Exploring Inverse Functions

Student Worksheet

Name ______________________

Class ______________________

Problem Statement

In this activity, you will explore inverse relations and functions. Two relations are said to be inverses if they "undo" each other. For example, if a relation maps 5 to 2, then the inverse relation maps 2 back to 5. In general for a function, if $f(a) = b$, then its inverse function is denoted $f^{-1}(b) = a$.

Inverse Point-by-Point

1. One way to find the inverse of a relation is to switch the mapping of the x- and y-coordinates of each ordered pair in the relation. That is, the inverse of the point (x, y) is (y, x) on the inverse relation. Find the inverse of this relation: $\{(2, 5), (-4, 5), (-9, -2), (0, -3)\}$.

2. Now that we have defined the inverse of a relation by the "undoing" nature it has for the mapping of points, what does this definition mean when we think of the relation graphically? On page 2.2 of the *CollegeAlg_Inverses.tns* file, you will find the function $f(x) = x^3$ graphed and a point on the function labeled. Under the Construction menu, use the **Measurement Transfer** tool to map the x-coordinate of the labeled point onto the y-axis and the y-coordinate of the point onto the x-axis. To do this, click on the number in the ordered pair followed by the axis you desire.

   Now "plot" the point on the inverse function by constructing a line perpendicular to the x-axis, passing through the point on the x-axis. Similarly, do the same for the point on the y-axis. Now mark the intersection point of the two perpendicular lines. This is the inverse point of the point on the original function, $f(x) = x^3$.

   Move the original point by dragging it. Describe how the inverse point moves.

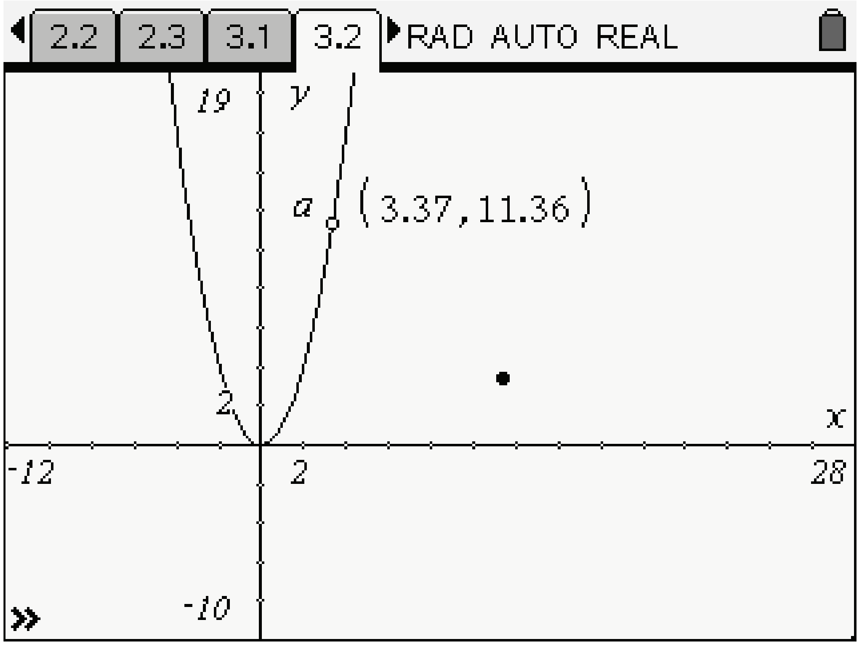
Inverses as a Graphical Relation

3. On page 3.2, you will find the graph of $f(x) = x^2$, along with point a on the graph. The inverse of point a is also shown, created in the same way we did in the last question. On this graph, we have hidden the perpendicular lines so that we can observe the movement of the inverse point more easily.

   a. Drag point a and describe how its inverse point moves.

   b. Does the point move in a manner consistent with your observations from question 2?

c. To display the entire collection of inverse points, we can use the **Locus** tool found under the Construction menu. To do this, after selecting the **Locus** tool, click on the inverse point followed by the original point on the function. You will see the "path" of the inverse point appear. Describe the shape of the inverse relation. Does it match your earlier sketch? Sketch a path for your point below.



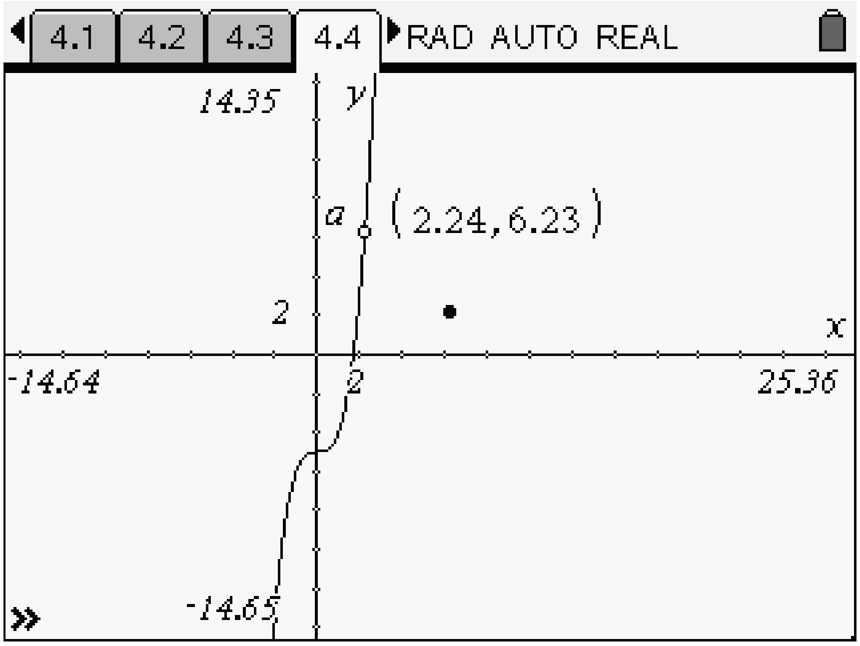
d. What relation does the locus of points appear to represent?

Inverses as Functions

4. In the previous problem, the graph of the inverse relation was not a function (evident by the fact that it does not pass the vertical line test). However, the original relation was a function (as it does pass the vertical line test).

a. Describe how you might test the graph of a relation to see if its inverse is a function.

b. Decide if the inverse of the graph of $y = x^3 - 5$ shown on page 4.4 of the *CollegeAlg_Inverses.tns* file is a function by checking for intersections with horizontal lines. Display the locus of the inverse points to confirm your conclusion. Make a sketch of your graph below.



Finding Inverses Algebraically

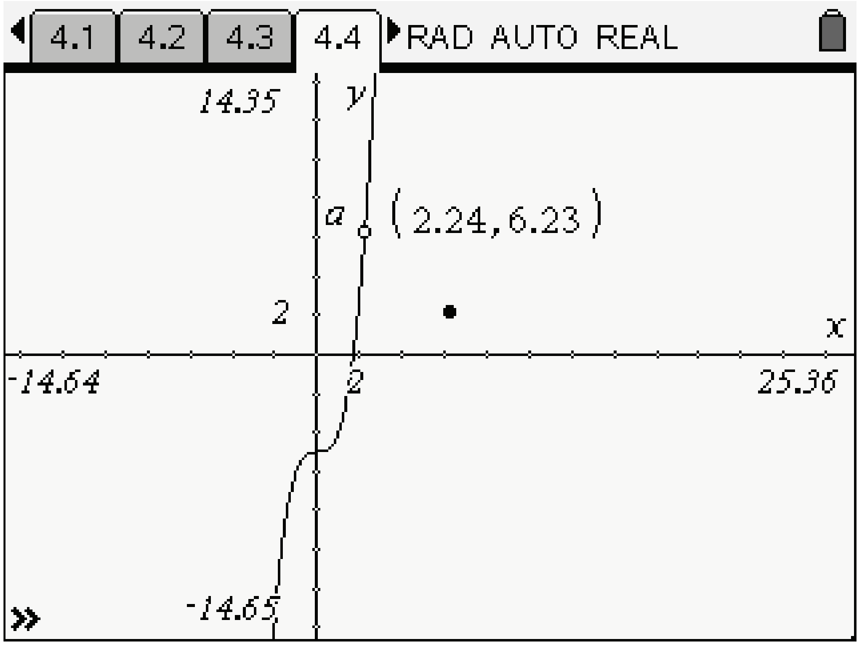
5. So far, this activity has focused on the method of switching *x*- and *y*-values on a local or graphical scale (one point at a time) to find the inverse of a function. It is now time to switch to a global scale—to algebraically calculate the inverse of a function. Here we will require that the inverse be a function as well.

To understand how to find the inverse function of a given function, we must first consider another way that the inverse is understood. Recall from elementary school that for even basic operations on numbers such as addition and multiplication, we have the relationship between inverse elements and the identity element. The identity element is the element under the operation that leaves everything alone. For example, under addition, 0 is the identity since $5 + 0 = 5$, leaving 5 unchanged. Under multiplication, 1 acts as the identity since $7 \cdot 1 = 7$, leaving 7 unchanged. So what are the inverses in these cases? Consider that $5 + -5 = 0$ and $7 \cdot \frac{1}{7} = 1$. Here the idea is that if you combine an element with its inverse, you get the identity element.

On the *Calculator* page 5.4, define a function, call it $e(x)$, that you think will leave all other functions unchanged when composed with them. Then test your "identity" function by composing it with other functions you define. Do your other functions remain unchanged? Continue until you find an identity function. Is your identity function surprising to you? Explain.

6. The idea of an inverse is to get the identity function when it is composed with the original function. Based on your observation from the last question, this would mean that $f(f^{-1}(x)) = x$, where $f^{-1}(x)$ represents the inverse function of $f(x)$. On the *Calculator* page 6.2, define $f(x) = x^3 - 5$. Now to find the inverse function, we would like $f(y) = x$, where y here is representing the inverse function (note the "switching" of the x- and y-coordinates as we did in the graphical swap in question 2).

a. Use the **solve(** command under the Algebra menu to solve the equation $f(y) = x$ for y. The syntax for the **solve(** command is **solve(f(y)=x,y)**. Give your inverse function and go back to page 4.4 and graph it. Does it match the locus you created? Sketch the graph of your inverse function below.



b. Now go back to page 6.2 and define your new inverse function as $g(x)$. Compose $f(g(x))$. Do you get the expected identity function? Explain why your composition result should yield just x.