Triangle Orthocenter Investigation

Investigating the locus of the orthocenter of a triangle as one vertex moves on a line parallel to the opposite side of the triangle -- fitting a quadratic function to this locus.

TI-Nspire instructions

Open Notes Page 1:

Investigation of the locus of the Orthocenter of a triangle.

Insert a Graphs & Geometry page.

Place 2 arbitrary points on the x-axis and label them A and B

Place an arbitrary point somewhere above the x-axis.

Construct a line through this point parallel to the x-axis.

Place an arbitrary point on this line and label it C.

Construct the triangle with vertices A, B & C.
Make the sides of the triangle thick (use the Attributes from the Geometry Menu and select each side in turn and change the line thickness to Medium).

Construct the altitudes from each vertex of the triangle (perpendicular to the opposite side through the vertex point).

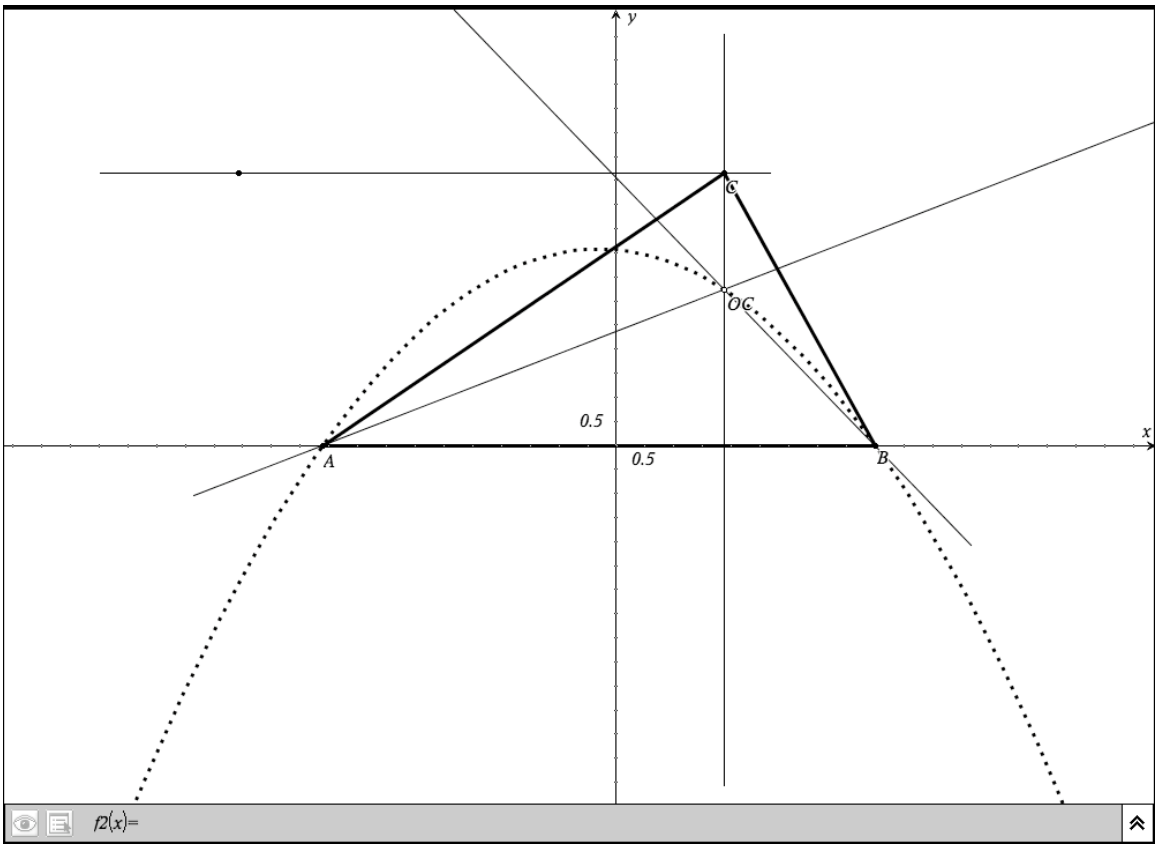
Construct the intersection point of two of the altitudes.
Does the third altitude pass through this point?
Label this point OC (orthocenter).
Construct the locus of point OC with respect to point C.
What shape is this locus? (See next page)

Open Notes Page 2:

Grab and move point C along the line to see the point OC move along its locus.

Grab and move points A and B to see the locus change its shape.

Grab and move the arbitrary point that defines the parallel line on which C resides up and down.
What happens to the locus?


y
C
OG
0.5
x
A
0.5
B
f2(x)=

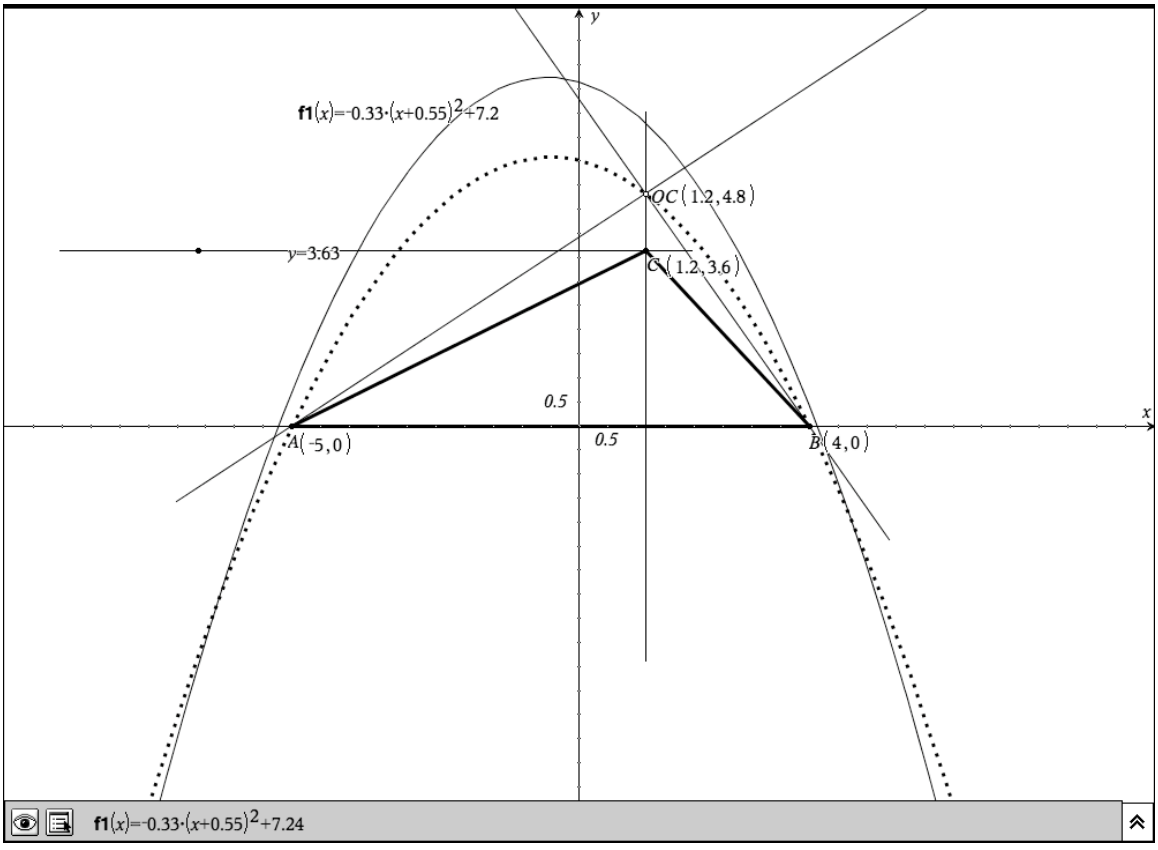
Part 2: Finding the function to fit the locus

Go to the page sorter and select the page with your construction. Use CTRL-C to copy this page.
Insert a new page, select it and paste your construction page on this new page. A copy of your construction page will appear AFTER the new page.
Select this copy and open it.

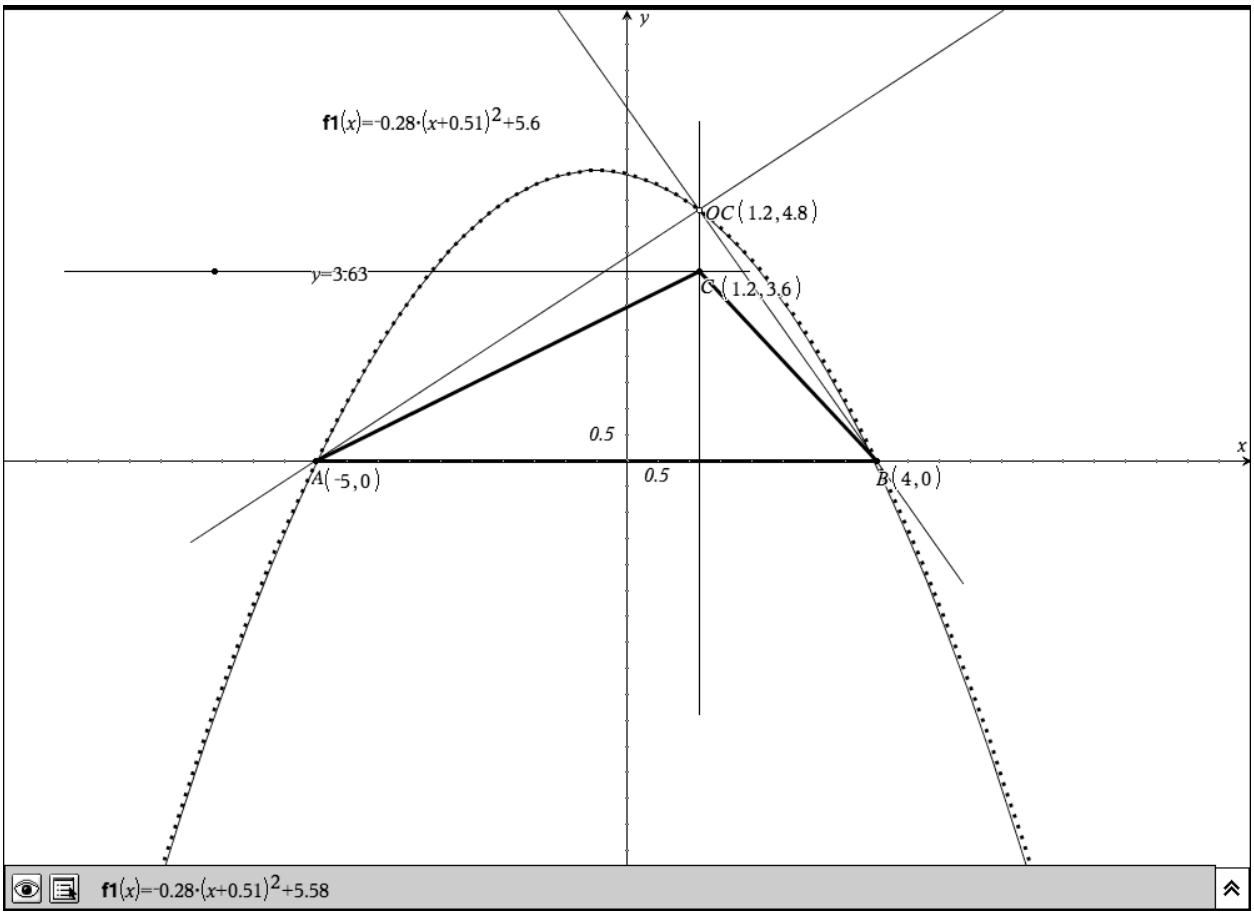
Go to the Menu of this new page and select 3:Graph Type, 1:Function.

In the function line enter an expression for a quadratic function that opens downward.

Move your cursor near the bottom edge of this parabola. The cursor should change to a short segment with arrows on each side of it. Grab the parabola and dilate it to coincide with the locus.



Move your cursor close to the vertex of your function parabola. The cursor should change to small cross hairs. Grab the vertex and move the parabola until the vertex coincides with the vertex of the locus parabola. You may now need to readjust the ends of your function parabola until they again coincide with the locus. You should be able to fit your quadratic function to the locus in this way (see last page).



On your new page with both function and locus parabolas, show the coordinates of your triangle vertices and the equation of the parallel line. See if you can find a connection among these measurements and the equation of the quadratic that fits your locus.