}{3}}$ is shown on the interval $3\le x\le 8$. The secant line through the endpoints has been drawn. How many times between 3 and 8 is the tangent parallel to the secant?



2.5 3.1 3.2 3.3 RAD AUTO REAL

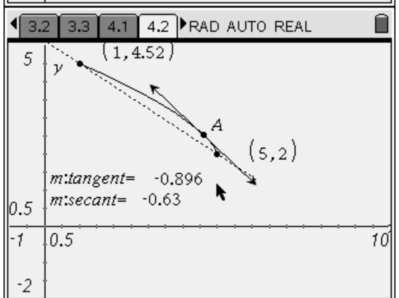
Question

Did you find a point where $\frac{f(8)-f(3)}{8-3}=f'(c)$? Why or why not?

Answer

3.1 3.2 3.3 4.1 RAD AUTO REAL

The function $\mathbf{f1}(x)=2+(x-5)^{\frac{2}{3}}$ from the previous problem is not differentiable at $x=5$. On the interval from 1 to 5, is there a tangent line parallel to the secant line? Use the graph on page 4.2 to investigate.



3.3 4.1 4.2 4.3 RAD AUTO REAL

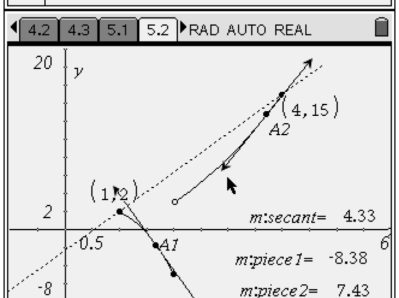
Question

The function is not differentiable at $x=5$ but there is a value of c between 1 and 5 where the tangent is parallel to the secant line. Why?

Answer

4.1 4.2 4.3 5.1 RAD AUTO REAL

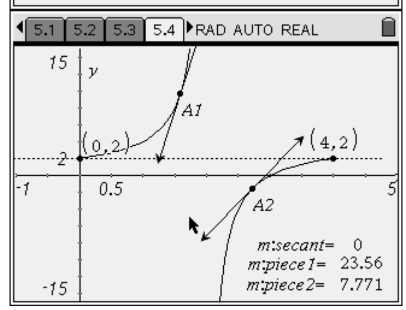
A piece–wise function is shown on page 5.2 On the interval from 1 to 4, is there a tangent line parallel to the secant line? Use the graph on page 5.2 to investigate.



4.3 5.1 5.2 5.3 RAD AUTO REAL

The function is discontinuous at $x=2$ yet there is still an $x=c$ where $\frac{f(4)-f(1)}{4-1}=f'(c)$.

Another graph with a discontinuity at $x=2$ is shown on page 5.4. On the interval from 0 to 4, is there a tangent line parallel to the secant line?



5.2 5.3 5.4 5.5 RAD AUTO REAL

If a function, $f(x)$, is continuous on $[a,b]$ and differentiable on (a,b), then there exists a c in (a,b) such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}.$$

With these hypotheses of continuity and differentiability, the derivative MUST assume a certain mean value.

5.3 5.4 5.5 5.6 RAD AUTO REAL

As seen from the examples, differentiability and continuity are not necessary conditions for the derivative to assume a certain mean value in a closed interval. Differentiability and continuity, though, guarantee that the derivative assumes a certain mean value in a closed interval.

