A Piece of Pi: An Investigation into Geometric Probability and Pi

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Activity Overview

Have you ever noticed in how many places π turns up? Students will take part in a Buffon's Needle experiment and see how close they can get to 3.14. To set up the lesson, there will be a brief historical tour of Pi as well as a clip from NUMB3RS, SEASON 2, Judgment Call, where Charlie is showing how throwing paperclips simulates the classical Buffon's Needle Problem. This episode can be downloaded from iTunes®. Next, the students will do an experiment and then transfer the data collected to the TI-nSpire where it will be analyzed and interpreted.

Concepts: Pi

- *Historical development of pi*
- *Relationships between real numbers and geometry*
- *Analyze aspects and properties of pi*

Teacher Preparation

Prior to the activity, the teacher needs to assemble all materials needed.

Materials needed:

- *Each team needs ten toothpicks,*
- *lined paper/poster board on which to drop the toothpicks (the distance between the lines needs to be equal to the length of the toothpicks you are using),*
- *the aPieceofPi.tns file and the aPieceofPiTeacherVersion.tns file,*
- *and the teacher/student activity sheets.*

You will need to load the student .tns file on the class handhelds. Be sure to note into what folder the file saved. Connect to Class® can be used to accomplish this quickly.

This investigation is set up to be a cooperative learning activity. If possible, pre-select the group to save time.

It would also be a good idea to familiarize yourself with Buffon's Needle experiment. Several sites have historical information as well as a discussion of the actual investigation.

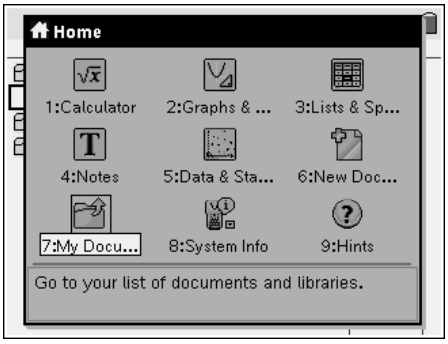
You can do the activity without showing the NUMB3RS clip; however, it is an excellent anticipatory set for your instruction.

The Classroom

Begin the classroom discussion by asking students what they know about the number pi. The first part of this document gives a brief historical overview of pi. Students can find out more by going to the Mac Tutor History of Mathematics Archives website: http://www-history.mcs.st-and.ac.uk/ . Discussing the history of the development of mathematics increases the student's interest in and appreciation of mathematics.

Most high school students should be familiar with the number pi. The history of pi is addressed in questions one through seven on the student activity sheet and Problem 1 of the document. This is a good opportunity to review the real number system and the classifications of different types of numbers. Students may need a refresher on this for question 7.

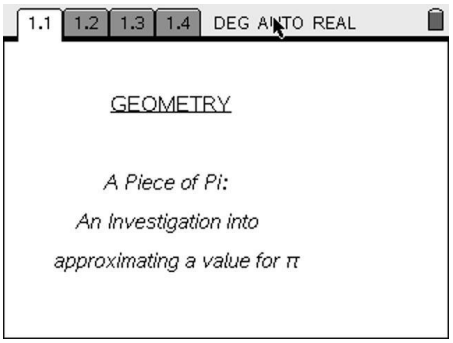
To view the document press: (home) 7



Using the Navigation Pad arrows, ◆▲▼, scroll down to the folder containing the document titled "A Piece of Pi."

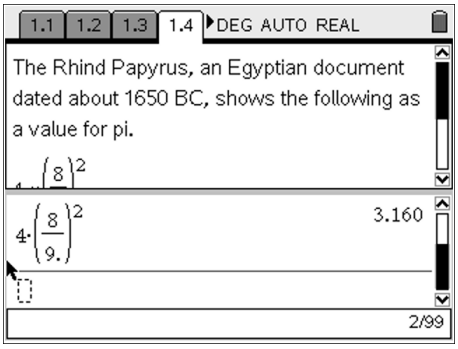
You will need to tell the students in which folder the .tns file is located. Select that folder.Then select the file.

If prompted with "Do you want to save…?" press (tab) to select "No" and press (enter). You should now see the screen to the right.



On page 1.4 of the document, the bottom half of the page is a calculator screen for the students to use to give an estimate for what value of pi was found in the Rhind Papyrus.

The students may need to know that to toggle between the notes part of the screen and the calculator part of the screen they press (ctrl) (tab).

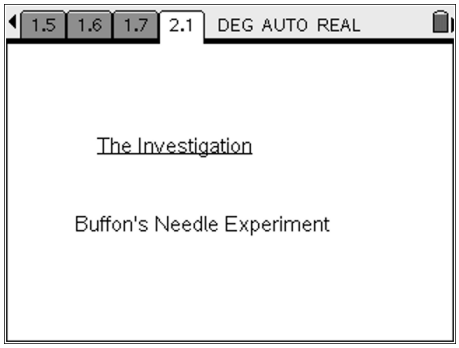


The students will also need to know if a decimal is placed at the end of any of the numbers the answer will be given in decimal form. Otherwise, the answer will be in fraction form.

To convert a fraction to a decimal press (menu) 2 1.

When the students finish with question seven there is a "PC – TC". This is a peer check and/or a teacher check. Be sure to take time to see if the students have any questions.

Before the students begin the investigation, make sure they have all the needed materials.



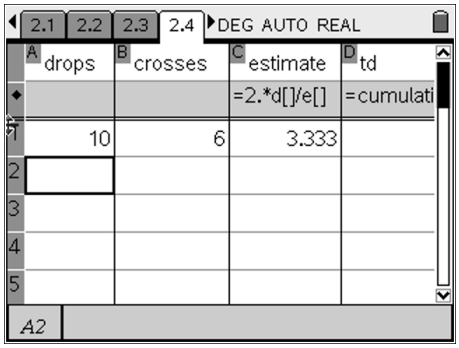
Set up the *NUMB3RS* clip for the students.

Don and Charlie investigate the murder of a judge's wife who was shot in her garage. It is unclear whether she was the intended target or her husband, who was hearing a death penalty case involving a gang leader. Charlie uses another example where needles randomly falls into line of sheet and everything relates to PI and it is called as Buffon's needle. PI seemingly appears in quantum mechanics, relativity theory and number theorem and remained unanswered. Charlie concludes same goes with this suspect, named Dolan.

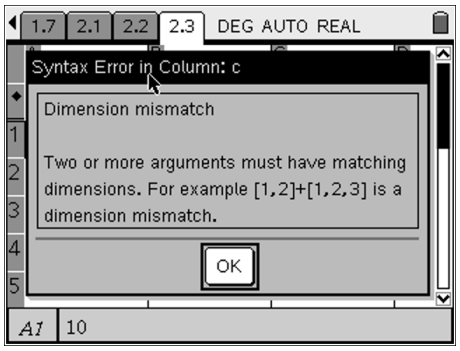
Show the *NUMB3RS* clip. Tell the students that they are now going to perform Buffon's needle investigation, but with toothpicks.

The formula $E = \frac{2d}{c}$ can be used to determine an estimate for pi. "E" is the estimate, "d" is the TOTAL number of drops, and "c" is the TOTAL number of crosses or touches. The students will first be asked to calculate their estimate for pi on screen 2.3, a calculator page.

Before the students work with screen 2.4 a discussion about how to read a spreadsheet application. Have them pay close attention to columns vs. rows.

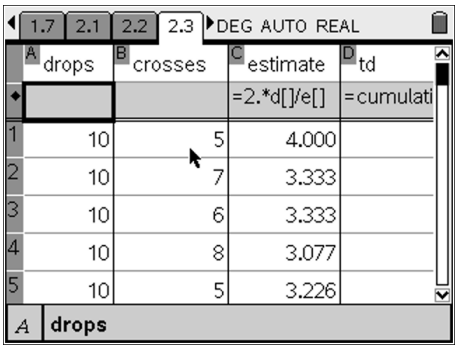


When they go to screen 2.4 on their handheld. the first row is already filled in with an example set of data. They will need to simply OVERWRITE the number of crosses or touches they had with the number in Column B, Row 1. They will see the calculation for the estimate of pi in Column C.

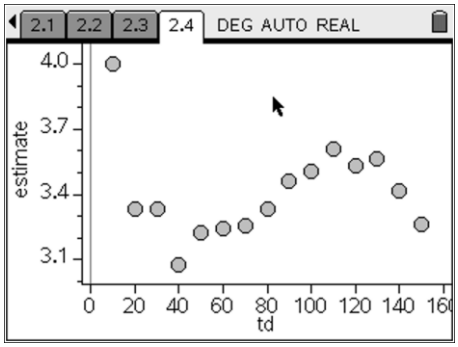


Because Column C has been defined to be the quotient of twice the number of total drops with the total number of crosses, the students will get an error message every time they fill in the first column's value. Just have them click OK and it will go away.

Give the students time to perform the investigation. A sample screen with data from a team with five members is shown.



After the investigation, the students are to collect the other teams' data in the class. Be prepared to give directions on how each group will report out.



Students will need to be monitored closely as they discuss and answer the last set of questions.

Solutions to questions

1. 3.14 or $\frac{22}{7}$
2. Pi is the ratio of the circumference of a circle to its diameter.
3. Answers may vary. Some students may have done the investigation in middle school where they wrap string around circular containers, measure the string and the diameter and then create the ratio of circumference to diameter to determine pi.
4. 3.16
5. 3.141, 3.143
6. Check students' answers. Be sure they give an explanation for why they think it would be true.
7. Irrational
8. No

9. No

10. Questions 10 – 23 will have varied student responses. Check their work and be sure to discuss these with them.

A Piece of Pi

Name: _____

Student Worksheet

Period/Block: _____

In this activity, you will explore the number pi, π.

Historical Overview:

1. One of the most well known "numbers" in mathematics is pi, π. As a student, you have been using π since probably elementary and middle school. What value do you commonly associate with π?

   $\pi \approx$ _____

2. Write a definition of π:

It is now time to turn on your TI-Nspire.

Press: (home) 7

Using the Navigation Pad arrows, ◆◆▲▼, scroll down to the folder containing the document titled "A Piece of Pi." Your teacher can tell you in which folder it is located if you do not see it. Select that folder. Then select the file.

If prompted with "Do you want to save…?"
Press (tab) to select "No" and press (enter).
You should now see the screen to the right

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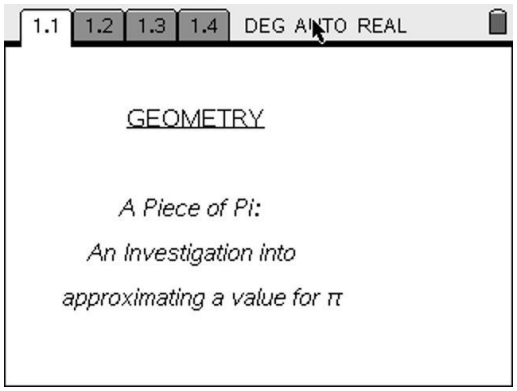
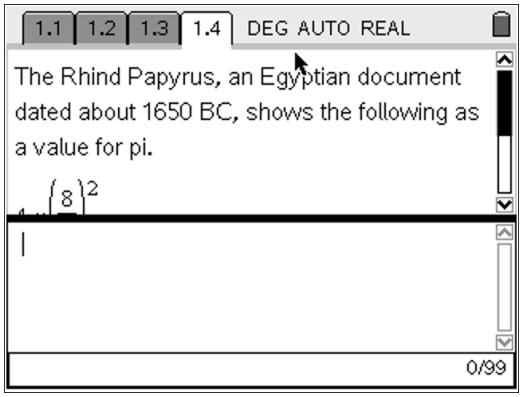


Note that you will first review some history of pi.

Press (ctrl)▶ to move to the next page in the document. Likewise, press (ctrl)◀ to move back in the document. Read pages 1.2 and 1.3. Then discuss these pages with your group.

3. A way you could estimate a value for pi using measurement is:

4. On page 1.4 of the document, the bottom half of the page is a calculator screen. Use the calculator to give an estimate for what value of pi was found in the Rhind Papyrus.

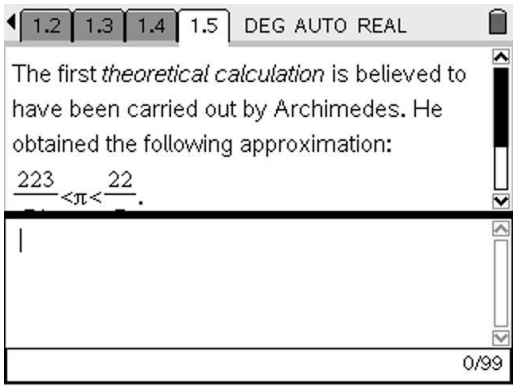


Press ctrl tab to toggle between the notes part of the screen and the calculator part of the screen. Round the answer to two decimal places.

Note: If you will place a decimal at the end of any of the numbers your answer will be given in decimal form. Otherwise your answer will be in fraction form. To convert to a decimal press menu 2 1.

$\pi \approx$ _____

5. On page 1.5 of the document, use the calculator part of the screen, at the bottom, to determine an estimate for the range of values between which Archimedes calculated pi.



_____ < π < _____

Move to screen 1.6 and 1.7 and read.

6. I think pi could be correctly computed to _____ decimal places. Give a brief explanation of why you think this would be true.

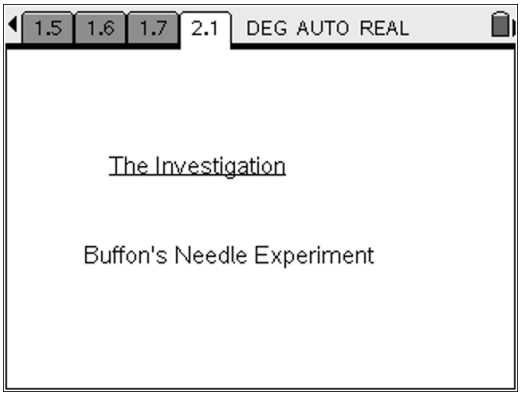
7. What type of number is pi, in other words, how is pi classified in the real number system?

Before beginning the investigation, discuss the questions above with a peer of your teacher.

_____ **PC** _____ **TC**

The Investigation:

Watch the clip from the episode *Judgment Call* from the *NUMB3RS* television show.



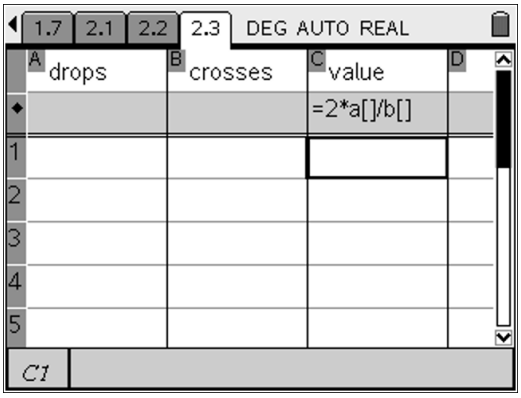
8. Charlie explains to Don and his team how pi can "…just show up in seemingly random places." Can he explain why?

9. Is there an explanation for why?

Buffon's Needle is a simple Monte Carlo method for the estimation of the value of pi.

The experiment is quite simple.

Suppose you have a piece of paper with a number of equally spaced parallel lines drawn on it. If you drop a "needle" that is the same length as the space between the parallel lines on the paper, you find that one of two things happens: (1) the "needle" crosses or touches one of the lines, or (2) the "needle" does not cross any lines. To perform the experiment you keep dropping this "needle" repeatedly on the paper, and record the statistics. You need to keep track of both the total number of times that the needle is randomly dropped on the table (we will call this d), and the number of times that it crosses or touches a line (we will call this c). If you keep dropping the needle, eventually you will find that the number 2d/c approaches the value of pi.



Your “needles” will be toothpicks. Your teacher will provide you with the paper on which you can drop the toothpicks. You will drop 10 toothpicks at a time and count the number of crosses or touches. Each of your team members will perform the experiment and keep track of all of your data.

When you have the materials you need for the investigation go to screen 2.3 in the “Piece of Pi” document. Begin the experiment and record your results in the chart above. Also record each of your team members results in the chart.

You should now have as many rows filled in as team members.

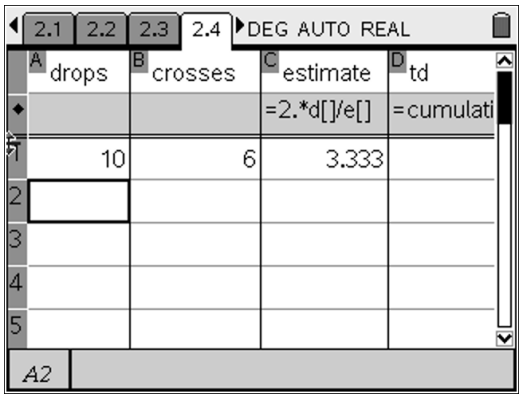
The formula $E = \frac{2d}{c}$ can be used to determine an estimate for pi. “E” is the estimate, “d” is the TOTAL number of drops, and “c” is the TOTAL number of crosses or touches.

Calculate your estimate of pi. Use Screen 2.3 to perform your calculations, if needed.

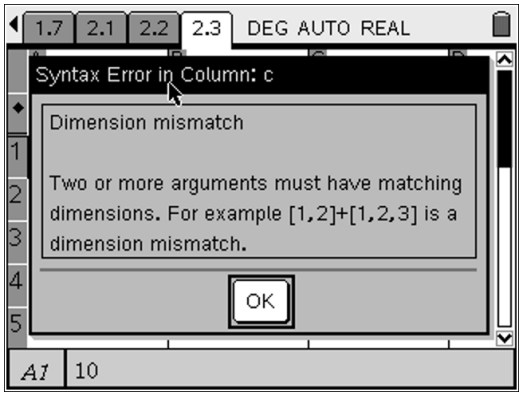
10. Record your estimate for pi here. :

$\pi \approx$ _____

Now go to screen 2.4 on your handheld. You will notice that the first row is already filled in with an example set of data. Simply OVERWRITE the number of crosses or touches you had with the number in Column B, Row 1. You will see the calculation for the estimate of pi in Column C.



Note: Because Column C has been defined to be the quotient of twice the number of total drops with the total number of crosses, you will get an error message every time you fill in the first column’s value. Just click OK and it will go away. Once you enter the data into Column B, you will see your estimate for pi in column C.



BE SURE to check here with your teacher if you have any questions.

Before continuing the investigation, discuss the investigation above with a peer or your teacher.

_____ **PC** _____ **TC**

Now, collect your team members' data and enter their data in the rows following yours.
Column C has been defined to keep a "running total" calculation.

11. Record your team's estimate for pi here, once you have input ALL of their data:

$\pi \approx$ _____

12. Discuss how close you are to the value of pi. Be sure to state reasons why you think your estimation is what it is.

13. Do you think if you had more data samples your value would change? If so, how would it change?

Now let's collect some more data. As you are finished with your data, report out so the rest of the class can see your results. You should only be reporting the *last row* of your teams' drops and crosses.

Your teacher will direct you on what method to use to accomplish this.

Input the other groups' data into the next row on your handheld.

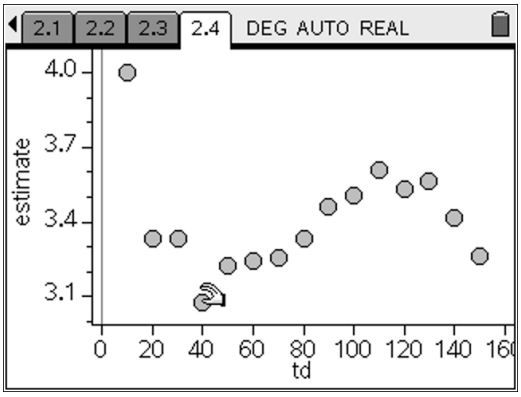
14. Once you have input ALL of your classes' data record the class value for pi below.

    $\pi \approx$ _____

15. Compare and contrast each of the other teams' estimates to yours in a few brief sentences.

16. Would you now adjust your answer to 13? Why or why not?

17. Now go to Screen 2.5. How does you graph of total drops vs. the estimate for pi compare to the graph below? Be specific.



Discuss each of the following questions in your group before answering. Be sure to look at your handhelds' graph as you answer. Give specific evidence to support your answers.

18. Does the approximate value for pi become increasingly more accurate with each successive 10 toothpicks dropped?

19. Does the approximate value for pi become increasingly more accurate with each successive 20 toothpicks dropped?

20. Does the approximate value for pi become increasingly more accurate with each successive 50 toothpicks dropped?

21. How close to the real value of pi would you expect the approximate value to be after 500 toothpicks were dropped? After 1000?

22. Is there an optimal number of toothpicks dropped that would produce the best approximation of pi? If so, can you determine that number?

23. Do you think Buffon's Needle experiment serves as a good approximation of pi? Why or why not?

Extensions

1. There are several Buffon's Needle simulations available on the Internet. The investigation you did assumed that the length of the "needle" was equal to the distance between the parallel lines on the paper. Use the following websites to investigate what would happen if the length of the needle was altered.

   http://www.ms.uky.edu/~mai/java/stat/buff.html

   http://www.mste.uiuc.edu/reese/buffon/buffon.html

2. Investigate Buffon's Coin Problem at the following website.

   http://lhome.wlu.edu/~mcraea/GeometricProbabilityFolder/Introduction/Problem1/Problem1.htm

3. Investigate how trigonometry and calculus could be used in deriving an approximation for the value of pi in the Buffon's Needle Experiment.