NUMB3RS Activity: Guarding the Goods Part II
Episode: “Obsession”

Topic: Securing art galleries **Grade Level**: 8 - 12
Objective: Explore a formula typically used for identifying the minimum number of security guards in an art gallery This formula introduces students to a floor function and the “Art Gallery Problems”
Time: 10 - 15 minutes

Introduction

Many famous and expensive art pieces are shown in art galleries around the world. The galleries can be represented as polygons, and you will find security guards in every gallery. However, how do you know how many guards are needed to secure the gallery? This question, also known as the “The Art Gallery Problem” asks for the minimum number of guards, and where they should be placed at vertices, so that the entire interior of a polygon can be viewed simultaneously. A well-known formula for identifying the minimum number of guards is as follows:
If a polygon contains ‘n’ vertices, the theorem states that the number of guards is “always sufficient and sometimes necessary to be $\left\lfloor \frac{n}{3} \right\rfloor$.” What this means is that a solution does not have to be more than $\left\lfloor \frac{n}{3} \right\rfloor$, but it could be fewer. The notation “$\left\lfloor \frac{n}{3} \right\rfloor$” refers to the floor function. $\lfloor n \rfloor$ means the largest integer that is less than or equal to n. This function is equivalent to the greatest integer function.

Example $\left\lfloor \frac{34}{3} \right\rfloor = 11$ since $\frac{34}{3} = 11.\bar{3}$ and the largest integer less than $11.\bar{3}$ is 11.

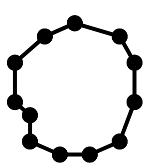
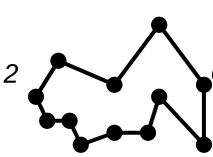
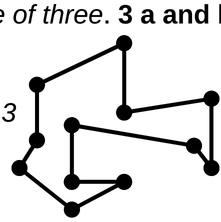
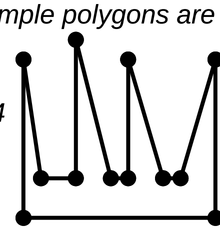
This theorem will tell you the greatest number of guards that you might need, but it does not tell you where to place them, or if you could use fewer than this number. This is because there are many factors that affect the answer, and because there is often more than one answer.

Discuss with Students

Explain these rules to the students. In “The Art Gallery Problem”, guards can see in all directions at once, but they can’t see through walls. Also, guards are placed in the corners (vertices) of the gallery so they don’t block anyone’s view of the art. The problem is to figure out where to place the guards so that you use as few guards as possible.

You can explore this in the classroom by asking students to stand in corners and describe what they can and can’t see. Another exploration to understand the rule is to draw a few simple polygons, and shade in the line of sight for any one vertex.

Student Page Answers: **1**. *1, 2, 2, 2, 3*. **2**. *Answers may vary, but generally should reflect that exactly n/3 are not required until n is a multiple of three*. **3 a and b**: *Sample polygons are given:*

G = 1

G = 2

G = 3

G = 4


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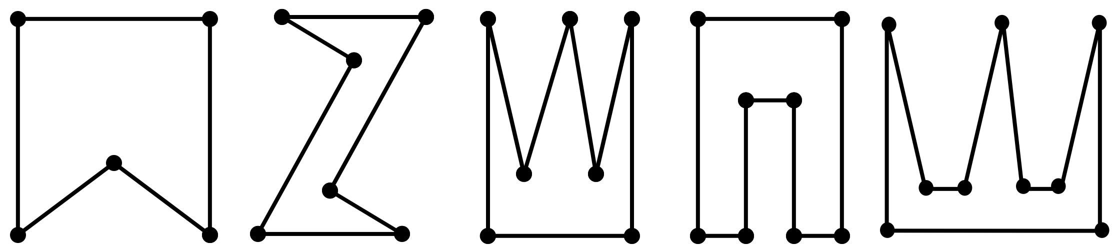
Name: ______________________________ Date: ________________

NUMB3RS Activity: Guarding the Goods II

1. For each of the polygons, circle the vertex where you would place each guard to ensure the entire interior of the polygon is visible by the fewest number of guards. Use different colored pencils to shade the region of the polygon that each guard can see. Write the number of guards you used in the space provided.



5 Sides	**6 Sides**	**7 Sides**	**8 Sides**	**9 Sides**
G = ___	G = ___	G = ___	G = ___	G = ___

2. Describe any pattern you see in your solutions. ______________________________

____

3. The Art Gallery Theorem states: "For a simple polygon with *n* vertices, $\left\lfloor \frac{n}{3} \right\rfloor$ guards are *always sufficient and sometimes necessary*." This means that $\left\lfloor \frac{n}{3} \right\rfloor$ is guaranteed to be enough, but the best answer might use fewer guards.

a. Draw a polygon with 12 vertices that requires 4 guards.

4 Guards

b. According to the theorem, not every polygon with 12 sides will require 4 guards. Draw polygons with 12 sides that need exactly 1, 2, and 3 guards.

1 Guard	**2 Guards**	**3 Guards**

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Activity: Variations of the Art Gallery Problem

Introduction

The Art Gallery problem is one example of many related problems. Explore these other problems in computational geometry by changing the parameters or purpose of the guards.

For the Student

- Suppose guards have limited vision (like 90º or 180º of peripheral vision).
- Suppose guards patrol one edge of the polygon, or that they patrol the edges in pairs and must remain in sight of each other at all times.
- Suppose the interior of the polygon is mirrored. Is there a place where a guard could stand and be able to see the entire interior?
- Consider The Fortress Problem in which the guards look outside the polygon and want to be able to see the entire exterior of the polygon.
- In The Prison Yard Problem, guards posted on the sides of the polygon can see both inside and outside simultaneously and must be able to view the entire exterior and interior of the polygon.

Additional Resources

http://www.cut-the-knot.org/Curriculum/Combinatorics/Chvatal.shtml
This site shows details on the actual construction of solutions to Art Gallery problems, along with an applet to experiment with different polygons.

http://www.cs.mcgill.ca/~cs507/projects/1998/eposse/
This site contains a discussion of The Fortress Problem and includes a discussion of "point" guards, those that can be located at points outside the polygon. There is also an applet that allows users to construct a fortress and solve it in steps by placing guards one at a time.

http://www.site.uottawa.ca/~jorge/openprob/
This site poses a collection of "Open Problems on Discrete and Computational Geometry," most problems are approachable for students, though the solutions have not been found.

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