

Activity 4

Exploring Quadratic Data with Transformation Graphing

In Activity 3, you explored the effect of the parameters A, B, and C on the graph of the general quadratic function in vertex form, $y = a(x - b)^2 + c$. In this activity, through translation and dilation of the general parabola, you will try to "eyeball" fit data that can be modeled with a quadratic function.

This activity will serve as practice with both the vertex form of a parabola and "eyeball" modeling. Its purpose is to continue your study of the quadratic function (parabola) and its properties by developing quadratic models.

This is also a good opportunity to think about what makes a good or reasonable model.

When you move to Data Model 2, think about the function properties of domain and range and how important they are in modeling.

If you have not completed Activity 3, you should do so before starting this activity.

Data Model 1 - Ball Bounce

Equipment Needed

- ◆ TI-83 Plus with CBL/CBR and Transformation Graphing applications installed
- ◆ CBR
- ◆ Ball - a racquet ball or a basketball will work fine and are the ones that are usually used for this activity. Do not use a "fuzzy" ball such as a tennis ball.

Experiment Set-up

1. Attach the CBR to the TI-83 Plus with the unit-to-unit link cable.
2. Press **[APPS]** and choose **CBL/CBR**.
3. Press any key to advance past the introduction screen.
4. Select **3:Ranger**.
5. Press **[ENTER]** to advance past the introduction screen.
6. Select **3:APPLICATIONS**.
7. Select either **1:METERS** or **2:FEET**.
8. Select **3:BALL BOUNCE**. Follow the instructions with the **BALL BOUNCE** program.

```

APPLICATIONS
1: Finance
2: ALGEBRA1
3: CBL/CBR
4: Transfrm

```

```

TEXAS
INSTRUMENTS
CBL/CBR (TM)
version 1.00
PRESS ANY KEY

```

```

CBL/CBR APP:
1: GAUGE
2: DATA LOGGER
3: RANGER
4: QUIT

```

```

TEXAS INSTRUMENTS
RANGER (V1.0)
PRESS [ENTER]

```

```

MAIN MENU
1: SETUP/SAMPLE
2: SET DEFAULTS
3: APPLICATIONS
4: PLOT MENU
5: TOOLS
6: QUIT

```

```

UNITS
1: METERS
2: FEET

```

```

APPLICATIONS
1: DIST MATCH
2: VEL MATCH
3: BALL BOUNCE
4: MAIN MENU

```

9. Once you have good data appearing on the calculator screen, exit the program by pressing **ENTER** **7:QUIT**. The data you will work with is stored in **L1** and **L2**.

```

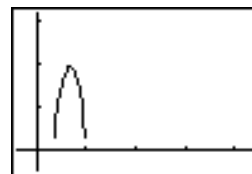
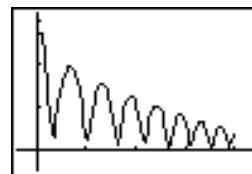
PROGRAM MENU
1: DIST-TIME
2: VEL-TIME
3: ACCEL-TIME
4: PLOT TOOLS
5: REPEAT SAMPLE
6: MAIN MENU
7: QUIT
  
```

```

L1=TIME
L2=DIST
L3=VEL
L4=ACCEL
Done
  
```

10. Use the built-in **Select()** feature of the TI-83 to select the first "good" bounce that is collected.

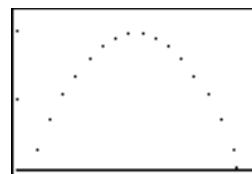
Note: Keystrokes are given below and on the next page.



11. The **CBL/CBR** program leaves the data connected, press **2nd** **[STAT PLOT]** **1:Plot1** and select the unconnected scatter plot option.

```

Plot1 Plot2 Plot3
Off Off Off
Type: [Scatter] [Line] [Bar]
Xlist: L1
Ylist: L2
Mark: [square] [circle] [plus]
  
```



Keystrokes for Using the Built-In Select Feature

1. With the plot displayed, press **2nd** **[LIST]** **8:Select(** and enter where you want to store the selected data. To use **L3** and **L4**, press **2nd** **[L3]** **,** **2nd** **[L4]** **)** **ENTER**.

```

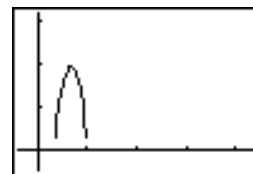
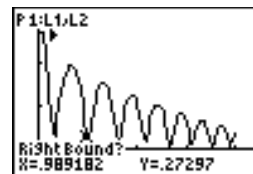
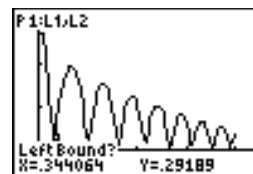
Select(
  
```

```

Select(L3,L4)
  
```

2. To actually select a part of the graph you will use, press \blacktriangleright to move to the left end of the data you want to keep. Press ENTER . This sets the left bound. Press \blacktriangleright to move to the right end of the data you want. Press ENTER . The selected data will be placed in L3 and L4, and then this data will be displayed.

Note: You can also find instructions for using `Select()` and setting up Stat Plots in the TI-83 Plus Guidebook.

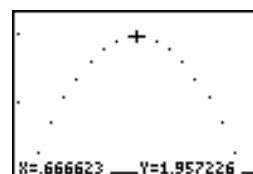


Developing the First Model

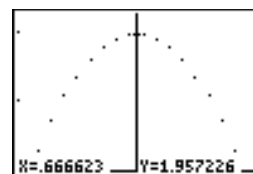
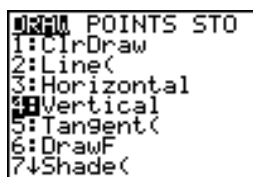
1. Using the free-moving cursor, try to position the cursor's + at the approximate location of the vertex. Record these coordinates.

To use the free-moving cursor, press the cursor keys (\blacktriangleleft \blacktriangleup \blacktriangleright \blacktriangledown) to move the cross hatch around the screen.

- Do you think the vertex will always be one of the points that is collected by the CBR? Why or why not?
- How does the fact that the parabola has an axis of symmetry help you estimate the location of the vertex?



2. Using the **DRAW** feature, sketch the axis of symmetry through the point you have selected as the vertex. Press 2^{nd} [DRAW] 4:Vertical. The line will be drawn by the TI-83 Plus at the same location as the cursor. Does the line appear to be the axis of symmetry? If not, move the line left and right with the cursor control keys until it appears where you envision the axis of symmetry.



If you need to move your axis of symmetry, be sure to update the starting location of the vertex accordingly.

Record the approximate coordinates of the vertex.

X = ____ Y = ____

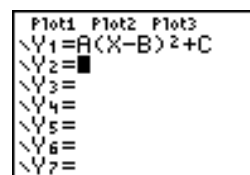
3. Press **[APPS]** and select Transformation Graphing by pressing the number at the left of **Transfrm**. Press any key (except **[2nd]** or **[ALPHA]**) to start Transformation Graphing.



Note: If you do not see the screen illustrated at the right, select 2:Continue).

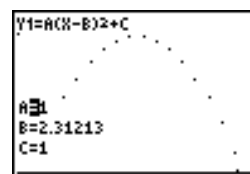


4. In **Func** mode, press **[Y=]** to display the **Y=** editor. Clear any functions that are listed. Enter the general vertex form of the quadratic function, $Y = A(X - B)^2 + C$. Press **[ALPHA]** **A** **[([X,T,θ,n] - [ALPHA]** **B** **) [x²] + [ALPHA]** **C**.



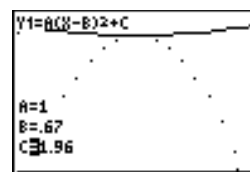
If the Play-Pause Mode is not selected at the left of **Y1** (**>||**), press **[◀]** until the cursor is over the symbol and then press **[ENTER]** until the correct symbol is selected.

5. Press **[GRAPH]** to display the plot and the graph. The graph will start with the most recent values of **A**, **B**, and **C**. These values do not necessarily relate to the problem you are working on and thus the graph might not appear on the screen.



- Why do you think you recorded the approximate coordinate of the vertex? How will you use them in this activity?
- The "eyeball" method to find a model for quadratic data works quickest when you are working with the vertex form of the parabola. You have already recorded a starting point for the vertex and thus have starting values for **B** and **C**. Which value is **B** and which is **C**? Why?

6. Press **[▼]** to highlight **B=**. Enter your estimate. In this example, you would press **[.] [6] [7] [ENTER]**.
7. Press **[▼]** to highlight **C=**. Enter your estimate. For this example, press **[.] [9] [6] [ENTER]**.

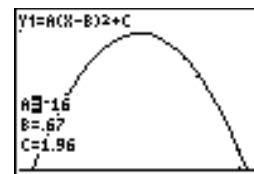


- You now have a parabola with its vertex matching the vertex of your data (in the example at .67, 1.96). You need a value for **A**. What would be a good starting value for **A**?

Because the parabola is opening down, you need a negative value. If you have taken Physical Science, you know that the coefficient of the quadratic term for a free falling body is half the acceleration due to gravity. The acceleration due to gravity is approximately 32 ft/sec^2 or 9.8 m/sec^2 . This may give you an idea for a starting value for **A**.

8. Press \square \square to highlight $A=$. Enter your estimate. In the example, press \square 1 6 \square [ENTER]. How good is your model?

If your model is not as good as the one displayed, you should refine your model by selecting new values for A, B, or C.



Internet Research: Once you finish this activity, look up free fall or projectile motion on the Internet and discuss how this activity relates to what you find.

Homework Page

Name _____

Date _____

Stamp Prices

In early 1999, the U.S. Post Office raised the price of a first class stamp to \$0.33. As it was in 1995 when the price was raised to \$0.32, many people complained that there was no reason to increase the stamp prices. The table shows each year that the cost of a first class stamp has changed since 1958. Enter the data into the Stat Lists of your TI-83 Plus and then use Transformation Graphing to develop a quadratic model for the relationship between year and price.

Use your model to predict the price of a first class stamp in 1999 and decide if the price increase seems justified.

Year	Cost of a First Class stamp
1958	4
1963	5
1968	6
1971	8
1974	10
1978	15
1981	18
1983	22
1988	25
1995	32

Source: Reprinted with permission from *World Almanac and Book of Facts, 1998*.
©1998, 2000 World Almanac Education Group, Inc.

1. Which variable is the independent variable?

- a. Enter the data in the Stat List of your TI-83 Plus. Press **STAT** Select **1:Edit**. Enter the values from the table into an empty list in the stat editor. Clear two lists if no lists are empty.

Note: the example shows the year data without the first two digits. (1974 is entered and 74.)

- b. Press **Y=** and deselect any functions that have been turned on.

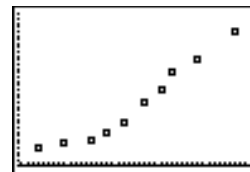
L1	L2	L3	Z
74	10		
78	15		
81	18		
83	22		
88	25		
95	32		
-----	-----		
L2(1) =			

Plot1	Plot2	Plot3
$\sqrt{Y_1 = A(X-B)^2 + C}$		
$\sqrt{Y_2 = \blacksquare}$		
$\sqrt{Y_3 =}$		
$\sqrt{Y_4 =}$		
$\sqrt{Y_5 =}$		
$\sqrt{Y_6 =}$		
$\sqrt{Y_7 =}$		

- c. To display the plot, press $\boxed{2\text{nd}}$ [Stat Plot]. Select **1:Plot1**. Turn the plot on and set the plot menu as shown at the right.



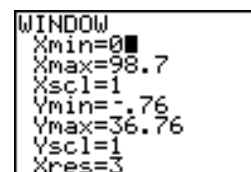
- d. Press \boxed{ZOOM} **9:ZoomStat** to display the plot.



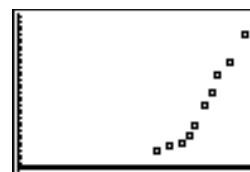
- Does there appear to be a relationship between the year and the cost of a first class stamp? Which point(s) seems the most "out of line?"
- Do you feel a quadratic is a reasonable model? Explain.
- Would you visualize a different model if the data stopped with 1983?

Use Transformation Graphing and the general vertex form of a parabola to develop a model for the data.

Important: Before you start, press \boxed{WINDOW} and set **Xmin = 0**. This will allow a better view of the data points and the coefficients with Transformation Graphing installed.



As a starting point, pick one of the given points as the vertex and then start refining your model.



Once you have found your model, answer the following questions.

- What is your model? Why do you think it is reasonable?
- What does your model predict for the price of a first class stamp in 1999? To have your calculator help answer this question, with the graph of your model showing, press \boxed{TRACE} $\boxed{\blacktriangle}$ **99** \boxed{ENTER} .

Note: your Xmax must be larger than 99 for this to work.

7. Using your model, do you think the cost increase was reasonable? Explain mathematically.

8. Some people would say that, in theory, an exponential model would be better. Why do you think they would make this claim?

9. The table listed the years as 58, 63, ... instead of 1958, 1963, Why do you think this was done? What type of operation was performed on the data?

10. What would you put in your table for the year 2000 or 2010?

11. Would you use your model to predict the cost of a first class stamp when the U.S. Post Office issued the first stamp? Hint: The first stamp was issued well before 1950. You might want to do an Internet search to learn the history of the Post Office Service.

Notes for Teachers

This activity is a perfect follow-up for the previous activity. In Activity 3, the student explores the effect of the various parameters on the graph of a parabola. In this activity, they will use what they have just learned to find a quadratic model using Transformation Graphing.

The main intent of this activity is to give the students practice with the parameters of the vertex form of the parabola and their effect on the shape of the curve. "Eyeball" fitting models helps reinforce these relationships and helps the student discover some of the properties of the particular curve being studied.

The methods used in this activity should be used before any more formal model development takes place. Doing "eyeball" fits also helps to develop a sense of when a model is reasonable.

Be careful not to look for one correct answer. Any reasonable model will have some strong points and can be justified even though no sort of formal optimization takes place.

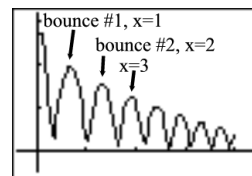
There are two data sets. Set 1 is collected by the students in the classroom. The instructions for this data set assume the use of a CBR but a CBL and motion detector can be used. If you are using the CBL, you will need the ball bounce program named **BOUNCE**. This program can be found on the TI website.

More detailed instructions on the ball bounce experiment can be found in the *CBL™ System Experiment Workbook* that comes with the CBL, or in the books *Real-World Math with the CBL™ System* or *Math and Science in Motion: Activities for Middle School*, available from Texas Instruments. The *CBL™ System Experiment Workbook* can also be downloaded for free from Guides section of the Texas Instruments web site at education.ti.com.

Hint: The instructions tell the student to use the free-moving cursor to approximate the vertex. It would probably be easier to go to the **DRAW** menu and use the vertical line to try to visualize the axis of symmetry. This was not done at the start of they activity as the attempt was to not bring in too many new or less familiar keystrokes early on, but to let them stay with familiar operations and thus concentrate more on the vertex form.

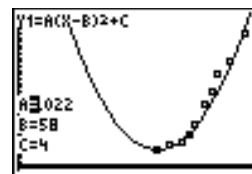
The models for the bouncing ball should all be very good and easy to find.

For the Future: You might want to save the ball bounce data and re-visit it when you study the exponential function. The data set formed by the number of the bounce and the maximum height of the ball for each bounce can be modeled with an exponential function.

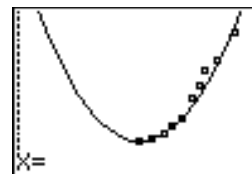


There are any numbers of reasonable models. Do not expect all students to get the same model, or in fact models whose predicted cost in 1999 are very close. Any reasonable model should be accepted.

Using the first data point (58, 4) as the vertex will be a very popular method. A reasonable model can be found using this approach. Both models shown here will predict a stamp cost of \$.41 for 1999.

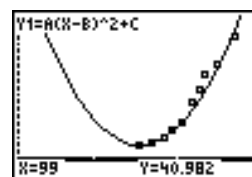


You might want to show your students how to use the "value" option under the CALC menu if you feel they do not already have this skill.

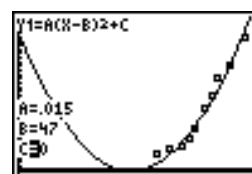


To use the value option:

- With the graph showing, press $\boxed{2nd}$ [CALC]1:value.
- Enter the number you want to use to evaluate the function (in this example, $x = 99$).
- Press \boxed{ENTER} .

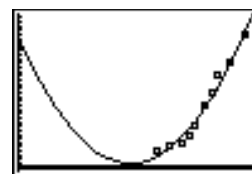


In the activity the same result was found using the Trace option.



Remember, there are any number of equally reasonable models. This model gives a predicted 1999 cost of \$.41.

Using the built-in regression gives a model in a different form ($y = ax^2 + bx + c$) that does not appear to be that much better than the eyeball models. The predicted cost for this model is \$.39.



An exponential regression on the data gives a model that predicts a cost of \$.50 in 1999, while a logistic regression predicts a cost of \$.34.

In any case, the cost increase does not seem to be out of line.

The quadratic model, no matter what they have, gives a perfect opportunity to discuss domain and range issues. If no restriction is placed on the domain any quadratic model will show that many years before those listed on the table the price of a stamp was much larger and that in fact the cost had dropped going into the 1940s or 1950s.

With unrestricted domains, the logistic or exponential models would make more sense for years previous to those in the table.

A couple of Internet searches are suggested during the activity. As a third Internet activity, you might have your students research the cost of stamps earlier than 1958, add that data to their plot, and then determine a new model. If you use this option, you might want to discuss the modeling concept of maintaining, updating, and revising a model.

Answers

Answers to most of the questions follow them in the activity. Those that do not are listed below.

Developing the First Model

1. a. There is no reason to believe that any particular point would be collected by the CBR. You will probably collect a point close to the vertex, which can be used as a starting place for the vertex, but not the vertex itself.
b. Since the vertex is on the axis of symmetry, if you visualize the axis of symmetry you can more easily find the x-coordinate of the vertex. It is often easier to start looking for your model by visualizing the axis of symmetry because you can use all the collected points.
5. a. You can use this point as a starting place for **B** and **C**.
b. **B** is the X-coordinate of the vertex and **C** is the Y.
7. Any value that is negative because the parabola opens down.

Homework Page

1. Date is independent, cost dependent.
2. Yes, there seems to be a relationship: as X increases, Y increases at an increasing rate. Looking at the total picture, the price increase in 1983 seems the most out of line.
3. A quadratic model seems reasonable, but it would have a restricted domain.
4. The year 1983 seems out of line with the total data set. If the data had stopped in 1983, you would probably have a steeper curve at the end.
5. Answers will vary. As an example $Y = .023(X - 58)^2 + 4$.
6. Answers will vary. For the example in #5, \$0.43.
7. Answers will vary. For the example in #5, very reasonable. For probably any model you might have, the model will predict a price greater than \$0.33.
8. Most people have heard that the cost of living grows exponentially. Thus they expect the cost of any particular item to grow exponentially over time also.
9. This performs a horizontal translation of the data to keep the numbers small. The quadratic relationship would hold before or after this translation.
10. The year 2000 would be 100 and 2010 would be 110. The data in the table has been shifted to the left 1900 years.
11. Answers will vary, but the model is working with a restricted domain. Many students will use the first data point (or somewhere close) as the vertex, which means any year before the vertex will have a higher cost. It is safe to assume the cost of a stamp did not drop in 1958.