Examining the Capabilities of TI InterActive!® and the TI-92 To Teach Programming Concepts:

A Mathematical Review of Transformations

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T^3 Conference

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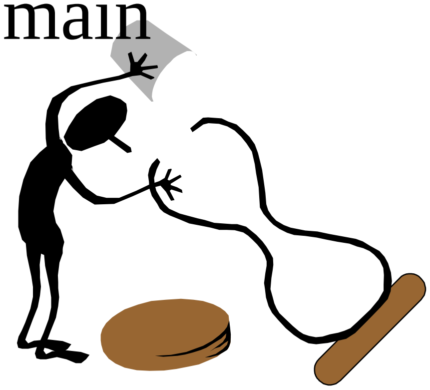
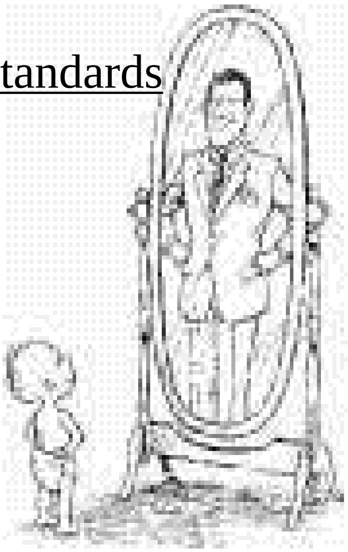
Objectives and Timeline

- Introduction (3 min.)
  - A brief history of transformational geometry
  - Its place in the curriculum
- Quick Demonstration (7 min.)
  - A demonstration of a completed TI-92 program and a similar Interactive page designed to enhance students' understanding of transformations through exploration and programming.
  - Flowchart discussion
- Build TI-Interactive pages that mimic the main capabilities and learning opportunities of programming on the TI-92. (40 min.)



A Quick History: Transformational Geometry in the Curriculum

- 1965
  - Secondary School Mathematics Curriculum Improvement Study (SSMCIS)
    - Suggested the inclusion of Affine, Vector, and Coordinate geometry
- 1967
  - Association for Supervision and Curriculum Development (ASCD)
    - Include motion geometry in the lower grades
- 1988
  - National Council of Supervisors of Mathematics (NCSM), Twelve Components of Essential Mathematics
    - Include transformations; **slides**, **flips**, and **turns**
- 2000
  - National Council of Teachers of Mathematics (NCTM), Principles and Standards
    - Middle-grades students should have had experience with such basic geometric transformations as **translations**, **reflections**, **rotations**, and **dilations**. In high school they will learn to represent these transformations with matrices, exploring the properties of the transformations using both graph paper and dynamic geometry tools. (p. 314)



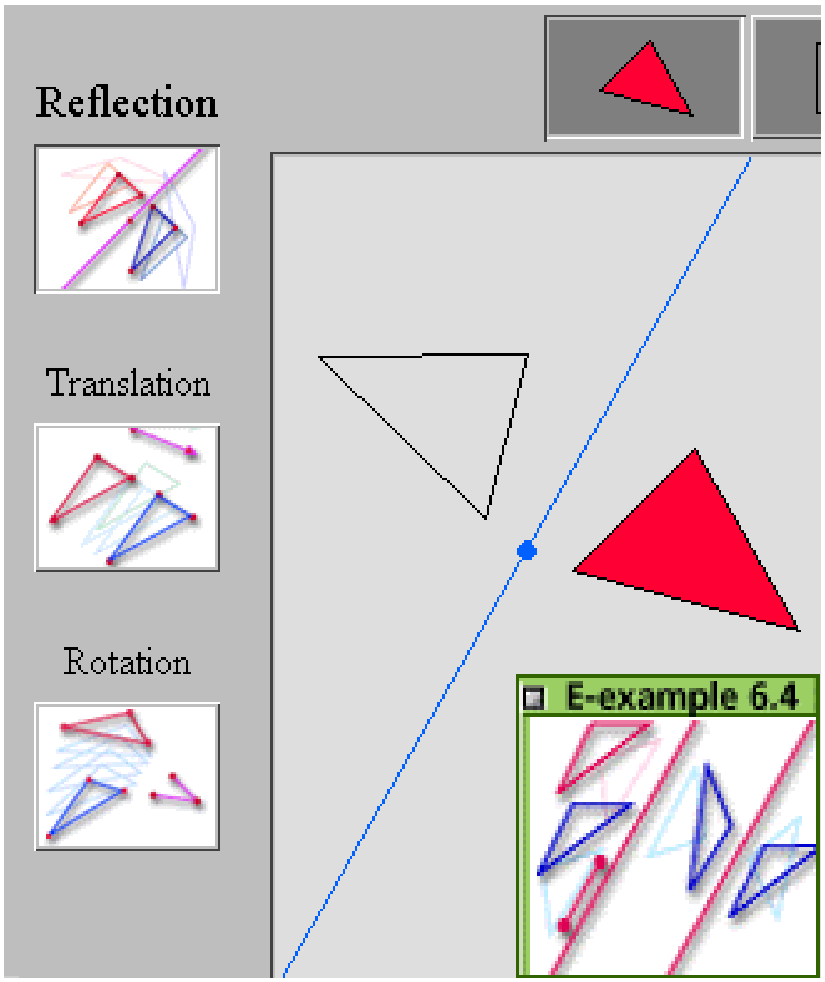
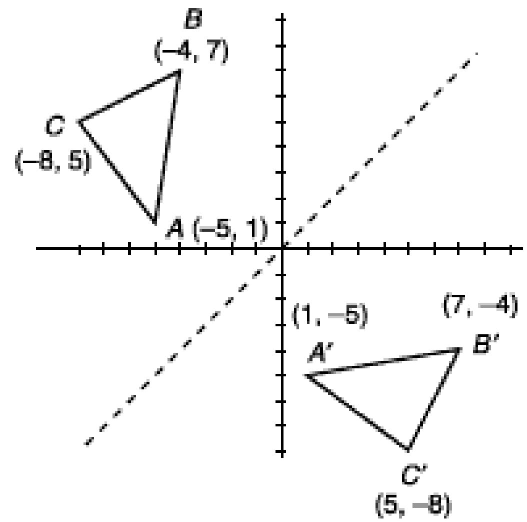


Fig. **7.17.** Representing a reflection using a matrix



Consider a triangle ABC with vertices $A = (-5, 1)$, $B = (-4, 7)$, and $C = (-8, 5)$.

Reflect the triangle over the line $y = x$ to obtain the triangle $A'B'C'$ as shown.

Determine a matrix M such that $MA = A'$, $MB = B'$, and $MC = C'$, where the points are represented as vectors.

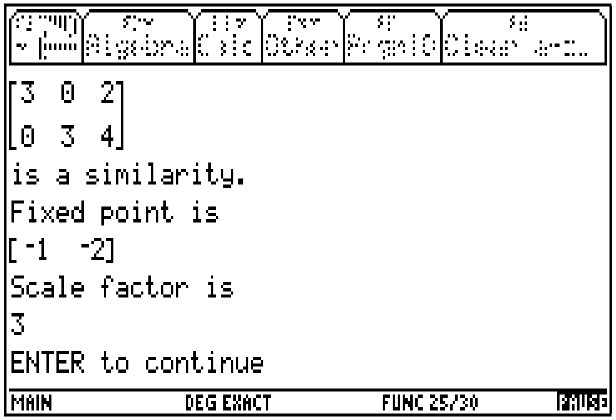
Explore the properties of the matrix M.

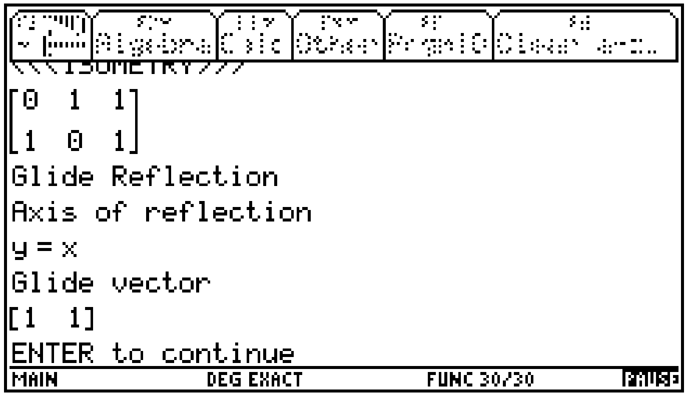
The completed TI-92: program Demo

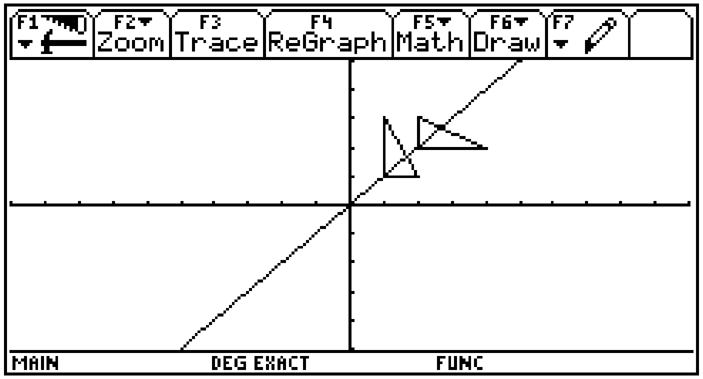
- **GRAFFINE()**

```
()
Prgm
¨Set modes
setMode("Angle","Degree")
setMode("Pretty Print","On")
setMode("Exact/Approx","Exact")
setMode("Split Screen","Full")
¨End setmode

¨Start Info Screens
ClrIO
Disp "This program will produce information"
Disp "about affine transformations of"
Disp "the form "
Disp ""
Disp "x≐ ax + by + h"
Disp "y≐ cx + dy + k"
Pause "ENTER to continue."
ClrIO
```





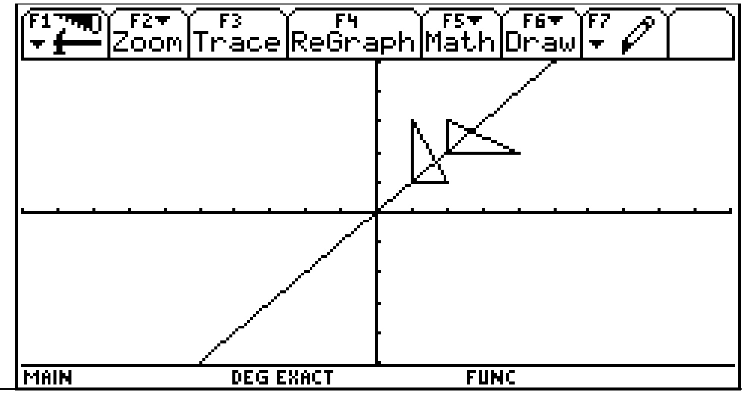


Equations:	Description:	Picture:

$x' = ax + by + h$
$y' = cx + dy + k$

$x' = 0x + 1y + 1$
$y' = 1x + 0y + 1$

Glide Reflection
Axis of reflection
y=x
Glide vector
[1,1]



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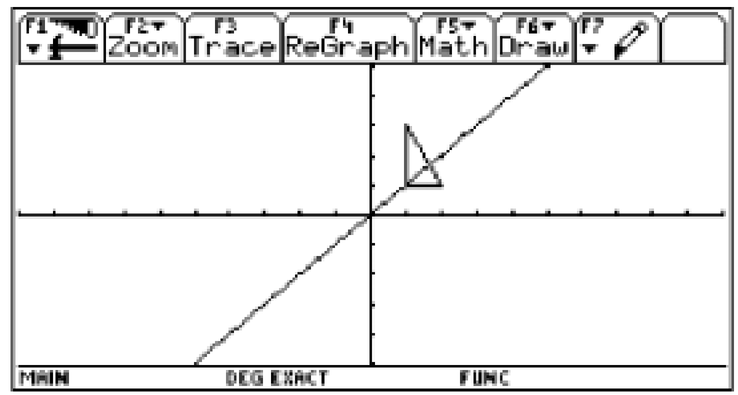

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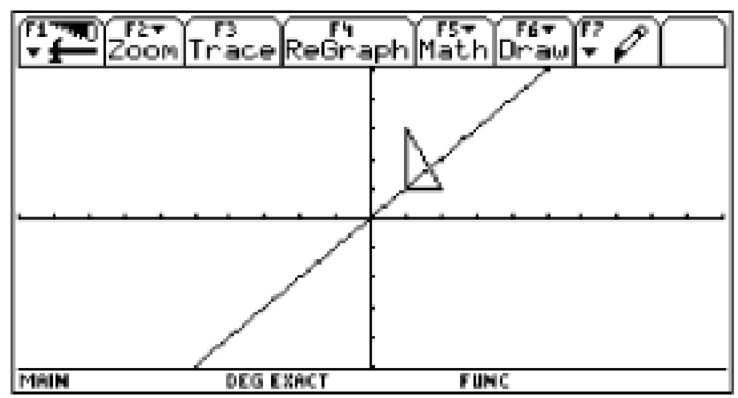
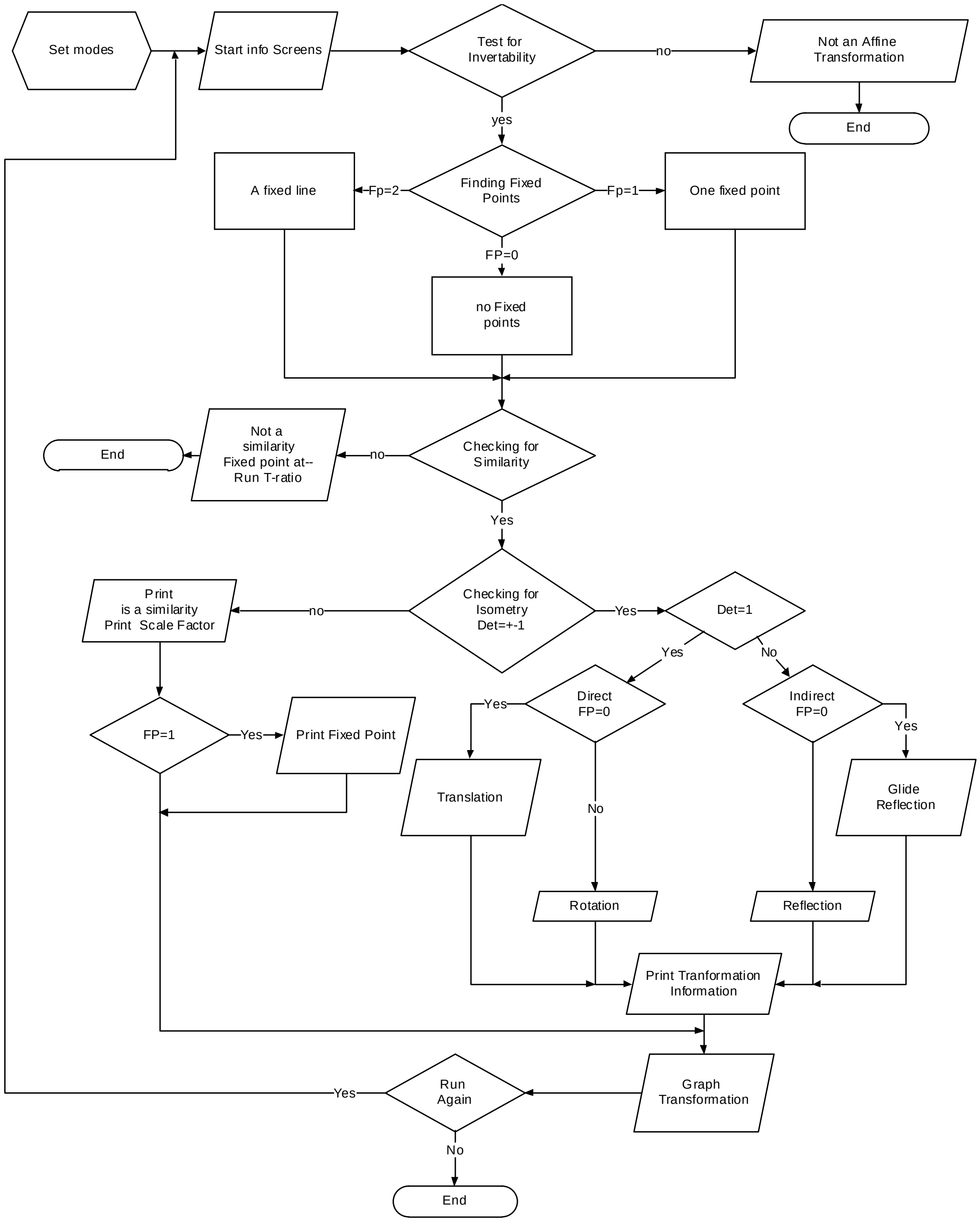


Flowchart


Set modes
Start info Screens
Test for Invertability
no
Not an Affine Transformation
End
yes
A fixed line
Fp=2
Finding Fixed Points
Fp=1
One fixed point
FP=0
no Fixed points
Not a similarity Fixed point at-- Run T-ratio
End
no
Checking for Similarity
Yes
Print is a similarity Print Scale Factor
no
Checking for Isometry Det=+-1
Yes
Det=1
Yes
No
Direct FP=0
Yes
Indirect FP=0
Yes
FP=1
Yes
Print Fixed Point
Translation
No
Glide Reflection
Rotation
Reflection
Print Tranformation Information
Run Again
Yes
Graph Transformation
No
End

A Similar Interactive Program

A quick demonstration.

Affine Transformations Change values by double clicking on them.

$0 \rightarrow a \quad 1 \rightarrow b \quad 3 \rightarrow c$

$1 \rightarrow d \quad 0 \rightarrow e \quad 4 \rightarrow f$

Input preimage points (x^*, y^*).

$0 \rightarrow x1 \quad 0 \rightarrow y1 \quad 5 \rightarrow x2 \quad 0 \rightarrow y2$

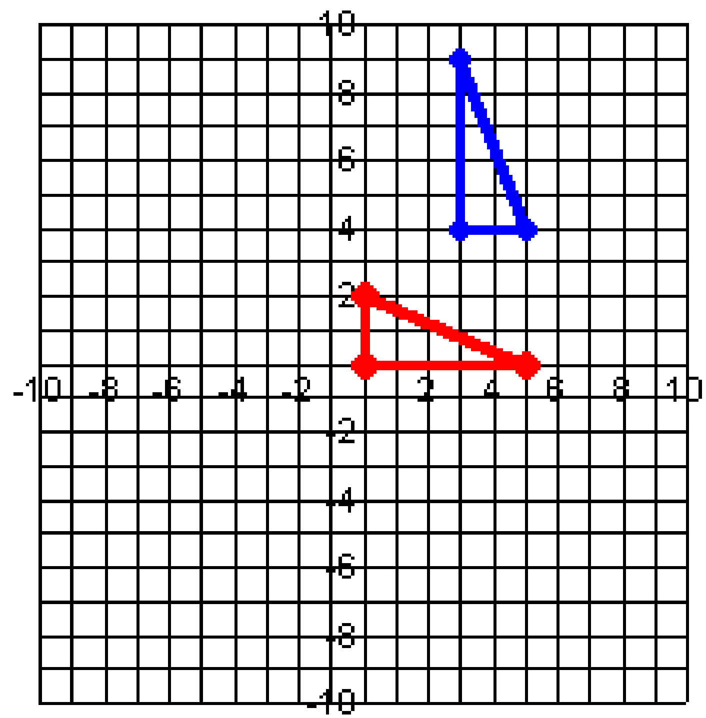
$0 \rightarrow x3 \quad 2 \rightarrow y3$

Transformation equations and new values:

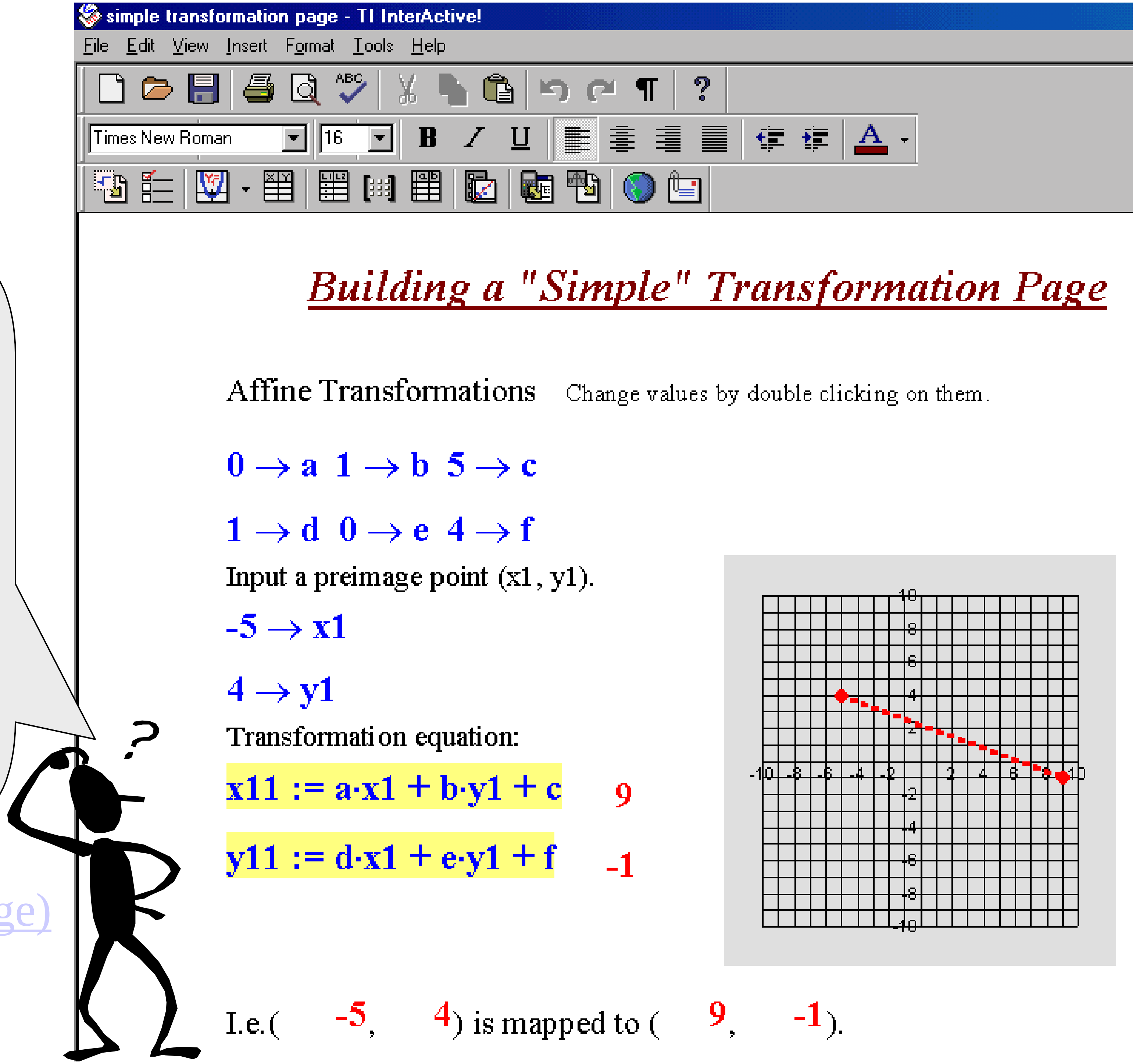
$x11 := a \cdot x1 + b \cdot y1 + c$ 3 $\quad x22 := a \cdot x2 + b \cdot y2 + c$ 3

$y11 := d \cdot x1 + e \cdot y1 + f$ 4 $\quad y22 := d \cdot x2 + e \cdot y2 + f$ 9

$y33 := d \cdot x3 + e \cdot y3 + f$ 4 $\quad x33 := a \cdot x3 + b \cdot y3 + c$ 5



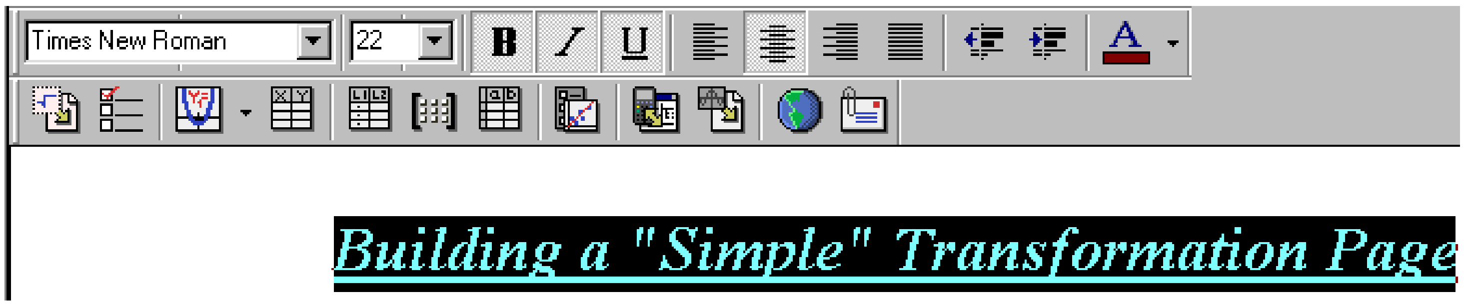
Our goal: To produce this and similar pages. First, lets see how this page works.



(Demo the page)

Lets get started!

- Open **TI InterActive!**®
  - Type
    - ***Building a "Simple" Transformation Page***

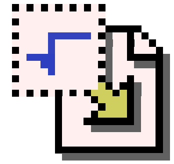


– Note: Font, Size, Bold, Italic, Underline, & Center. Press return

- Affine Transformations   Change values by double clicking on them.

(Font size 16)   (Font size 12)

- Simple word processor

Using the Math Box  Upper Left Corner

→ store to variable

$0 \rightarrow a \quad 1 \rightarrow b \quad 5 \rightarrow c$

$1 \rightarrow d \quad 0 \rightarrow e \quad 4 \rightarrow f$

Press return after each entry. Use the backspace key to get the proper alignment.

Plain word processing.

Input a preimage point (x1 , y1).

Math Box

$-5 \rightarrow x1$

$4 \rightarrow y1$

$\rightarrow$

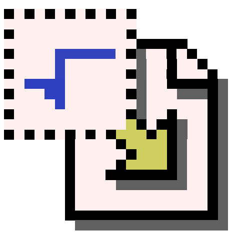
:= variable assignment

Standard Text 14p

Transformation equation:

$$x11 := a \cdot x1 + b \cdot y1 + c \qquad 9$$

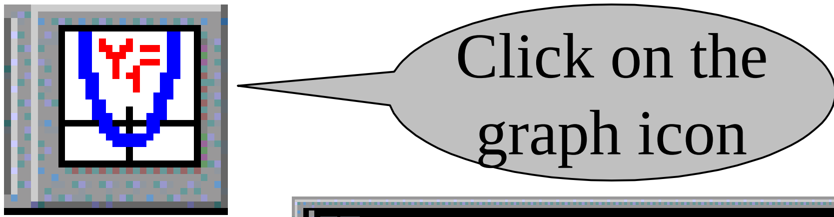
$$y11 := d \cdot x1 + e \cdot y1 + f \qquad -1$$

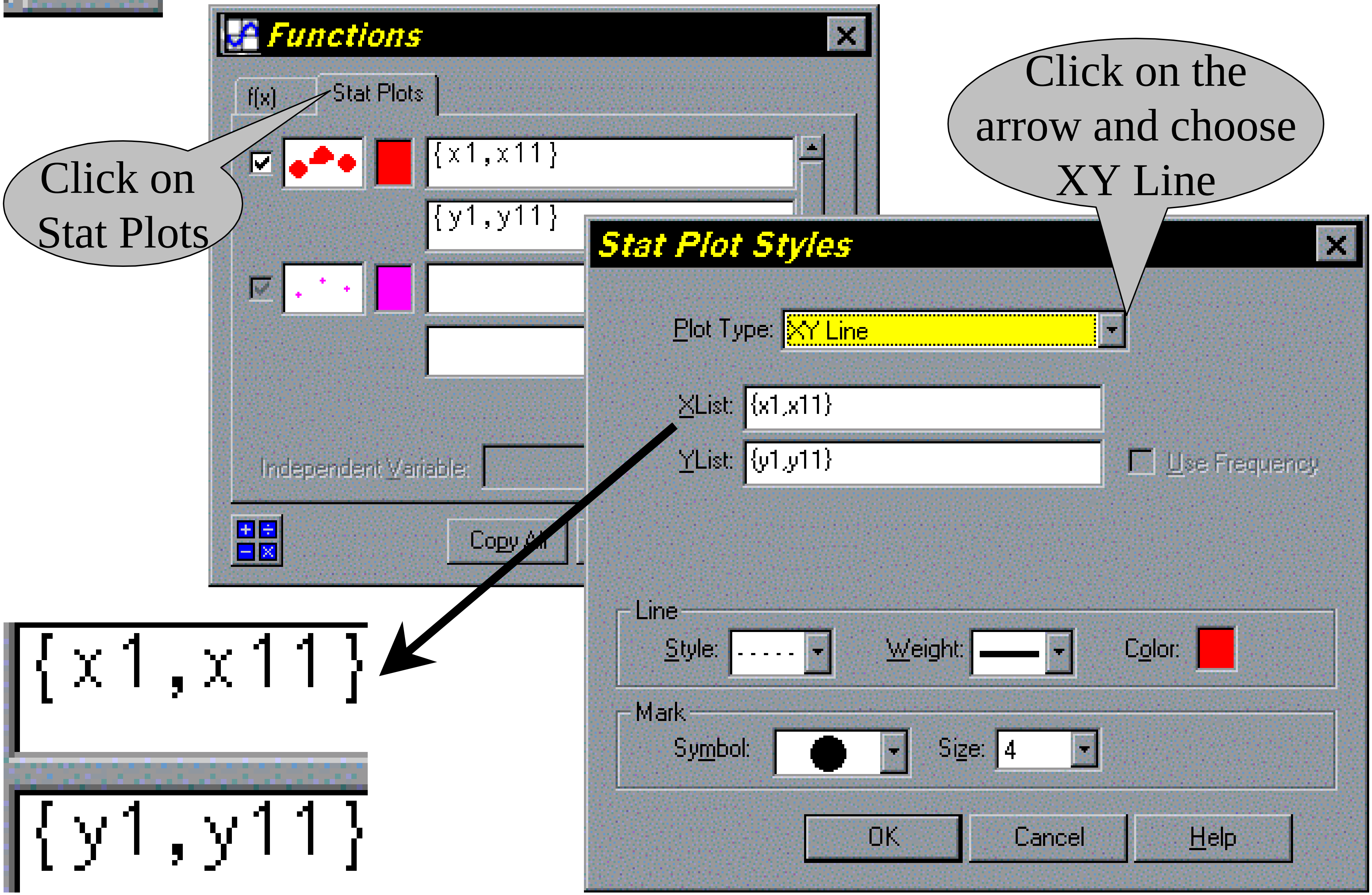


Use the Math Box for this entire line.

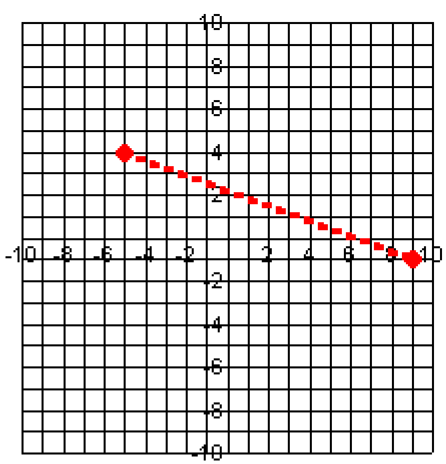
:=

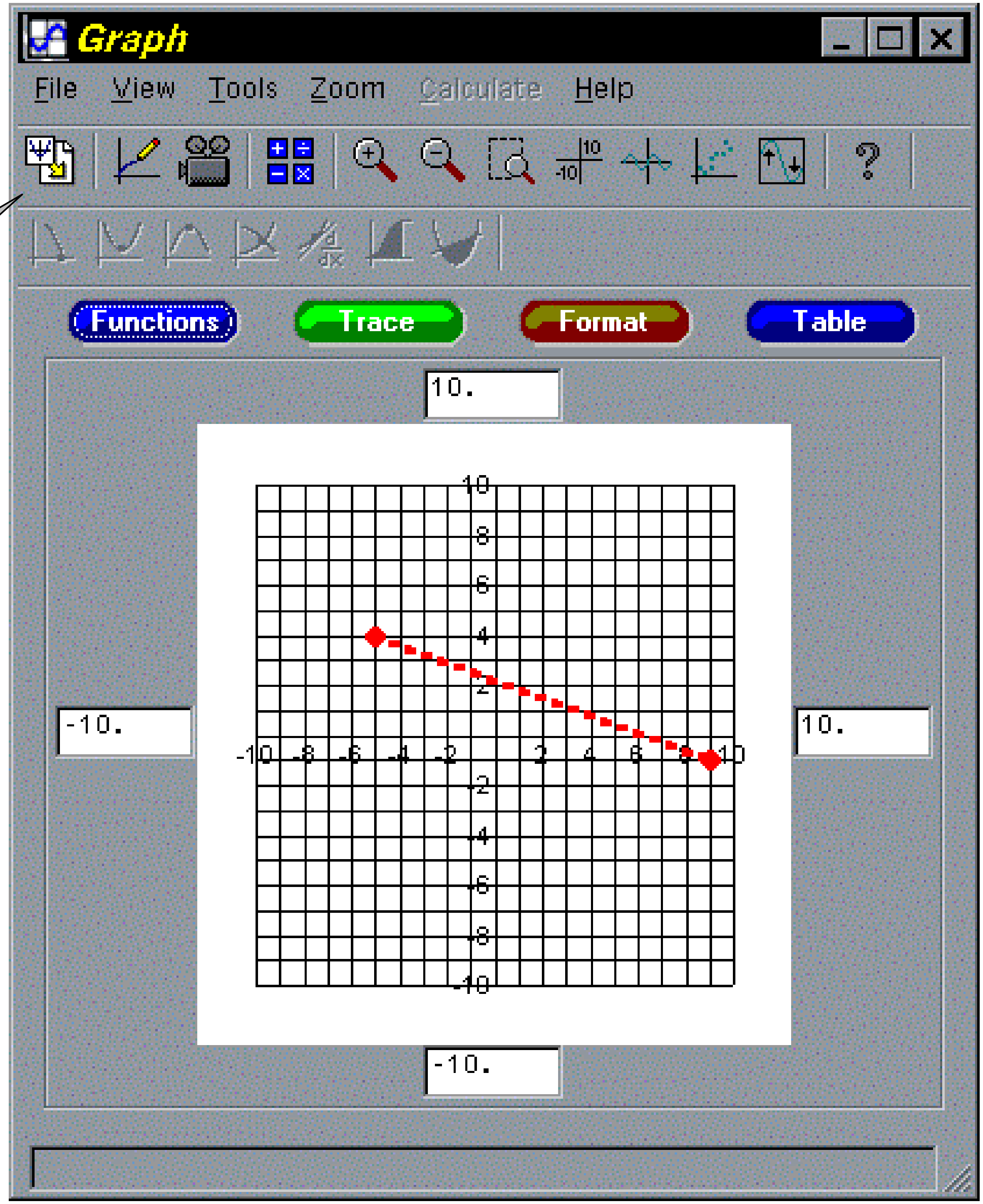
Creating the Graph



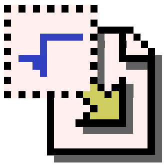


Click on
Save to Document





Integrated Word Processing



Use the math box for the blue text

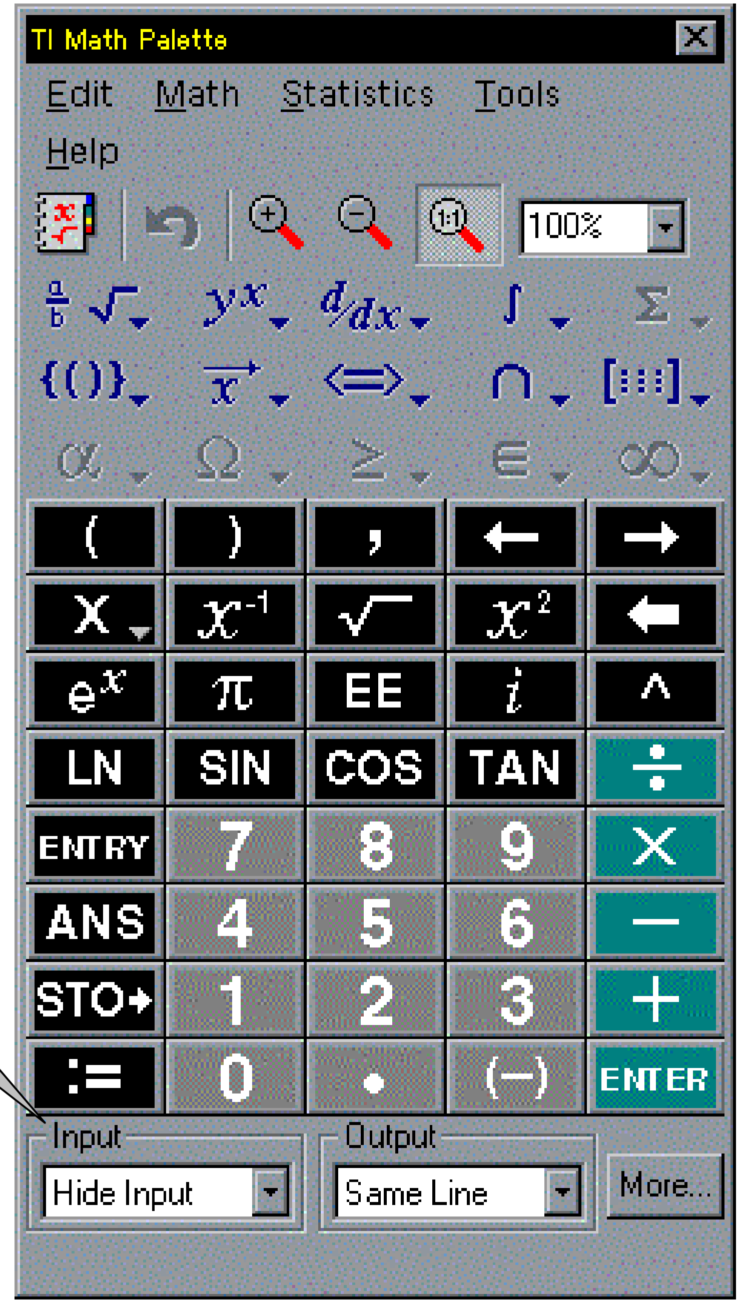
I.e.(x1 -5, y1 4) is mapped

(x11 9, y11 -1).

Click on Hide Input

I.e.(-5, 4) is mapped to

(9, -1).



Extensions, Building the Complete Transformation Page

Affine Transformations

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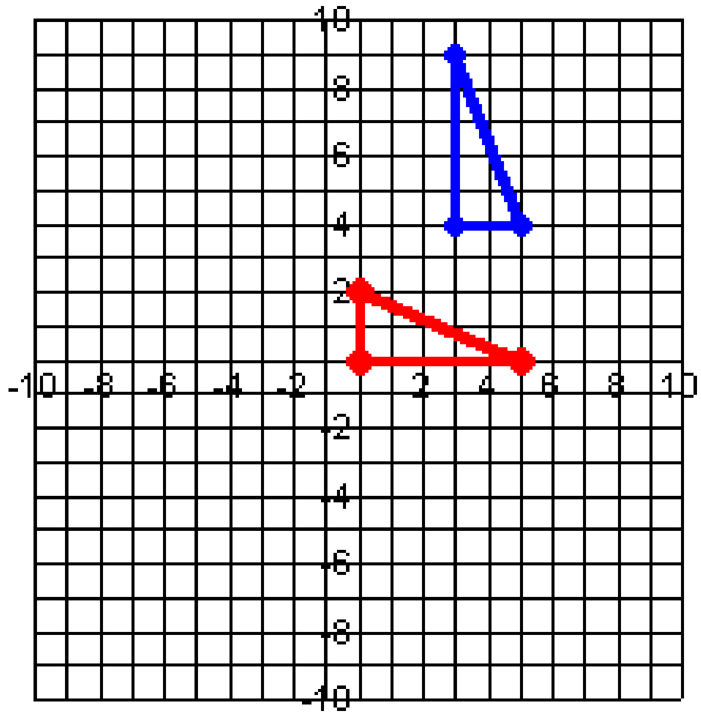

Transformation: Standard Equations

- Most General Linear Transformation (**Projective**)

$$x' = \frac{a_1 x + b_1 y + c_1}{a_3 x + b_3 y + c_3} \quad y' = \frac{a_2 x + b_2 y + c_2}{a_3 x + b_3 y + c_3} \quad where \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$$

- **Affine**, specialized to the case $a_3=b_3=0$ and $c_3=1$

$$x' = a_1 x + b_1 y + c_1 \quad y' = a_2 x + b_2 y + c_2 \quad where \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$$

- **Euclidean**
  - Affine with $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \pm 1$

Affine and Euclidean Standard Forms (Simplified)

- Sheer

$$x' = a_1 x \_ \; y' = b_2 y$$

- Dilation

$$x' = kx \_ \; y' = ky$$

- Translation

$$x' = x + c_1 \_ \; y' = y + c_2$$

- Rotation

$$x' = x\cos\alpha - y\sin\alpha \_ \; y' = x\sin\alpha + y\cos\alpha$$

- Reflection

$$x' = x\cos 2\alpha + y\sin 2\alpha \_ \; y' = x\sin 2\alpha - y\cos 2\alpha$$

the line of reflection is y=tanα

- Bibliography

  - Ayres, Frank. (1967). Theory and Problems of Projective Geometry. Schaum Publishing Co. New York.
  - Horadam, A. F. (1970) A Guide to Undergraduate Projective Geometry. Pergamon Press, Australia.
  - Kay, David (1960). College Geometry. Holt, Rinehart and Winston, INC. New York.
  - Allendoerfer, Carl & Oakley, Cletus. (1965) Fundamentals of Freshman Mathematics. McGraw-Hill, Inc. New York.

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  - T^3 presentation
  - TI-92 program
    - Word document
    - TI-92 program format
    - Flowchart
  - Interactive pages