Thursday Night Precalculus
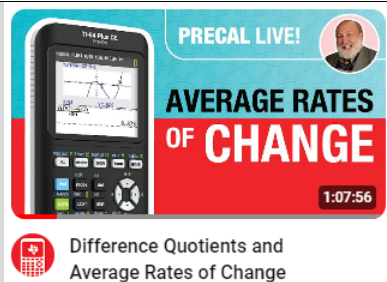**November 30, 2023**
In this AP Precalculus Live session, we will explore average rates of change.

Difference Quotients and Average Rates of Change

About the Lesson

- This Teacher Notes guide is designed to be used in conjunction with the AP Precalculus Live session and Student Problems document that can be found on-demand:
  - https://www.youtube.com/live/hwPPPgbyrms?si=8kk18sB-jH53hWLQ
  - *Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the "Live Session" but can be enriched by that resource.*
- This session involves difference quotients and average rates of change.
- Students should be able to use the TI-Nspire to visualize secant lines and to calculate the slopes of secant lines.

- ✖ **Class Discussion**: Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

Materials:

TI-Nspire document
- AROC.tns
Student document
- problems_11_30_23
Solutions
- problems_solutions_11_30_23
YouTube
- https://www.youtube.com/live/hwPPPgbyrms?si=8kk18sB-jH53hWLQ
- ***Documents and materials can be downloaded from this site.***

AP Precalculus Learning Objectives

- 1.2.A: Compare the rates of change at two points using average rates of change near the points.
- 1.3.A: Determine the average rates of change for sequences and functions, including linear, quadratic, and other function types.
- 1.3.B: Determine the change in the average rates of change for linear, quadratic, and other function types.
- 1.4.A: Identify key characteristics of polynomial functions related to rates of change.

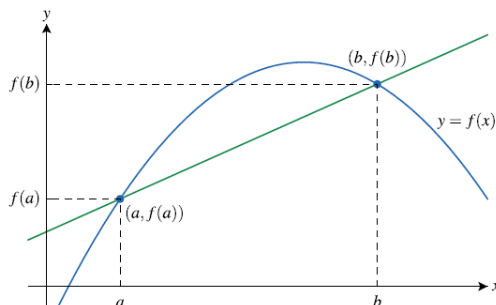*Source: AP Precalculus Course and Exam Description, The College Board*

Introduction – A Geometric Interpretation of Average Rate of Change

Average Rate of Change is the change in y divided by the change in x.

Average Rate of Change $= \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{f(b) - f(a)}{b - a}$

The average rate of change finds the slope of a secant line from $(a, f(a))$ to $(b, f(b))$.

Using the TI-Nspire to demonstrate the geometric interpretation.
In the video, the demonstration uses the TI-84. The "Using the TI-Nspire" notes in this document provide directions and screen shots that are similar to the use of the TI-84.

Graph $f1(x) = 3x - 1 - \dfrac{x^2}{2}$. To display a secant line, select menu

Geometry>Points & Lines>Segment. Make sure the prompt says "point on" to assure that the points are on the graph of the function.
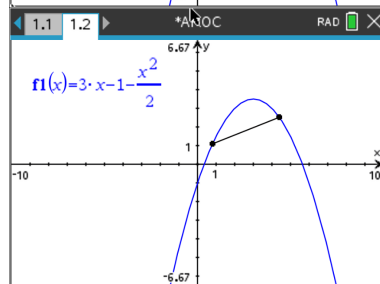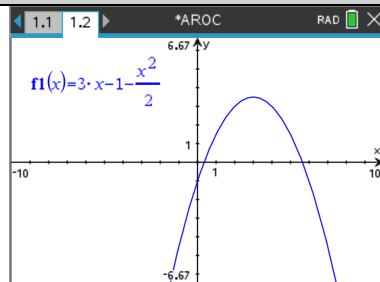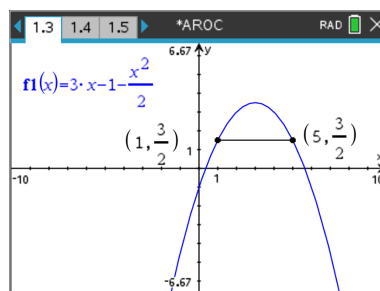

Suppose we want to graph the secant line from $x = 1$ to $x = 5$.

Add a new page, ctrl doc▾. Select Add Graphs and move up to f1(x) and enter.

Select menu Geometry>Points & Lines>Point by Coordinates. Enter $(1, f1(1))$. Repeat with $(5, f1(5))$.

Since the y-coordinates are equal, the average rate of change is 0. It is the slope of the horizontal line.

Technology Tip: Storing different values for a and b gives a quick way to visualize the secant line.

To use stored values, change the points to $\left(a, f1(a)\right)$ and $\left(b, f1(b)\right)$.

On the calculator application page, type $\dfrac{f1(b) - f1(a)}{b - a}$ to find the slope of the secant line.

Technology Tip: To create a fraction template, select ctrl ÷.

Technology Tip: On the graphing application page, select menu Geometry>Measurement>Slope. Select the Segment to find the slope of the secant line.
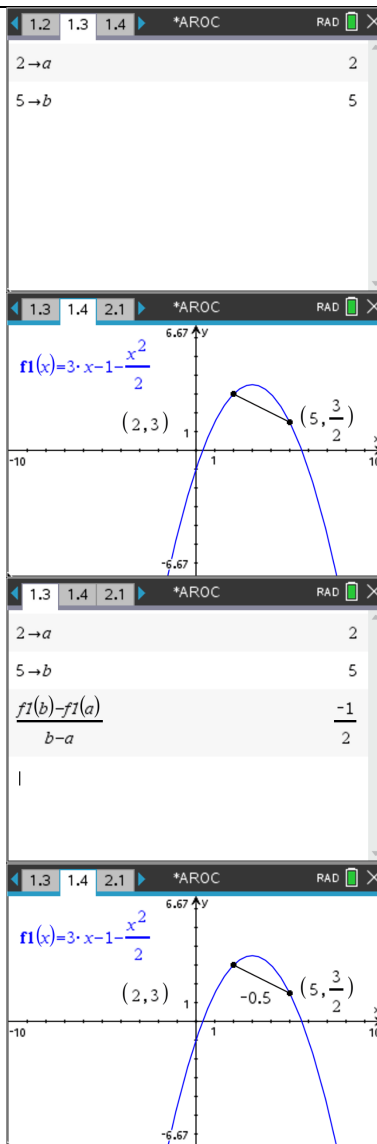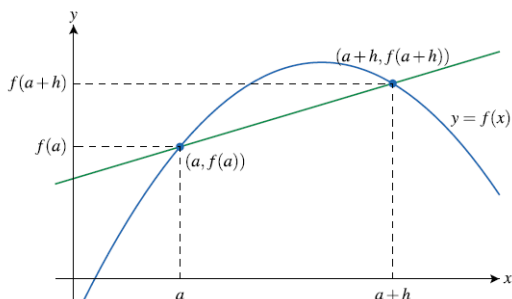
Introduction – A Geometric Interpretation of a Difference Quotient

Difference Quotient is the measure of the average rate of change of a function over an interval of length h. We think of h as a small value.

Difference Quotient $= \dfrac{f(a+h) - f(a)}{h}$

A more general difference quotient in terms of x.

Difference Quotient $= \dfrac{f(x+h) - f(x)}{h}$

Teacher Note: The form of the difference quotient used most often in AP Precalculus is

Average Rate of Change $= \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{f(b) - f(a)}{b - a}$. The form used in Problem 1 is a

preview of the development of a derivative in AP Calculus.

Problem 1. (a) – (d)

For each of the following functions, simplify the expression $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$ as far as

possible.

(a) $f(x) = 3x^2 - 5x$

Sample Solution:

   Refer to the Teacher Solutions Document for the full solution to this problem.

📌 **Class Discussion:**

What is the average rate of change for consecutive equal-length intervals? Do you see a
pattern in the given table?

Interval	Difference Quotient
$[a, a+h]$	$6a + 3h - 5$
$[a+h, a+2h]$	$6a + 9h - 5$
$[a+2h, a+3h]$	$6a + 15h - 5$
$[a+3h, a+4h]$	

Possible Answers: The difference quotient for the interval $[a+3h, a+4h]$ is $6a + 21h - 5$.

The pattern is to add $6h$ for each consecutive equal-length interval.

(b) $f(x) = \sqrt{x^2 - 1}$

Sample Solution:

   Refer to the Teacher Solutions Document for the full solution to this problem.

(c) $f(x) = \dfrac{1}{x^2}$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

(d) $f(x) = \dfrac{x}{1 + x^2}$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 2

The number of pounds (in millions) of lobster caught by Maine commercial fisherman is given by $P(t)$, where t is measured in years. Selected values for $P(t)$ are given in the table.

t	2010	2012	2014	2016	2018	2020	2022
$P(t)$	255.8	318.3	306.9	302.9	285.8	205.1	107.1

Find the average rate of change in pounds of lobsters caught (**i**) from 2016 to 2018; (**ii**) from 2018 to 2020.

Indicate units of measure. *What do your answers suggest about the change in the number of pounds of lobster caught in recent years?*

Sample Solution:

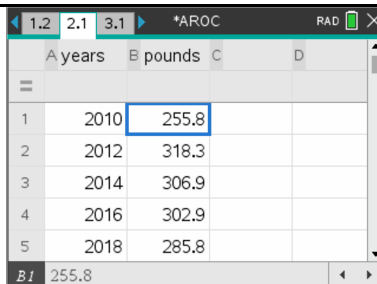Refer to the Teacher Solutions Document for the full solution to this problem.

Teacher Note: We encourage students and teachers to avoid abbreviations, including AROC for average rate of change. Students should present both a difference and a quotient when finding an average rate of change. Either of these expressions would be acceptable for demonstrating this: $\dfrac{P(2018) - P(2016)}{2018 - 2016} = \dfrac{285.8 - 302.9}{2018 - 2016}$

Teacher Note: The interpretation of the average rate of change is important in both AP Precalculus and AP Calculus. Including units of measure whenever working with average rates of change in a contextual problem is encouraged.
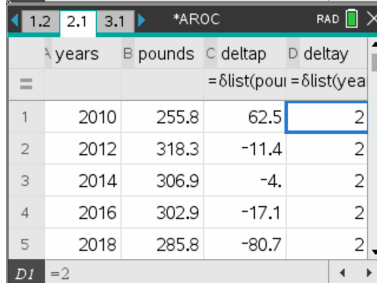
Using the TI-Nspire with Problem 2. *In the video, the demonstration uses the TI-84. The "Using the TI-Nspire" notes in this document provide directions and screen shots that are similar to the use of the TI-84.*

On a new page or new document, select [menu], Add Lists and Spreadsheet. Label column A as years and column B as pounds. Enter the data in the table.

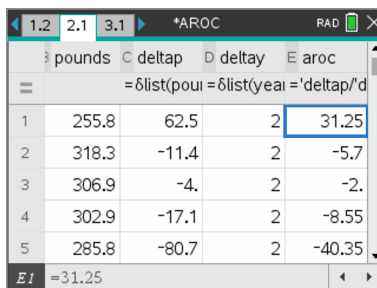|	A years	B pounds	C	D
=				
1	2010	255.8		
2	2012	318.3		
3	2014	306.9		
4	2016	302.9		
5	2018	285.8		

B1 255.8

Label column C as **deltap** and column D as **detay**. In column C find the successive differences in pounds, and in column D find the successive differences in years. In the formula cell for column C, select [menu] Data>List Operations>Difference List and type **pounds**. In the formula cell for column D, select [menu] Data>List Operations>Difference List and type **years**.

	A years	B pounds	C deltap	D deltay
=			=δlist(pou	=δlist(yea
1	2010	255.8	62.5	2
2	2012	318.3	-11.4	2
3	2014	306.9	-4.	2
4	2016	302.9	-17.1	2
5	2018	285.8	-80.7	2

D1 =2

Technology Tip: You may also access the names of the columns (lists) using [var].

Label column E as **aroc**. In the formula cell for column E, select [=] and type **deltap/detay**.

	B pounds	C deltap	D deltay	E aroc
=		=δlist(pou	=δlist(yea	='deltap/d
1	255.8	62.5	2	31.25
2	318.3	-11.4	2	-5.7
3	306.9	-4.	2	-2.
4	302.9	-17.1	2	-8.55
5	285.8	-80.7	2	-40.35

E1 =31.25

Problem 3

A particle moves along the horizontal number line. Its position at time $t \geq 0$ is given by $s(t) = t^2 - 7t + 2$ where t is measured in seconds and s is measured in meters.

  (a) Find the average rate of change in the particle's position from $t = 0$ to $t = 8$ seconds.
  (b) Use your answer in part (a) to determine if the particle is to the left or the right of its initial position at time $t = 8$.

Class Discussion:

How can we use the average rate of change in part (a) to answer the question in part (b)?

Possible Answers: Since the average rate of change over the given interval is positive, the particle is to the right of its initial position. The average rate of change means that on average the particle moved 1 meter to the right each second.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 4

The figure shows the graph of the altitude of a plane (h, in feet) from takeoff, $t = 0$ minutes, to $t = 45$ minute.
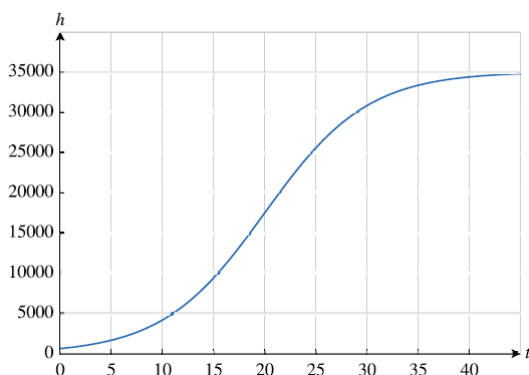
Use the graph to determine on which five-minute interval, 0-5, 5-10, 10-15, etc., the average rate of change in height is greatest.

Class Discussion:
How can we use the graph to answer this question? If we had a function, we could evaluate the functions values to find the average rates of change. In this case we only have a given graph.

Possible Answers: Use a ruler and draw in the secant lines. Choose either the 15-20 or 20-25 five-minute interval. Find the slopes of these secant lines.

Using the TI-Nspire with Problem 4. *In the video, the demonstration uses the TI-84. The "Using the TI-Nspire" notes in this document provide directions and screen shots that are similar to the use of the TI-84.*

Graph $f1(x) = \dfrac{35000}{1 + e^{-0.2(x-20)}}$ and $f2(x) = \dfrac{f1(x+5) - f1(x)}{5}$.

Notice that $f2(x)$ is the average rate of change of the function in $f1(x)$.

To display a graph similar to the one in the video, use the window in the screen shot to the right.
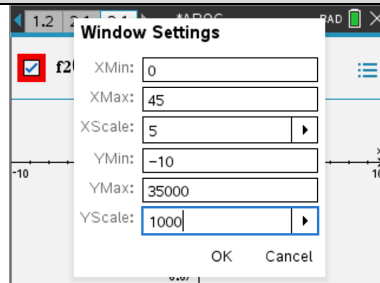
The red graph represents the average rates of change. With the Xscl = 5, it appears that the maximum is in the 5-minute interval 15-20.

Trace along the f2 graph to see the values of the average rates of change.

The table would also confirm that the greatest rate of change is in the 5-minute interval 15-20.

Technology Tip: Select [ctrl][T] to open a table. Select [ctrl][T] again to go back to the graph.
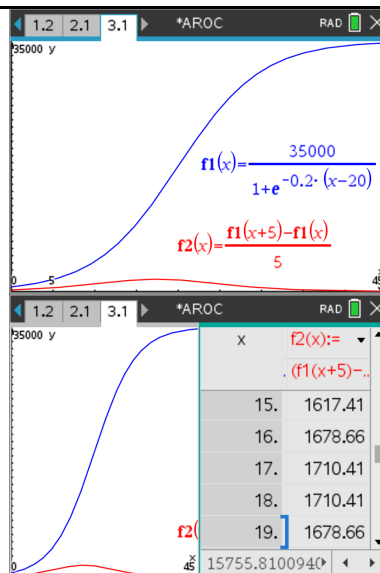
Note: *The following problems in 5 are not discussed in the video.*

Problem 5 (a) – (b)

For each of the following functions, simplify the expression $\dfrac{f(x+h)-f(x-h)}{2h}$

(a) $f(x) = 2x + 8$

(b) $f(x) = x^2 + 3x + 4$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to these problems.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The graphing application can be used to graph a function and a related secant line.
- The calculator application can be used to find average rates of change.

For more videos from the AP Precalculus Live series, visit our playlist
https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI