

## Minimizing Distance

by – Vicki Carter

### Activity overview

Students are presented with a classic optimization problem of the distance from a function and a point. They will explore the concept of minimum distance graphically, numerically, and algebraically.

### Concepts

Distance Formula

Data collection – representation and interpretation using scatter plots

Derivatives and critical points

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### Teacher preparation

This investigation could be used as an introduction to optimization (max-min) problems in precalculus or calculus. Students should be familiar with polynomial and radical functions. They should also be able to represent coordinates in terms of functions.

- Download the *Minimizing Distance.tns* file.

### Classroom management tips

This activity is intended to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Students will answer the questions posed on the Q&A Notes pages.

- As all questions are posed in the .tns file, the intent of this activity is for the teacher to collect the document from the students at the conclusion of the activity. As an alternative, you may wish to have the class record their answers on a separate sheet of paper or simply use the questions posed to engage the students in a class discussion.

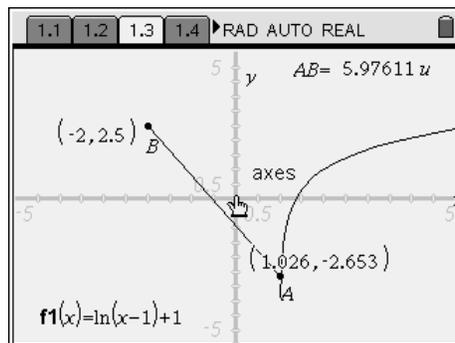
### TI-Nspire Applications

Graphs & Geometry, Lists & Spreadsheet, Notes, Notes with Q&A templates, Calculator

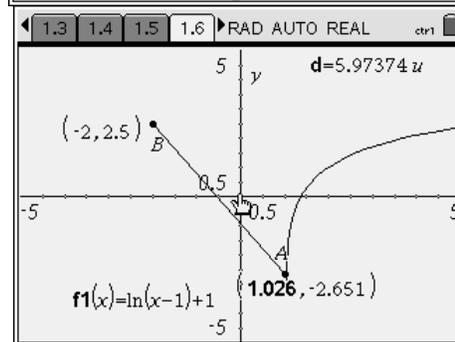
**Step-by-step directions**

**Investigating the distance from a point to a curve graphically and numerically**

**Step 1:** Students should grab point A and observe the changes in the length of side AB. After moving the point A to various positions, they should be able to find the minimum distance and answer the question on page 1.4



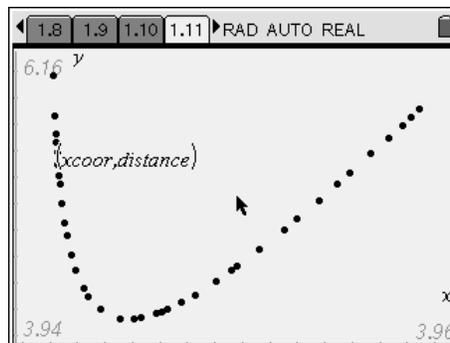
**Step 2:** Students are instructed to again drag point A along the logarithmic function on page 1.6.



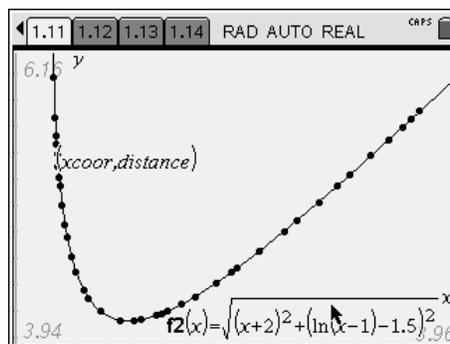
**Step 3:** As the students move point A, a table of values for the area and distance AB are captured automatically. Here is an example of what their table should look like. Students are instructed to investigate the distance column, column B, and answer the question on page 1.9.

A	xcoord	B	distance	C	D	E
=capture(x=capture('d,1))						
1	1.02597		5.97374			
2	1.03759		5.66434			
3	1.04439		5.52849			
4	1.04774		5.46969			
5	1.05295		5.38695			
A7	=1.0259746996					

**Step 4:** The resulting scatter plot is produced on page 1.11. The window will need to be adjusted to view the scatter plot. Discuss the type of function that produces this graph.



**Step 5:** On page 1.12, students are instructed to return to the scatter plot to graph the distance function. Students will need to display the Entry Line. **MENU > 2:View > 6:Show Entry Line** or use **Ctrl+G**. The distance function will be typed into  $f2(x)$ . The students should be instructed to trace on the graph. **MENU > 5: Trace > 1: Graph Trace**. Care should be taken to insure that the students are tracing on the function and not the scatter plot. Pressing up on the Navigation Pad should toggle them between the scatter plot and the function trace. They should trace on this function to find the minimum,  $m$ . You may want to have the students compare the minimum distance recorded from the graphical investigation and from the table with this minimum value.

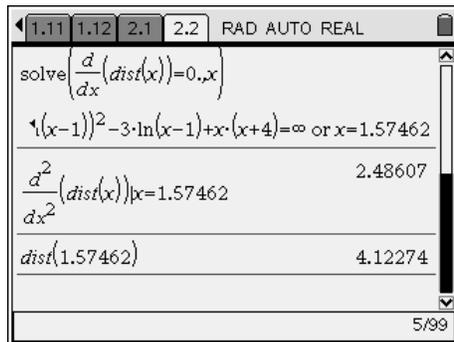


**Extension: Investigating the distance between a point and a curve algebraically**

The second part of this activity (Problem 2) is intended to be **teacher-led**. The use of the TI-Nspire™ CAS is necessary for this part of the activity. This is an investigation for calculus students.

**Step 6:** On page 2.1, students are instructed to investigate the calculus part of this problem. With your assistance, students should discuss the mathematics they need to calculate a minimum value of a function. The function is already defined on page 2.2. The recommended procedure is:

- **Menu > 4: Algebra > 1: Solve**   Select the derivative template. Find the zeros of the derivative.
-   Select the  $n^{\text{th}}$  derivative template to evaluate the 2<sup>nd</sup> derivative of  $dist(x)$  at  $x = 1.57462$  to justify the minimum via the Second Derivative Test.
- Evaluate the  $dist$  function at  $x = 1.57462$  to find the minimum distance.



Expression	Result
$\text{solve}\left(\frac{d}{dx}(dist(x))=0, x\right)$	$\{(x-1)^2 - 3 \cdot \ln(x-1) + x \cdot (x+4) = \infty \text{ or } x = 1.57462\}$
$\frac{d^2}{dx^2}(dist(x)) _{x=1.57462}$	2.48607
$dist(1.57462)$	4.12274

**Step 7:** You may want to have the students compare the minimum distance recorded from the graphical investigation, from the table, and from the function graph with this minimum value.

**Assessment and evaluation**

The teacher could collect the document from the students at the conclusion of the activity to check for understanding. As an alternative, you may wish to have the class record their answers on a separate sheet of paper or simply use the questions posed to engage the students in a class discussion.

### Student TI-Nspire Document

#### Maximizing Area.tns

1.1 1.2 1.3 1.4 ▸ RAD AUTO REAL

Minimizing the distance from a given point to a function

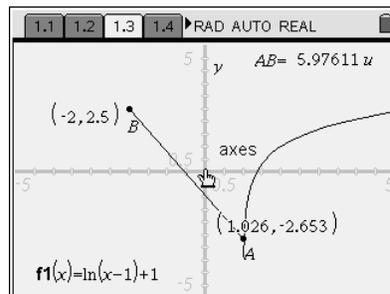
AP Calculus

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1.1 1.2 1.3 1.4 ▸ RAD AUTO REAL

We will explore distance from the point  $(-2, 2.5)$  to the curve  $f_1(x) = \ln(x-1)+1$

On the next page, drag point A to investigate how the distance changes as point A moves along the curve.



1.1 1.2 1.3 1.4 ▸ RAD AUTO REAL

**Question**

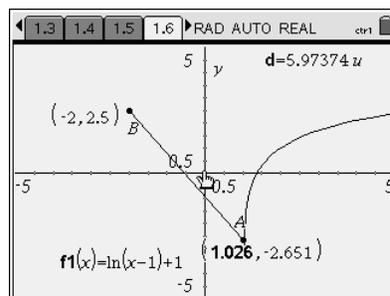
What was the minimum value of the distance in your investigation?

**Answer** ▾

1.2 1.3 1.4 1.5 ▸ RAD AUTO REAL

On the next page, drag point A so that the x-coordinate of the point on the graph of  $f$  increases. Data will be collected automatically as you drag point A.

The distance from point A to point B will be represented by  $d$ .



1.4 1.5 1.6 1.7 ▸ RAD AUTO REAL

In the spreadsheet on the following page, investigate the distance column (column B).

1.5 1.6 1.7 1.8 ▸ RAD AUTO REAL

A	xcoord	B	distance	C	D	E
1	1.02597		5.97374			
2						
3						
4						
5						

$A1 = 1.0259746996$

1.6 1.7 1.8 1.9 ▸ RAD AUTO REAL

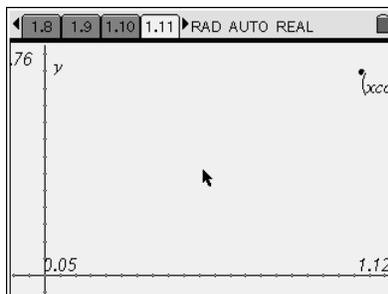
**Question**

What is the minimum distance recorded in your spreadsheet? What is the value of the x-coordinate of point A at this minimum?

**Answer** ▾

1.7 1.8 1.9 1.10 ▸ RAD AUTO REAL

On the next page, look at the scatter plot of x-coordinate and the distance from  $(-2, 2.5)$  to the curve.



1.9 1.10 1.11 1.12 ▸ RAD AUTO REAL

Since the distance formula is

$$\text{dist} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

then

$$\text{dist} = \sqrt{(x+2)^2 + (\ln(x-1)-1.5)^2}.$$

Return to the scatter plot page and graph this function. Trace on this function to find the minimum (m).

## Minimizing Distance

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Grade level: 9-12

Subject: Precalculus, Calculus

Time required: 45 minutes

Materials: Minimizing Distance.tns

