# Transformations of Exponential Functions

## Math Objectives

- Students will explore the family of exponential functions of the form $f(x) = c \cdot b^{x+a}$ and be able to describe the effect of each parameter on the graph of $y = f(x)$.
- Students will be able to determine the equation that corresponds to the graph of an exponential function.
- Students will understand that a horizontal translation and a vertical stretch of the graph of an exponential function are essentially the same.
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

## Vocabulary

- exponential function
- translation
- vertical dilation
- parameter
- reflection

## About the Lesson

- This lesson involves the family of exponential functions of the form $f(x) = c \cdot b^{x+a}$.
- As a result students will:
  - Manipulate parameters, and observe the effect on the graph of the corresponding exponential function.
  - Conjecture and draw conclusions about the effect of each parameter on the graph of the exponential function.
  - Compare horizontal translation and vertical dilations and manipulate equations to demonstrate they are the same.
  - Match specific exponential functions with their corresponding graphs.

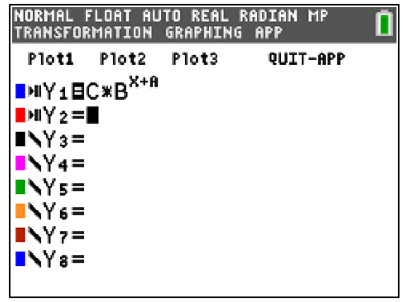
## Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems.

## Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

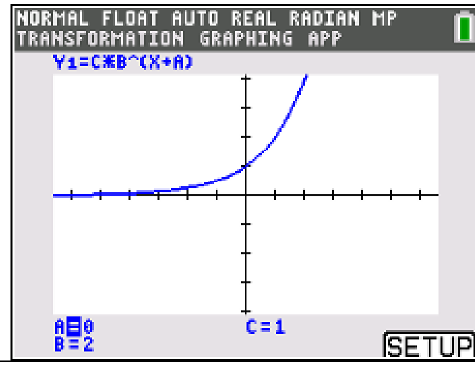



**Tech Tips:**

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

**Lesson Files:**
*Student Activity*
Transformations of Exponential Functions_84_Student.pdf
Transformations of Exponential Functions_84_Student.doc

In this activity, you will examine the family of exponential functions of the form $f(x) = c \cdot b^{x+a}$ where $a$, $b$, and $c$ are parameters. You will use the **Transformation App** (Transfrm) on your handheld to manipulate these parameters in Questions 1 - 3.




**Discussion Points and Possible Answers**

**Tech Tip:** To change the parameters throughout this activity using the **Transformation App** on the handheld, you will be using the arrow keys. Up and down move you from parameter to parameter, left and right change the value of each parameter. While on the graph, you can press Setup (graph) to manually change the parameters, including using decimals, or just type in the number you want while your cursor is on the parameter. You can use up to two functions in the app and they can be typed into $Y_1$ or $Y_2$.

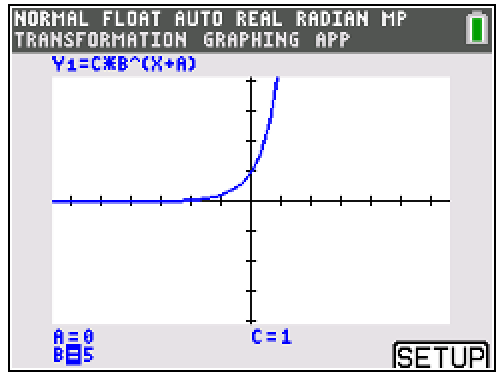
The parameter $b$ is the base of the exponential function and $b > 0, b \neq 1$. Using the transformation application change the value of a parameter by entering the equation for each question into $Y_1$ or $Y_2$, and press the arrow keys to manipulate each parameter of the function on the graph.

**Question 1**

1. Graph the following function: $Y_1 = B^x$. Press the arrows to change the value of $B$, and observe the changes in the graph of $Y_1$.
   a. Explain why for every value of $B$ the graph of $Y_1$ passes through the point $(0,1)$.

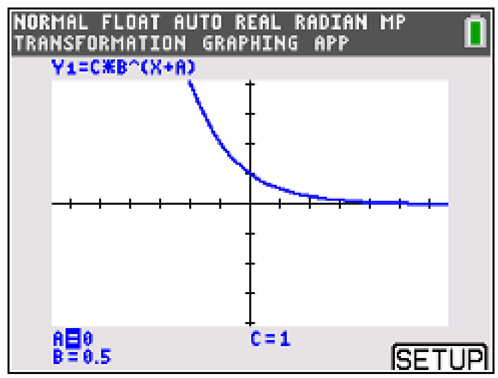
**Sample Answers:** The graph of $y = f(x) = b^x$ passes through the point $(0,1)$ for all values of $b > 0$ because $f(0) = b^0 = 1$. The *y*-intercept of the graph of $f$ is 1.

b. For $B > 1$, describe the graph of $Y_1 = B^x$.




**Sample Answers:** The graph is above the $x$-axis and is always increasing. As $x$ takes on smaller and smaller negative values ($-10, -100, -1000, \ldots$), the values of $f$ get closer to 0. In more precise mathematical language, we would say as $x$ decreases without bound, $b^x$ approaches 0. As $x$ gets larger and larger (10, 100, 1000,…) the values of $f$ get larger and larger. In more precise mathematical language, as $x$ increases without bound, $b^x$ also increases without bound. As $b$ gets larger, the graph becomes steeper, or increases more rapidly. As $b$ gets closer to 1, the graph becomes less steep approaching the graph of the line $y = 1$.

c. For $0 < B < 1$, describe the graph of $Y_1 = B^x$.




**Sample Answers:** The graph is above the $x$-axis and is always decreasing. As $x$ gets smaller and smaller $(-10, -100, -1000, \ldots)$ the values of $f$ increase without bound. As $x$ gets larger and larger (increases without bound), the values of $f$ get smaller and approach 0. As $b$ gets closer to 0, the graph becomes steeper. As $b$ gets closer to 1, the graph becomes less steep and approaches the graph of the line $y = 1$.

**Teacher Tip:** Teachers might need to remind students that a negative exponent inverts the fraction $b$. The reciprocal of a fraction between 0 and 1 is a number greater than 1.

d. Find the domain and range of function $Y_1 = B^x$ for all possible values of $B$.

**Answer:** The domain is all real numbers, and the range is all positive real numbers: $(0, \propto)$.

e. Does the graph of $Y_1 = B^x$ intersect the $x$-axis? Explain why or why not.

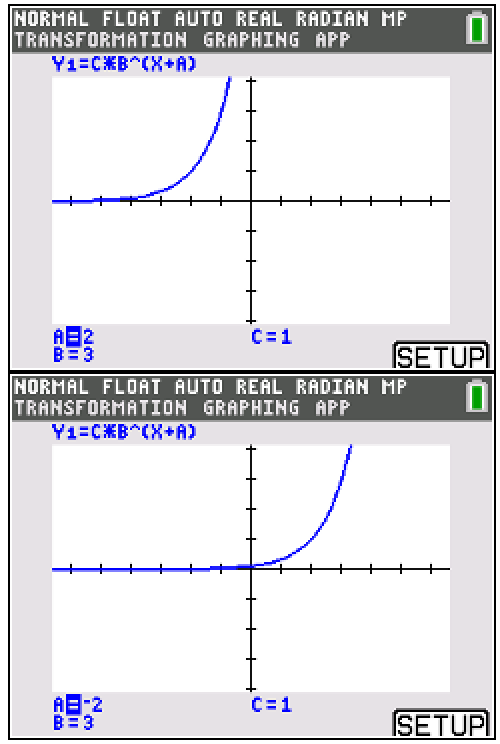
**Answer:** For $b > 1$: as $x$ decreases without bound, the graph of $y = b^x$ approaches the $x$-axis but never touches it. In limit notation, $\lim_{x \to -\infty} f(x) = 0$. For $0 < b < 1$: as $x$ increases without bound, the graph of $y = b^x$ approaches the $x$-axis but never touches it. In limit notation, $\lim_{x \to \infty} f(x) = 0$. The $x$-axis, the line $y = 0$, is a horizontal asymptote to the graph of $y = b^x$.

**Tech Tip:** The limited resolution on the handheld screen might result in a graph that appears to intersect the $x$-axis. This presents an opportunity to trace the graph and/or create a table of values to show that values of the function are small, but not equal to zero.

**Question 2**

2. Graph the following function: $Y_2 = B^{x+A}$. For a specific value of $B$, click the arrows to change the value of $A$, and observe the changes in the graph of $Y_1$. Repeat this process for other values of $B$. Describe the effect of the parameter $A$ on the graph of $Y_2 = B^{x+A}$. Discuss the effects of both positive and negative values of $A$.

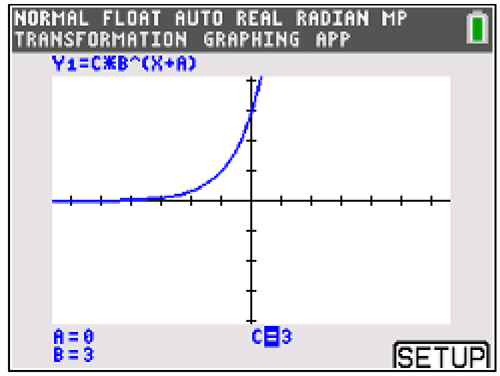
   **Answer:** For $a > 0$, the graph of $y = b^x$ is translated horizontally, or moved, left $a$ units. For $a < 0$, the graph of $y = b^x$ is translated right $a$ units.




**Teacher Tip:** This left/right translation occurs for any value of $b$.
Horizontal translations of the graph of an exponential function are difficult to recognize because students often focus on the $y$-intercept and vertical shifts. Emphasize that the horizontal asymptote ($y = 0$) did not change (move up or down) which would have happened if there were a vertical shift in the graph.

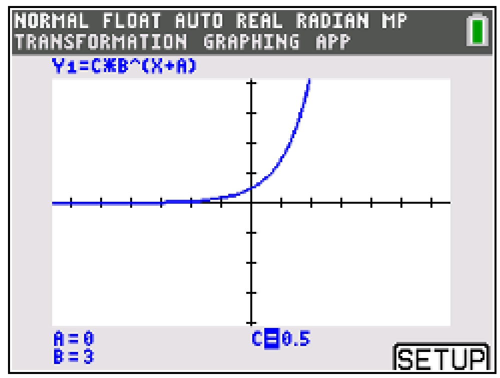
**Question 3**

3. Graph the following function: $Y_2 = C \cdot B^{x+A}$.. For specific values of $A$ and $B$, click the arrows to change the value of $C$, and observe the changes in the graph of $Y_1$. Describe the effect of the parameter $C$ on the graph of $Y_2 = C \cdot B^{x+A}$. Discuss the effects of both positive and negative values of $C$.

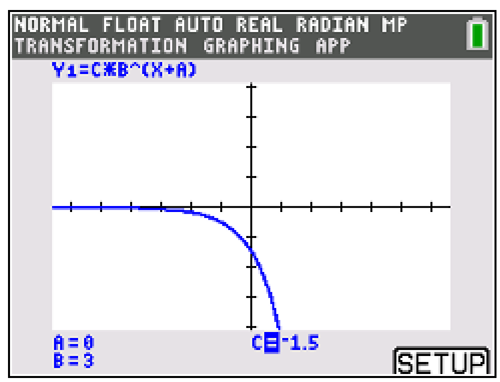



**Answer:** The graph has a vertical dilation. For $|c| > 1$, the graph of $y = b^{x+a}$ is stretched vertically.

For $|c| < 1$, the graph of $y = b^{x+a}$ is compressed vertically.

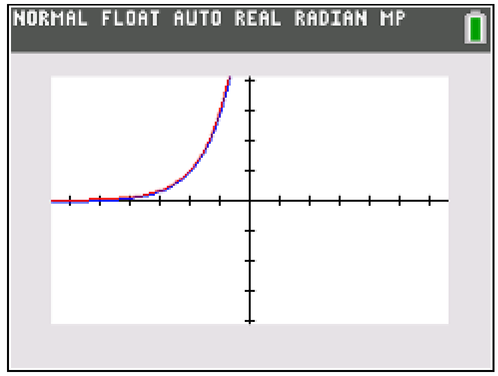



If $c < 0$, the graph is reflected across the $x$-axis.



**Question 4**

4. Turn off the Transformation App by selecting Quit-App on the $y =$ screen. Graph each function given and answer the following questions.




a. Display the graphs of $Y_1 = 3^{x+2}$ and $Y_2 = 9 \cdot 3^x$.

(i) How is the graph of $Y_2$ related to the graph of $Y_1$?

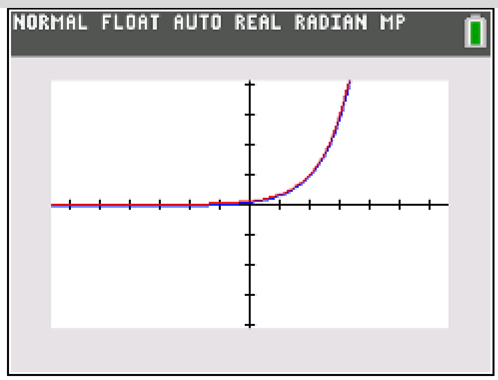
**Answer:** The graphs of these two exponential functions are the same.

(ii) Use the properties of exponents to justify your answer.

**Answer:** $Y_1 = 3^{x+2} = 3^x \cdot 3^2 = 9 \cdot 3^x = Y_2$

**Teacher Tip:** The thickness of the second function could be changed to thin so that both graphs are visible.

b. Display the graphs of $Y_1 = 3^{x-2}$ and $Y_2 = \left(\frac{1}{9}\right) \cdot 3^x$.




(i) How is the graph of $Y_2$ related to the graph of $Y_1$?

**Answer:** The graphs of these two exponential functions are the same.

(ii) Use the properties of exponents to justify your answer.

**Answer:** $y_1 = 3^{x-2} = 3^x \cdot 3^{-2} = \left(\frac{1}{9}\right) 3^x = Y_2$.

c. Use your answers to parts (a) and (b) to explain the relationship between a horizontal translation and a vertical dilation of the graph of an exponential function.

**Answer:** A horizontal translation and a vertical dilation of the graph of an exponential function are essentially the same. Consider the following expression to show this analytically.

$$y = b^{x+a} = b^x \cdot b^a = c \cdot b^x$$

This demonstrates that any horizontal translation can also be considered a vertical dilation.

**Question 5**

5. Without using your calculator, match each equation with its corresponding graph.
   Check your answers by graphing each function on your calculator.

(a) $f(x) = 3^{x-4}$

(b) $f(x) = -\left(\frac{1}{3}\right)^{x}$

(c) $f(x) = (0.7)^{x-4}$

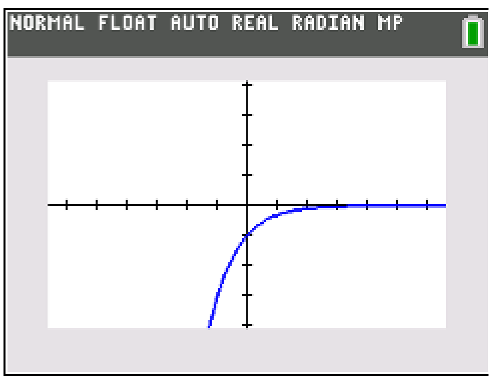
(d) $f(x) = -2\,(0.1)^{x+3}$

(e) $f(x) = e^{x}$

(f) $f(x) = -\left(\frac{1}{2}\right) \cdot \pi^{x}$

Note: The function in part (e) is the "natural" exponential function and involves the number $e \approx 2.71828...$

(i)

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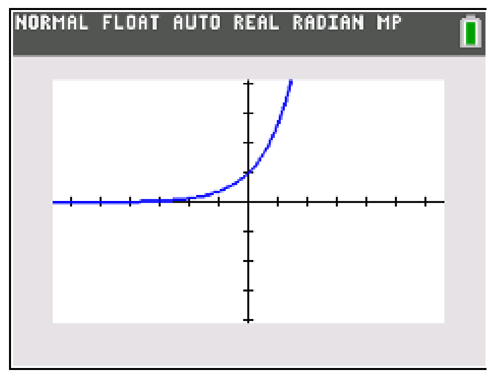


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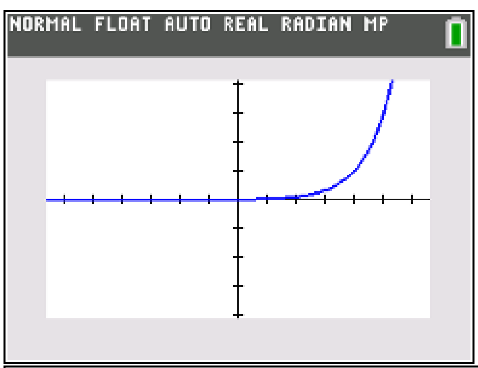


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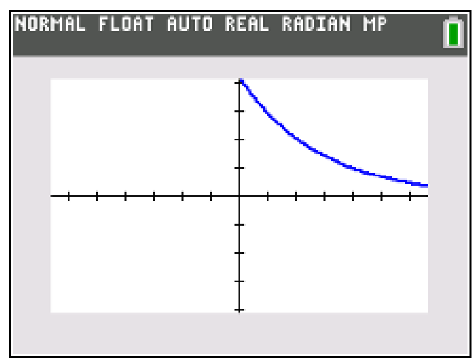


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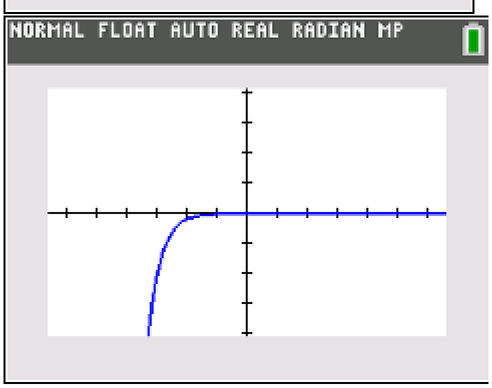


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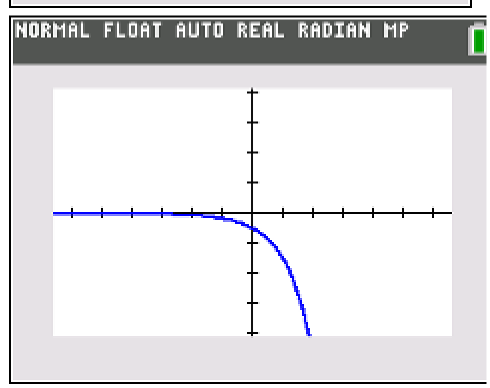

**Answers:** (a) → (iii) (b) → (i) (c) → (iv) (d) → (v) (e) → (ii) (f) → (vi).

**Extensions**

1. Ask students to display the graph $f(x) = 3^{2-x}$ and to find the domain.
2. Ask students to display and compare the graphs of $f(x) = \left(\frac{1}{3}\right)^{-x}$ and $f(x) = -\left(\frac{1}{3}\right)^{x}$.

**Wrap Up**

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to graph an exponential function of the form $f(x) = c \cdot b^{x+a}$.
- How to explain the concepts of dilations and translation.