



Zooming In On Local Linearity

Student Activity

Name _____

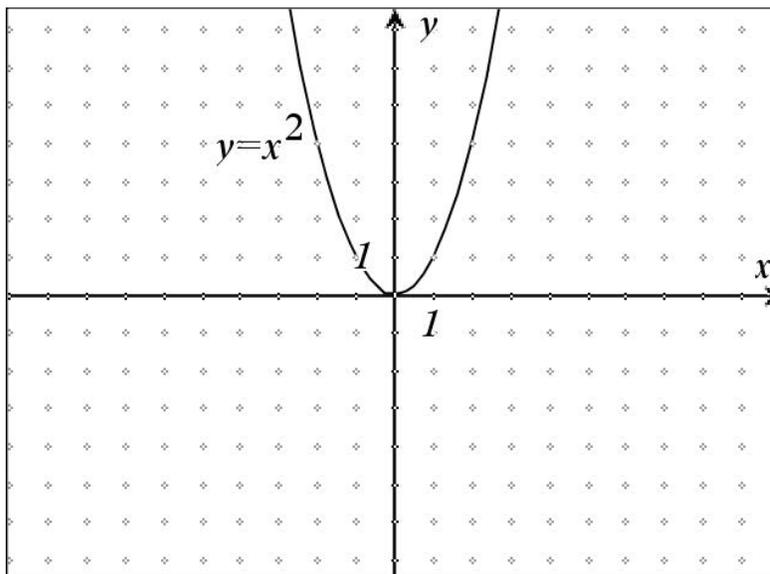
Class _____

Problem 1 – Draw a Tangent Line by Hand

On the graph to the right, draw a line tangent to $y = x^2$ in the first quadrant.

Step 1: Approximate the slope of the line. Show your work.

Step 2: Write the equation of your line.



Problem 2 – Draw and Explore Tangent Lines Using Technology

Press $\boxed{y=}$ and enter the function $Y1=X^2$.

Press $\boxed{\text{zoom}}$ **6:ZStandard** to view the graph.

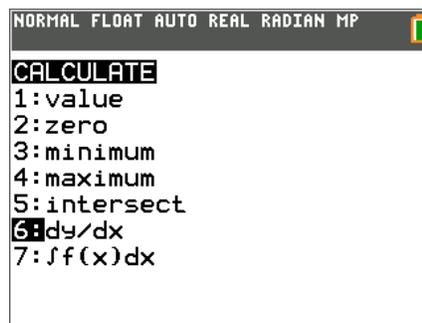
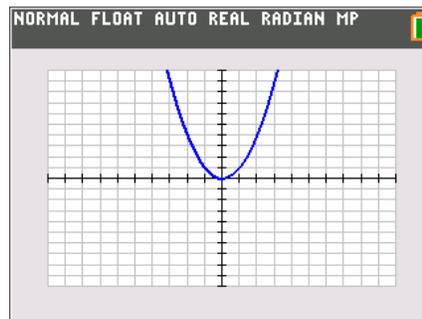
Next, recreate the tangent line you drew in Problem 1. To do this, you will first need to calculate the slope of the tangent at the value of x you previously chose. To do this, from the graphing screen, press $\boxed{2nd}$ $\boxed{\text{trace}}$ [calc] and select option **6:dy/dx**.

Let's zoom in and observe the behavior. Press $\boxed{\text{zoom}}$ and select the option **2:Zoom In**.

Move the cursor near the point of tangency you chose for Problem 1 and press $\boxed{\text{enter}}$.

Zoom in again by repositioning the cursor to the desired x value (if needed) and pressing $\boxed{\text{enter}}$ again.

Repeat the process of zooming in on the point of tangency a couple more times to observe what happens.





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1. Write your observation of how your tangent line and the graph $Y_1(x)=x^2$ compare when examined close up.
2. Will this type of behavior occur for all other functions? Explain your reasoning.

You may want to try it for another function. You can choose your own or try $y = \sin(x)$.

Problem 3 – Graph a Piecewise Function to Explore Local Linearity

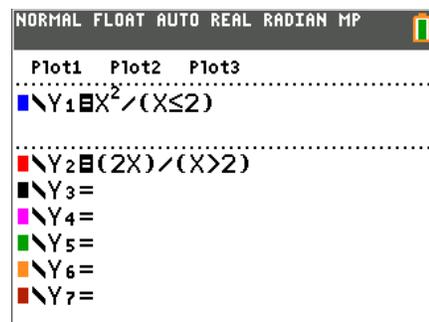
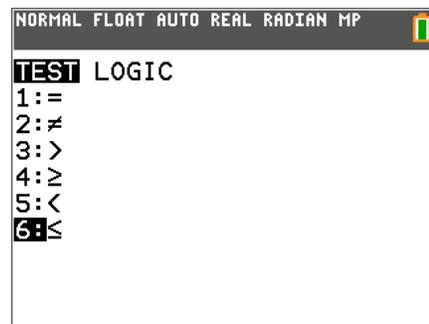
A function is said to be linear over an interval (i.e. locally linear over a small interval) if the slope is constant. Let's discover if all functions have a constant slope when they are examined in a small enough interval.

On your calculator, graph $y = \begin{cases} x^2, & x \leq 2 \\ 2x, & x > 2 \end{cases}$.

To do this you will have to use both the $Y_1=$ and $Y_2=$ entry lines. Enter the two parts of the piecewise function as shown in the screenshot below and to the right.

To type an inequality, press $\boxed{2nd} \boxed{math} \boxed{[test]}$ and select the desired symbol.

First, observe the graph by pressing $\boxed{zoom} \boxed{6:ZStandard}$.





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Discover if all functions have the property of local linearity by zooming in on the point (2, 4) several times.

3. What is happening in the neighborhood of (2, 4) for the piecewise function, $y = \begin{cases} x^2, & x \leq 2 \\ 2x, & x > 2 \end{cases}$?

Does this function demonstrate local linearity at that location?

Problem 4 – Graphing Another Piecewise Function

Graph the function $f(x) = \begin{cases} x^2, & x < 2 \\ 4x - 4, & x \geq 2 \end{cases}$ using the same procedure from Problem 3.

To explore if all piecewise functions lack the property of local linearity, beginning with **Zoom Standard**, and then zoom in on (2, 4) a few times of the function

4. Does this function appear to be locally linear in the neighborhood of (2, 4)? Compare and contrast this function to the one graphed and explored in Problem 3.

Problem 5 – Conclusion

You know the slope is $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

For the function $f(x)$, this can be written as $\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$.

5. If you were finding the slope of function in the interval of a repeatedly zoomed in graph, describe what happens to $\Delta x = x_2 - x_1$.