According to the Standards:

Instructional programs from preK-grade 12 should enable students to:

- Recognize and use connections among mathematical ideas
- Use the language of mathematics to express mathematical ideas precisely
- Select, apply and translate among mathematical representations to solve problems

In grades 9-12 students should

1. Students should develop an increased capacity to link mathematical ideas and a deeper understanding of how more than one approach to the same problem can lead to equivalent results.

Calculus Scope and Sequence: Applications of Definite Integrals

Keywords: area, bounded area, area between curves

Description: This activity will illustrate the idea of using the definite integral to represent the area bounded by two curves.

Calculate the area bounded by the curves $y = x^2 - 3$ and y = 2x + 1. The standard way to do this symbolically is:

"right intersection" $\int_{\text{"left intersection"}} (\text{top curve-bottom curve}) d(\text{variable})$

- 1. Go to Y= and input the two functions
- 2. Graph them in a standard window (found in F2-Zoom-#6)

F1+ F2+ F3 F4 F5+ F6+ 50 ToolsZoomEdit / AllStyle3001	F1+ F2+ F3 F4 F5+ F6+ 50 Tools Zoom Edit / All Style 2005	F1+ F2+ F3 F4 F5+ F6+ F7+8:: ToolsZoomTraceReGraphMathDrawPen:<
-PLOTS -Yy1=x ² - 3	-PLOT 1:ZoomBox √y1: 2:ZoomIn 3:ZoomOut	
√y2=2·×+1 y3=∎ u4=	yu21 3:200Modic yu21 4:ZoomDec yu21 5:ZoomSgr	L
97- 95= 96=	95 6:200MStd 95 7:200mTrig 96 8#ZoomInt	
U/= U3(x)= Main rad auto func	U3(X)= MAIN RAD AUTO FUNC	MAIN RAD AUTO FUNC

Find the intersections on the home screen by setting the functions equal:

- 1. Press HOME
- Go to F2-Algebra-#1 (solve) and use the following arguments: Solve(first function = second function, variable) *Here we'll use y1(x) and y2(x) since that's where we stored the functions, we could also have just typed them in directly.*

F1+ F2+ F3+ F4+ F5	F1+ F2+ F3+ F4+ F5
ToolsAl9ebraCalcOtherPr9mlOClean UP	ToolsA19ebraCa1cOtherPr9mIOClean UP
<u>solve(y1(x)=y2(x),x) </u> MAIN BAD AUTO FUNC 0/30	• solve($y1(x) = y2(x), x$) $x = -(\sqrt{5} - 1) \text{ or } x = \sqrt{5} + 1$ solve($y1(x) = y2(x), x$) MAIN RAD AUTO FUNC 1/30

Now set up the integral in the "traditional" sense:

$$\int_{-\sqrt{5}+1}^{\sqrt{5}+1} (2x+1) - (x^2 - 3)dx$$

(The integral is found in F3-Calc-#2 and requires the following arguments: (function, variable, lower bound, upper bound)



• Users tip: you can change from the exact answer to a decimal approximation by hitting green diamond key before you press enter



Now, we can show the same result graphically if we create a third function that is the result of the difference of the two originals just as we've done symbolically: (be sure to shut off by using F4, the original functions)



- Use the same "solve" routine and find the roots (note: they should be the same as the x-values of the intersection if you want to pass up this step ⁽²⁾)
- Using the F5-Math-#7 calculate the bounded area



• This shows the same results as the calculation before



Warning: As is the case with multiple intersections symbolically, you will have to take that into account when you show it graphically, since the graph will only produce the net area with reference to the horizontal axis.