Scale Factor

ID: 8299

Time required
45 minutes

Activity Overview
Students will dilate polygons and find the perimeter and area of both the pre-image and image. Then they find the ratios of the perimeters and areas. When they change the scale factor (or polygon) all of these values automatically update and students can see that the ratio of the perimeters equals the scale factor and the ratio of the areas equals the square of the scale factor. Students will see that this is true for both enlargements and reductions.

Topic: Transformational Geometry
- *Given a center of dilation, a scale factor k, and a geometric figure, dilate the figure to discover that angles and shapes are preserved under dilations, but lengths are increased by a factor k and areas by k^2.*

Teacher Preparation and Notes
- *This activity is designed for a high-school geometry classroom. In can be used while teaching similar figures or transformations.*
- *Although the definition is reviewed, students should already know that a scale factor is the ratio of the corresponding sides of similar polygons.*
- *This activity is designed to be primarily teacher-led, with breaks for student practice.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files), and student worksheet, go to education.ti.com/exchange and enter "8299" in the keyword search box.***

Associated Materials
- *ScaleFactor_Student.doc*
- *ScaleFactor.tns*
- *ScaleFactor_Soln.tns*

Suggested Related Activities
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.
- *Dilations (TI-Nspire technology) — 8487*
- *Transformers (TI-Nspire technology) — 8772*
- *Similarity and Dilations (TI-89 Titanium) — 1288*

A dilation is a transformation under which the pre-image and image are *similar*, meaning they have the same shape (but not size). Corresponding angles are congruent, and the lengths of corresponding sides are proportional.

Problem 1 – Enlargements

Allow students to explore the figure on page 1.3. Students should investigate, on their own, how the ratios of the lengths of the sides change when a figure is dilated.

The **Text** and **Calculate** tools (from the Actions menu) can be used to display and evaluate the ratios of corresponding sides as shown in the screenshot to the right. Students can click the up arrow to verify that they have computed the correct calculations.
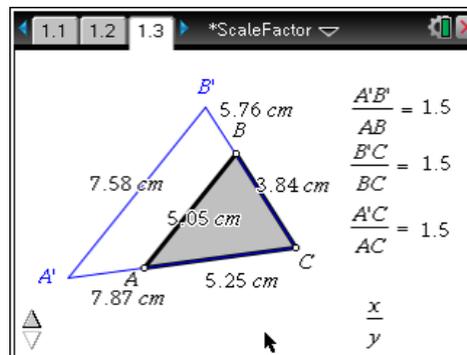

TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 1 at the end of this lesson.

Quadrilaterals *ABCD* and *A′BC′D′* appear on page 1.6. First have students estimate the scale factor that was used to create *A′BC′D′* from *ABCD*. They should record their guesses on their worksheets.

Now, students will find the lengths of each side of both quadrilaterals by using **Length** tool from the Measurement menu. Identify the segment you wish to measure by clicking on its endpoints or by pointing the cursor at it, pressing [tab] until it flashes, and then pressing [enter].
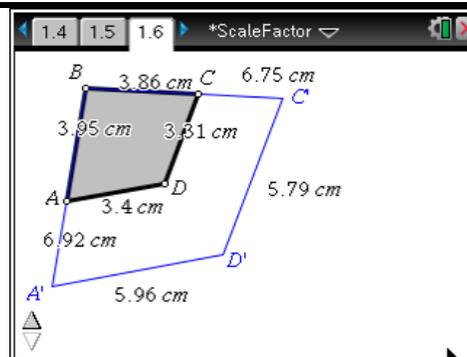
Once more, have students use the **Text** tool to display the ratios of corresponding sides as shown. A "dummy" ratio such as $\frac{x}{y}$ must also be displayed to be used as the ratio that is repeatedly evaluated using the **Calculate** tool. It is important that students then grab and drag the vertices of quadrilateral *ABCD* to observe what happens to these ratios.
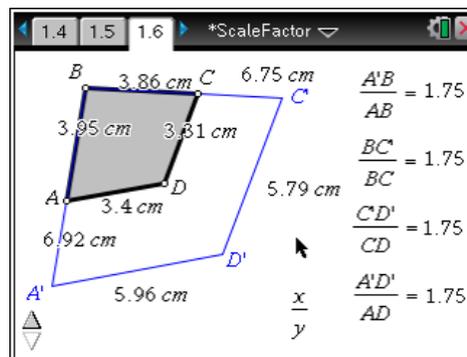
Each of these four values should be equal, as well as remain the same regardless of how quadrilateral *ABCD* is changed. This value is the *scale factor*. Have students record this value on their worksheets.

Next, students should find the perimeter of each quadrilateral by again using the **Length** tool and selecting each quadrilateral.
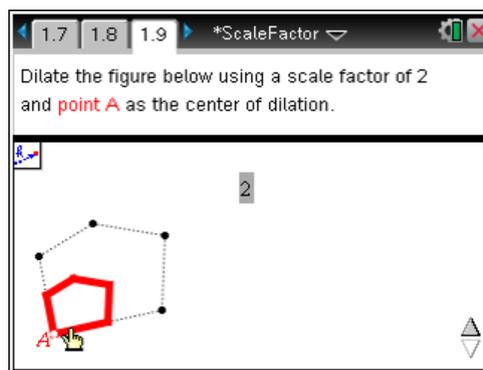
Instruct them to use the ratio expression $\frac{x}{y}$ to find the ratio of the perimeter of the image to that of the pre-image. This information should be recorded on their worksheets.

Students can click the up arrow to verify that they have computed the correct calculations.
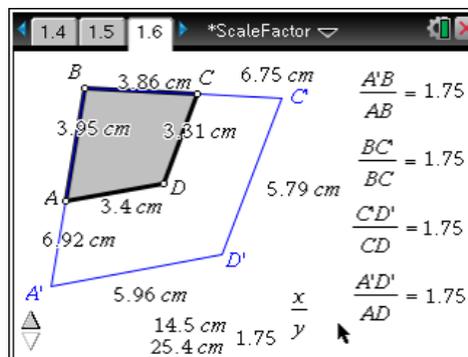
A pentagon is shown on page 1.9. Students will dilate the figure using point *A* as the center of dilation and 2 as the scale factor. (Press `ctrl` + `tab` to move between applications in a split screen).

Before dilating a figure, they must use the **Text** tool and display the value of the scale factor on the screen. Next, select **Dilation** from the Transformation menu. Then they should select, in order, the scale factor, the center of dilation, and the figure that is to be dilated.

Note: Dilating the *figure* and using one of the vertices as the center of dilation (as opposed to dilating the vertices and connecting the image points to form the polygon) may restrict the ability to drag the vertex that was used as the center of dilation. For the purposes of this activity, however, dilating in this manner is more than sufficient.

Students should find the ratio of the perimeter of the image to that of the pre-image. Additionally, they should find the ratio of the *areas* of image to pre-image.

They can use any variables to set up and find expressions for the ratios. Both ratios should be recorded on their worksheets. Again, it is important for students to drag the vertices of the pre-image and observe any changes.
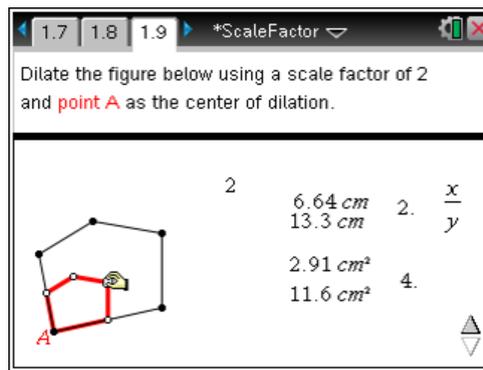
Once students have these values recorded, they should change the scale factor from 2 to 3. When the scale factor is changed, the diagram, perimeters, areas, and ratios will all be updated to reflect that change. These new ratios should be recorded on their worksheets.
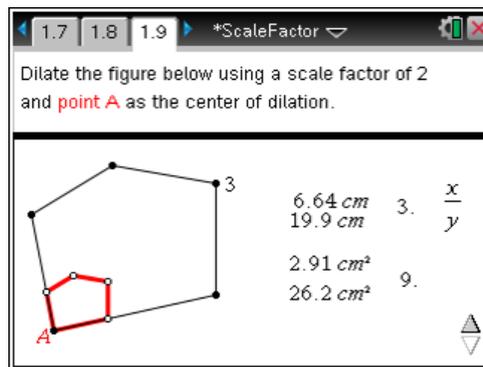
Once more, students should change the scale factor—this time, to 1.5. The displayed measurements and ratios, again, update automatically.

At this point, students should be able to draw a conclusion about the relationship among the scale factor, the ratio of perimeters, and the ratio of areas. That is, the ratio of the perimeters is equal to the scale factor, and the ratio of the areas is equal to the *square* of the scale factor.

Students can click the up arrow to verify that they have computed the correct calculations.
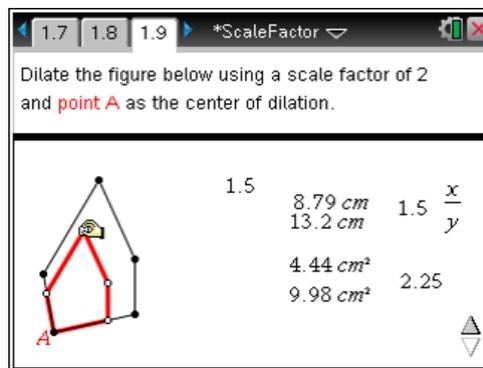
Problem 2 – Reductions

On page 2.1, students will create a hexagon. If students want to create a regular hexagon, they can select **Regular Polygon** from the Shapes menu, press to place the center point, move away from the center point, and press again when the hexagon is the desired size.
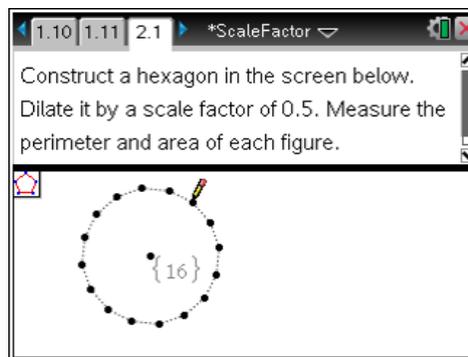
They will then need to move the cursor (using the TouchPad) *clockwise* until the figure has six sides, and press again to set the hexagon in place.
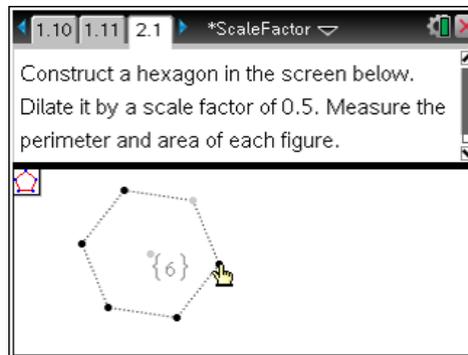
Students can also create their own hexagon using the **Polygon** tool by pressing to place down each vertex of the figure, and pressing on the first vertex to finish the polygon.

Next, students will dilate the figure using a scale factor of 0.5 and the center of the hexagon as the center of dilation. As before, they should find the ratio of the perimeter of the image to that of the pre-image, as well as the ratio of the area of the image to that of the pre-image. Results should be recorded on their worksheets.
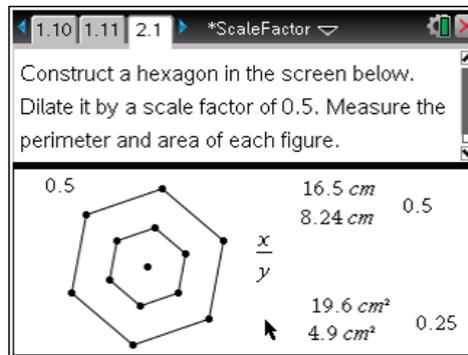
Students might need a reminder that in this instance, the *pre-image* is now the *larger* figure and the *image* is the *smaller* figure.

Now, students should then change the scale factor to 0.3 and then again to 0.7 in an attempt try to draw a conclusion about the relationship between the scale factor and the ratios of perimeters and areas.
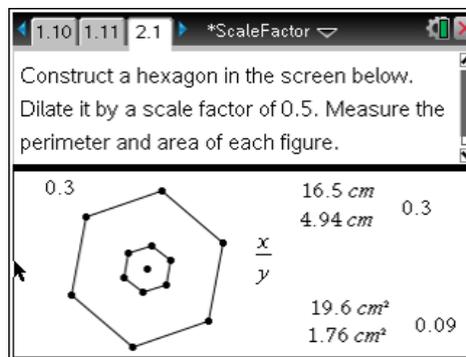
For reductions, as with enlargements, the ratio of the perimeters is equal to the scale factor and the ratio of the areas is equal to the square root of the scale factor.
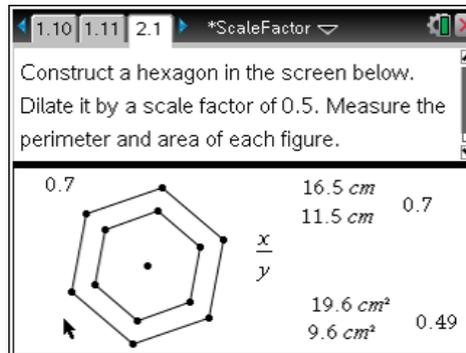
On page 2.4, students will see a problem at the top of the page (the same as Exercise 1 of Problem 2 on the worksheet). The *Scratchpad* should aid in answering the question, as well as the others posed on the worksheet.
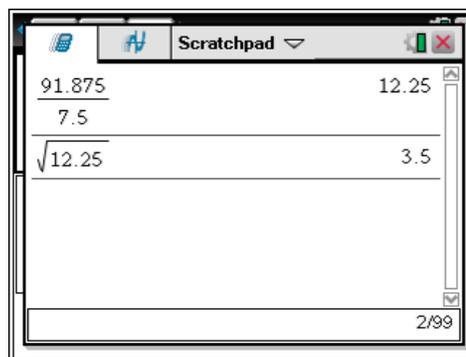
Student should provide explanations for their answers, such as the following (for Exercise 1): *The ratio of the areas is 12.25. This is the square of the scale factor. Take the square root of 12.25 to find that the scale factor is 3.5.*

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Allow students time to answer the remaining problems on their worksheet.

Student Worksheet Solutions:

1. 3.5   **2.** 97.5 ft, 591.5 sq. ft   **3.** 0.2   **4.** 4.5   **5.** 5.25 sq. units

Problem 3 – Extension

By now, students are aware that a scale factor greater than 1 enlarges the figure and that a scale factor between 0 and 1 reduces the figure. Have students dilate the two figures on page 3.1 by the values shown to make a conclusion about dilating figures with a negative scale factor. (The figure is both enlarged/reduced *and* rotated 180°.)

This extension is often best done in the coordinate plane in conjunction with rotations about the origin and reflections over the axes.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Screen Capture*
This would be a good place to do a screen capture to verify students are computing the correct calculations.

Note 2
Problem 2, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.