Teacher Notes

Activity 1

Approaching Limits

Objectives

- Develop an intuitive understanding of the limiting process
- Estimate limits from graphs and tables of values

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 50 minutes

Abstract

This activity will provide students with various views of how a function behaves as the input approaches a particular value.

Management Tips and Hints

Prerequisites

Students should:

- be able to produce and manipulate graphs and tables of values manually and with the graphing handheld.
- have a basic understanding of function language.
- be able to identify rational, exponential, and trigonometric functions.

Student Engagement

Working in pairs or small groups is recommended for maximum student engagement.

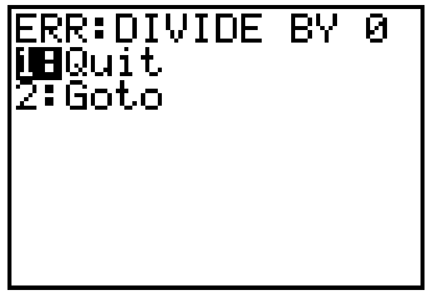
Evidence of Learning

- Given a function, students will state and explain the limit at particular values.
- Given a graph, students will state and explain the limit at particular values.
- Given a table of values, students will state and explain the limit at particular values.

Common Student Errors/Misconceptions

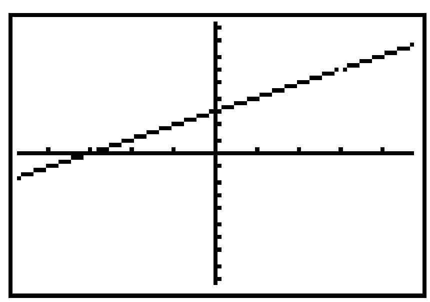
- Students may inadvertently make the table increment so small that the display in the table shows a value that does not exist.
- Students may often use misleading viewing windows that appear to show functions defined when they are not.
- Students may incorrectly assume that in connected mode the “jump lines” produced are part of the graph.

Activity Solutions



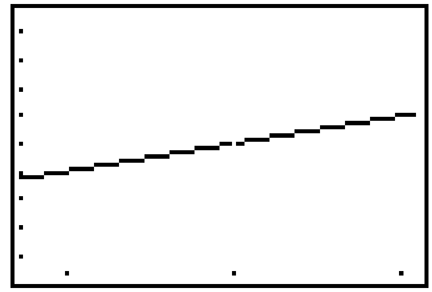
1. n/a
2. {5, ERROR, 7}
3. Answers will vary. However, most students should recognize that at 3, the numerator is being divided by zero, creating the ERROR.
4. n/a
5. {5.5, ERROR, 6.5}
6. {5.75, ERROR, 6.25}
7. {5.9, ERROR, 6.1}
8. {5.99, ERROR, 6.01}
9. Answers will vary. Most students should, at this point, be comfortable stating that the function is getting “very near” 6 as the input value gets closer to 3. Some students may even try to relate the size of the “error” to the size of the increment. This provides an opportunity to discuss the symbolic result of “removing the problem” by factoring and how the result is linear, thus producing a linear relationship between the input and output increments. This will, of course, not always be the case.
10. Note that the “hole” visible at $x = 3$ should be part of students’ sketches.

    Note: *The reason for setting a particular viewing window is to make the hole visible. The graphing handheld will only show the hole if it is asked to light up that particular pixel and it does not exist at that spot. In a standard viewing window, the point (3, 6) would NOT be one that the graphing handheld would try to graph, so the graph produced would appear to be continuous.*



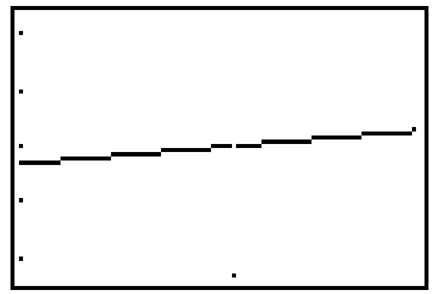
11. n/a

12. The line still seems to have a "hole" at $x = 3$:



13. Again, the "hole" still appears:

Note: *Care must be taken when looking at the screen to remember that the* y = 6 *that appears is NOT a function value; it represents a pixel value on the screen. Remind students to press* TRACE *to look at actual function values as they move around the screen.*



14. Answers will vary, but a correct answer should contain the idea that as the input value approaches 3, the output value (in fact, the limit) is very near 6.

15. $\lim\limits_{x \to 3} f(x) = 6$ OR $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3} = 6$

16. {8, ERROR, 0}

17. increment of 0.5:

X	Y1	
0	4.5	
.5	5.8333	
1	8	
1.5	13.5	
2	ERROR	
2.5	-5.5	
3	0	

X=2

increment of 0.25:

X	Y1	
1	8	
1.25	9.9167	
1.5	13.5	
1.75	23.75	
2	ERROR	
2.25	-15.75	
2.5	-5.5	

X=2

increment of 0.1

X	Y1	
1.7	20.367	
1.8	28.8	
1.9	53.9	
2	ERROR	
2.1	-45.9	
2.2	-20.8	
2.3	-12.37	

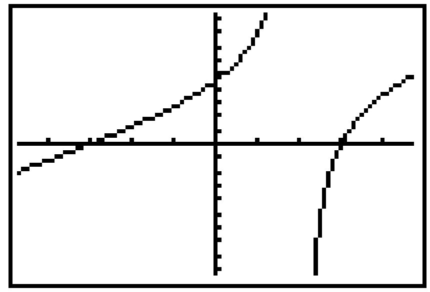
X=2

increment of 0.01

X	Y1	
1.97	170.64	
1.98	253.98	
1.99	503.99	
2	ERROR	
2.01	-496	
2.02	-246	
2.03	-162.6	

X=2

18. A correct answer should contain the idea that the values are NOT getting near any one value; in fact, it seems as if they are getting farther and farther apart. A very good answer would suggest that on one side the function is getting larger, and on the other side the function value is getting smaller. This will be looked at further in an activity on one-sided limits and can also be examined with regard to one of the conditions of continuity.



19. The graph seems to go one way below $x = 2$ and the other way above $x = 2$.

20. Zooming in does not help the function appear to approach a value. As you zoom in, it becomes more difficult to keep both branches of the graph visible.

21. The graph confirms what the numerical values seemed to indicate. As the input of the function approaches 2 from below, the graph heads toward positive ∞; as the graph approaches 2 from values above it, the graph plummets toward negative ∞.

22. $\lim\limits_{x \to 2} g(x)$ does not exist OR $\lim\limits_{x \to 2} \dfrac{x^2 - 9}{x - 2}$ does not exist.

Note: *In many cases, an acceptable answer would also be* $\lim\limits_{x \to 2} \dfrac{x^2 - 9}{x - 2} = \infty$.

Although this is accepted, the graph clearly indicates that the function, as a whole, does not go toward ∞ as x *approaches 2, but depending on which direction 2 is approached from, the graph can go toward either $+\infty$ or $^-\infty$. Care should be taken to stress that although ∞ and "no limit" are used almost interchangeably, there is a difference. This will be further seen when the topic of one-sided limits is explored.*

23. $^-2$

24. 0.693

25. 0.25

26. The limit does not exist. Using the graph or the table of values would show this function moving toward $^-\infty$ as $x \to 1$ from the right side and $+\infty$ as $x \to 1$ from the left side.

27. 0.2795