

Exploring Complex Roots

Time required
00 minutes

Activity Overview

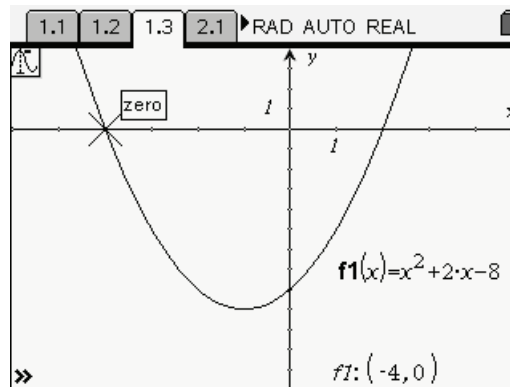
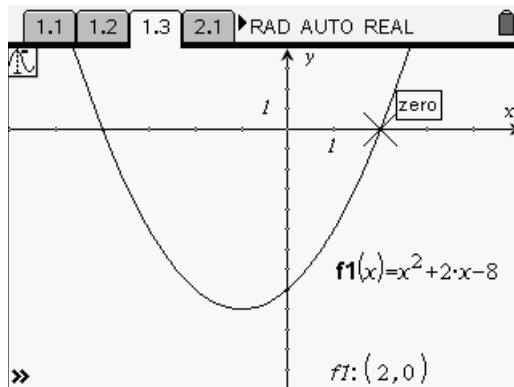
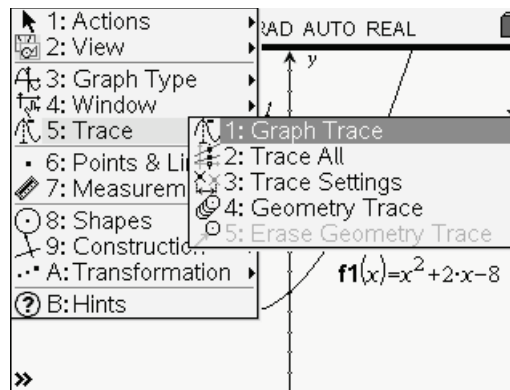
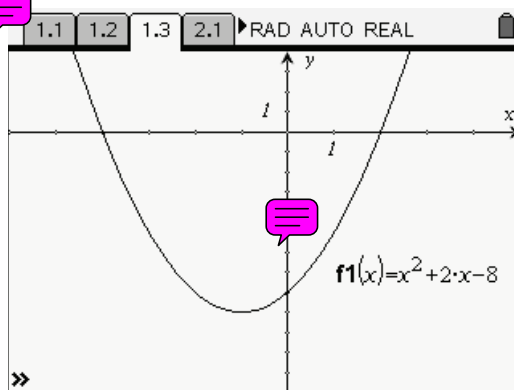
In this activity, you will explore the relationship between the complex roots of a quadratic equation and the related parabola's graph. Open the file *CollegeAlg_ComplexRoots.tns* on your TI-Nspire handheld device to work through the activity.

Materials

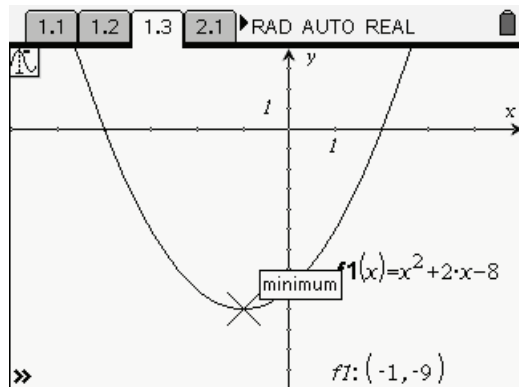
- **Technology:** TI-Nspire CAS handheld or TI-Nspire CAS computer software
- **Documents:** Complex_Roots.tns, Complex_Roots_Student.doc

Exploring Complex Roots — Student Solutions

1. a. $x = -4$ and $x = 2$

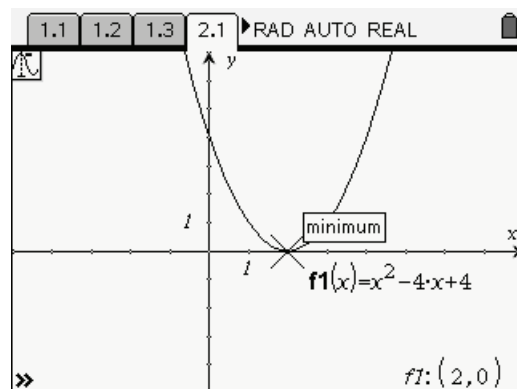
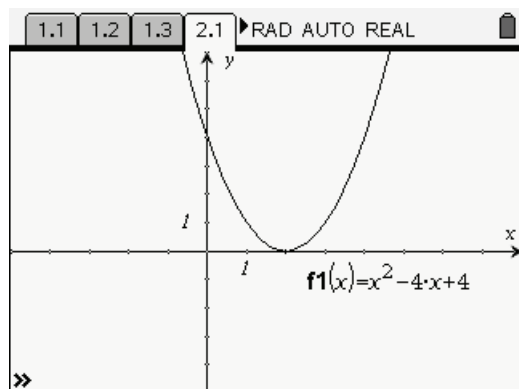


b. Axis of symmetry: $x = -1$; Vertex: $(-1, 9)$



c. The line segment joining the real zeros is perpendicular to the axis of symmetry. The real zeros are equidistant from the axis of symmetry and the vertex.

2. a. One real zero, $x = 2$



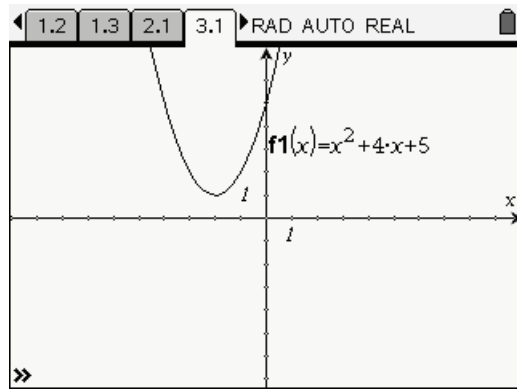
b. Axis of symmetry: $x = 2$; Vertex: $(2, 0)$

c. The real zero lies on the axis of symmetry and passes through the vertex

Exploring Complex Roots

Finding Complex Solutions with Imaginary Parts

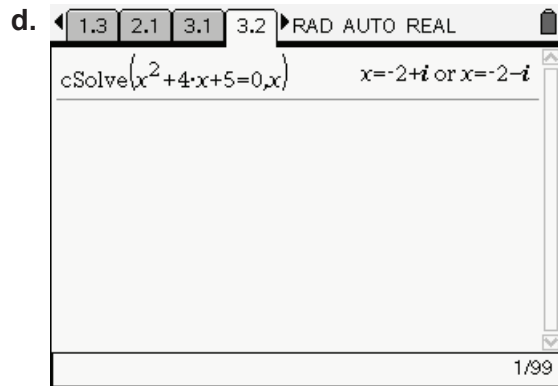
3. a. Axis of symmetry: $x = -2$; Vertex: $(-2, 1)$



b. No real solutions (never intersects x-axis)



c. Real zeros exist if the parabola intersects the x-axis. Complex zeros exist if the parabola does not intersect the x-axis.




Visualizing Complex Roots

4. a. So here we simply need to change the a in front of the $a(x - h)^2$ part of the expression.

The sequence of screenshots illustrates the following steps:

- Graphing the functions $f_1(x) = x^2 + 4x + 5$ and $f_2(x) = -(x+2)^2 + 1$.
- Using the **Point** tool to mark the vertex $(-2, 1)$ and a point on the x-axis.
- Using the **Segment** tool to draw a horizontal line from the vertex to the x-axis.
- Using the **Perpendicular Bisector** tool to draw a vertical line through the midpoint of the segment.
- Using the **Point** tool to mark the intersection of the perpendicular bisector and the parabola.
- Using the **Intersection Point(s)** tool to find the x-intercepts $(-3, 0)$ and $(-1, 0)$.

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- b. At the x -intercepts of the reflected parabola
-  c. $(-3, 0)$ and $(-1, 0)$
- d. They coincide.
- e. If the parabola does not intersect the x -axis, reflect it vertically over its vertex. Find the x -intercepts of this reflection. Rotate these intercepts 90° about their midpoint. The coordinates of these rotated points, written as complex numbers of the form $a + bi$ and $a - bi$ using their x -coordinate as a and their y -coordinate as b , will be the desired roots.

Notes