EXPLORATIONS™

Activity 5

Objectives

- Investigate the changes on the graph of a quadratic equation that result from changes in *A, B,* and *C*
- Locate the vertex of a parabola when given its quadratic equation expressed in standard form

Investigating the Parabola in Vertex Form ($y = ax^2 + bx + c$)

Introduction

In Algebra class you study two forms of the quadratic function: the standard and the vertex forms. Any quadratic function can be expressed in either form. In Activities **3** and **4** you studied the vertex form, $y = a(x - h)^2 + k$. In this activity you will investigate the standard form of the quadratic function, $y = ax^2 + bx + c$.

A Look Back At Activity 3

Note: *If you have not completed Activity* ***3****, you can either go to that activity first or read through the questions that follow. If you have already completed Activity* ***3****, you may want to skip to the next section.*

1. What is the vertex of the parabola $y = (x - 3)^2 + 5$? Does it open up or down? Enter the equation in your graphing handheld and check your answer.

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2. What happens to the graph if the equation is changed to $y = 2(x - 3)^2 + 5$?

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How about $y = {}^-5(x - 3)^2 + 5$?

____

Enter the equations in your graphing handheld and check your answers.

3. What effect does changing the value of *A* in the vertex form of the function have on the vertex of a parabola?

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4. What are the coordinates of the vertex of the parabola $y = 2(x + 3)^2 + 5$?

$x =$ ____________________ $y =$ ____________________

When you remove the parentheses and rewrite the vertex form to the standard form, ($y = ax^2 + bx + c$), you get:

$y = 2(x + 3)^2 + 5$

$y = 2(x^2 + 2*3*x + 9) + 5$

$y = 2(x^2 + 6x + 9) + 5$

$y = 2x^2 + 12x + 18 + 5$

$y = 2x^2 + 12x + 23$

5. What relationship exists between the x-coordinate of the vertex and the values of A, B and C in $y = ax^2 + bx + c$?

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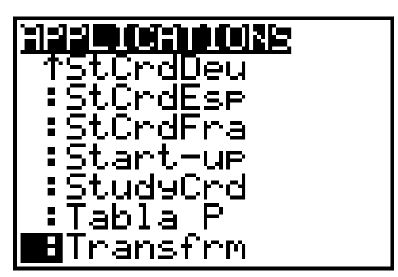
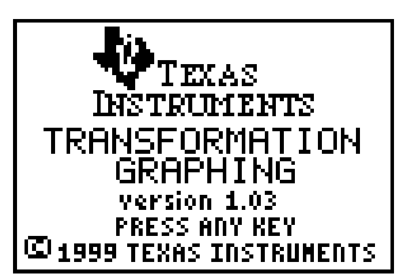
Investigating the Effects of *A*, *B*, and *C*

1. Press [APPS] and select **Transfrm** and press [ENTER].



2. Press any key (except [2nd] or [ALPHA]) to start the Transformation Graphing App.

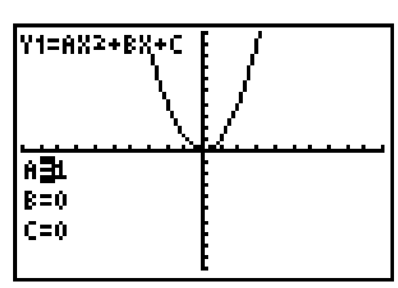
Note: *If you do not see the screen shown, select* **Continue**.



3. In **Func** mode, press [Y=] to display the **Y=** editor. Clear any functions that are listed, and turn off any plots.

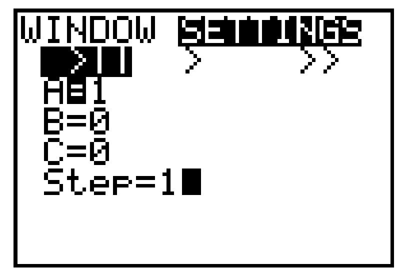
4. At **Y1** enter **AX²+BX+C**, the standard form of the quadratic function. (Press [ALPHA] **A** [X,T,Θ,n] [x^2] [+] [ALPHA] **B** [X,T,Θ,n] [+] [ALPHA] **C**.)

If **Play-Pause** mode (**>||**) is not selected at the left of **Y1**, press [◄] until the cursor is over the symbol; then press [ENTER] until the correct symbol is selected.



5. Press [WINDOW] [▲] to display the **SETTINGS** screen for the Transformation Graphing App.

   Enter the values shown. These settings define the starting values for the coefficients and the increment by which you want the coefficients to change.

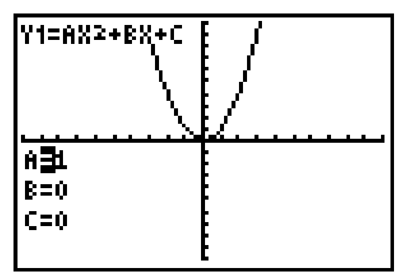


   Before you start your investigation, what effect do you suspect changes in the value of *C* will have on the graph of the parabola?

   ____

Investigating the Effect of *C*

1. Press [ZOOM] **6** to select **6:ZStandard** and display the graph. The graph will show the pre-selected values of *A*, *B*, and *C*. Both the *x*- and *y*-axis will be set to display the graph between $^{-}10$ and 10 with a scale of 1.



2. Press [▼] [▼] to highlight **C=**. Press [▶] to increase the value of *C*. What happens to the graph as the value of *C* increases?

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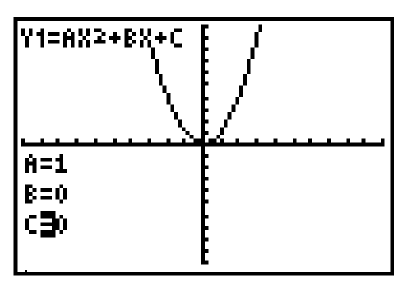

3. Press [◀] to decrease the value of *C*. What happens when *C* decreases?

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   Describe the change that you would expect to see if *C* were changed to $^{-}2$ while **A=1** and **B=0**. Set **C=$^{-}$2**. Was the change what you expected?

   ____

4. Use the [▲] key to highlight **B=** and change the value of *B* to any value. Highlight **A=** and then change the value of *A* to any positive value. Highlight **C=**. Increase and decrease the value of *C*.

   Is the effect on the graph different from what it was when **A=1** and **B=0**? Explain your answer.

   ____

Questions for Discussion

1. When $A = 1$ and $B = 0$, what is the relationship between the vertex and the *y*-intercept?

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2. Does the same relationship hold if $A \neq 1$ or 0, but $B = 0$? If $B \neq 0$?

   ___

3. By knowing the value of *C*, what point on the graph do you know?

   ___

4. What is the *y*-intercept of a parabola if $C = 3$? If $C = {}^{-}1$? If ${}^{-}0.5$?

   ___

5. Do the values of *A* and *B* affect the *y*-intercept of a parabola?

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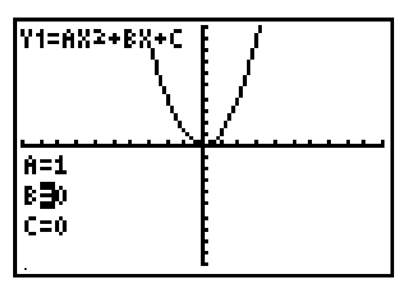
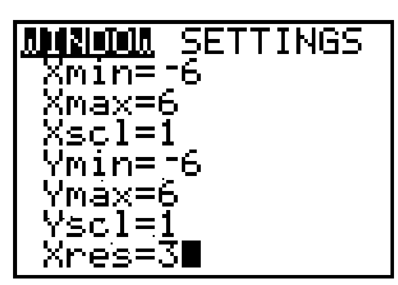
Investigating the Effects of *A* and *B* Together

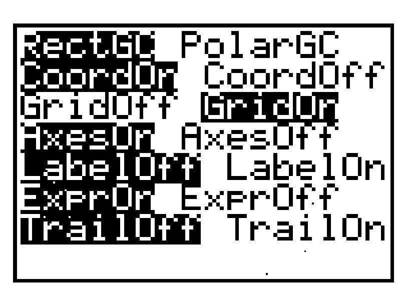
1. Reset the values of *A, B*, and *C* as shown.



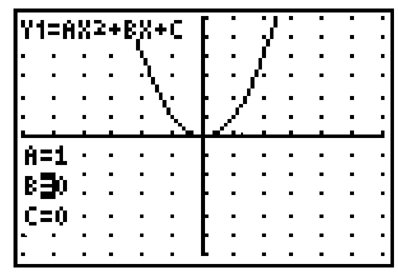
2. Press [WINDOW] and change the **WINDOW** settings to those shown.



3. Press [2nd] [FORMAT]. Using the cursor keys, highlight **GridOn** and press [ENTER]. With the **GridOn**, you can see the next segment of this activity better.



4. Press [GRAPH] to return to the graph screen.



Questions for Discussion

1. With **A=1**, increase the value of *B*. As *B* increases what happens to the location of the vertex of the parabola?

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2. Decrease the value of *B* a few times. What happens to the location of the vertex as *B* decreases?

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3. Did the *y*-intercept change as *B* changed?

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4. Change the value of *B* to fill in the table.

Value of *B*	-2	-1	0	1	2	4
Value of *A*	1	1	1	1	1	1
***x*-coordinate of the vertex**						

5. What relationship did you notice between the value of the *x*-coordinate and the value of *B* (with $A = 1$)? Use that relationship to find the *x*-coordinate of the vertex when $B = -4$ and when $B = 3$.

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6. Set **A=2, B=0**, and **C=0**. When the value of *A* is changed from **1** to **2**, what happens to the graph of the parabola? Does the vertex change?

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7. Use the Transformation Graphing App to change the value of *B* and complete the table.

Value of *B*	-2	-1	0	1	2	4
Value of *A*	2	2	2	2	2	2
***x*-coordinate of the vertex**						

8. Now investigate the effect of doubling the value of *A* on the *x*-coordinate of the vertex of a parabola. Fill in the table below.

Value of *B*	-2	-1	0	1	2	4
***x*-coordinate when *A* = 1**						
***x*-coordinate when *A* = 2**						

9. Make a hypothesis about the effect on the *x*-coordinate of the vertex when the value of *A* is doubled, assuming the value of *B* does not change.

10. Write an expression relating the values of *A* and *B* to the *x*-coordinate of the vertex.

11. Test your hypothesis by using your expression to complete the table below.

Value of *B*	1	2	3	4	6	10
Value of *A*	2	5	1	-2	0.5	-4
What your expression says the *x*-coordinate should be						
Actual *x*-coordinate of the vertex	-0.25	-0.2	-1.5	1	-6	1.25

12. The relationship between *A*, *B*, and the *x*-coordinate of the vertex can be expressed as: $\frac{-b}{2a}$ = the *x*-coordinate of the vertex. Does this equation match the one you discovered? Use this equation to check the values in the table above.

Quick Check of What You Have Learned

As a way of reviewing what you have observed, complete the table below. Check your answers by graphing the parabola on your graphing handheld.

Equation	A	B	C	*x*-coordinate of the vertex	*y*-intercept
$y = x^2 + 3x - 1$					
$y = 2x^2 + 3x - 1$					
$y = {}^-2x^2 + 4x + 5$					
$y = \frac{1}{2}x^2 - 2x + 3$					
$y = {}^-3x^2 - 6x - 10$					

Discuss and Extend What You Have Learned

1. If you know the *x*-coordinate of the vertex, how do you find its *y*-coordinate?

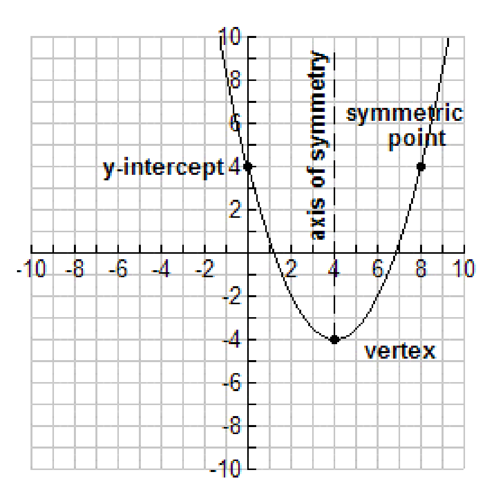
2. Find the coordinates of the vertex of $y = 2x^2 + 8x + 3$. What is the *y*-intercept?

3. What happens when $A < 0$? How about if A = 0?

Putting It All Together

You can sketch the graph of a parabola if you know its vertex and at least one other point. You know that the parabola is symmetric around its axis of symmetry.

To graph a parabola in $y = ax^2 + bx + c$ form, plot its vertex, *y*-intercept, and a symmetric point to the *y*-intercept. Connect the points to form a parabola.



As practice, complete the table.

	Vertex	*y*-intercept	Symmetric point
1.	(1, 3)	(0, 5)	
2.	(1, ⁻4)	(0, 0)	
3.	(⁻3, ⁻2)	(0, ⁻3)	
4.	(⁻4, 2)	(0, ⁻1)	
5.	(4, 2)	(0, 1)	

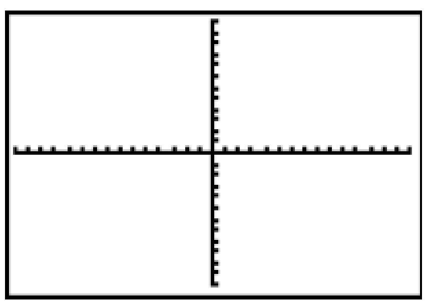
Student Worksheet

Name ______________________

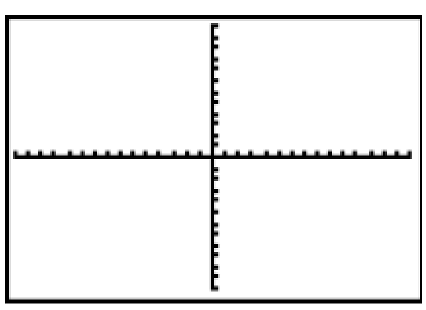
Date ______________________

List the vertex, the *y*-intercept and sketch the graph for each of the equations.

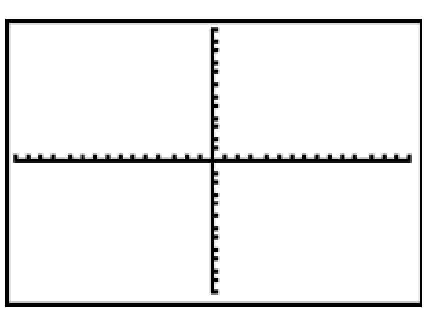
1. $y = x^2 + 4x - 1$



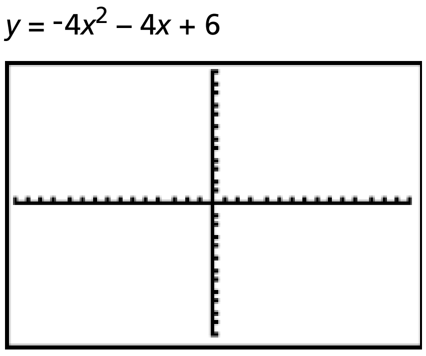
2. $y = 2x^2 + 8x - 3$



3. $y = 0.5x^2 - x + 3$



4. $y = {}^{-}4x^2 - 4x + 6$



Discuss and extend what you have learned.

5. What is the maximum value of the equation $y = {}^{-}x^2 - 6x + 20$?

6. How do you know the equation $y = 2x^2 + 5x + 15$ has no absolute maximum value?

7. For what values of *A* will the function have an absolute maximum value? Absolute minimum?

8. A method was given for sketching the graph of a parabola by plotting the vertex, the *y*-intercept, and a symmetric point. For what value or values of *B* will this method, as it is given, not work? Explain your answer.

Teacher Notes

Activity 5

Investigating the Parabola in Vertex Form ($y = ax^2 + bx + c$)

Objectives

- Investigate the changes on the graph of a quadratic equation that result from changes in *A, B,* and *C*
- Locate the vertex of a parabola when given its quadratic equation expressed in standard form

Materials

- TI-84 Plus/TI-83 Plus

Time

- 60 minutes

This activity is intended to allow students to study the effects of changes to the values of *A*, *B*, and *C* on the graph of the quadratic function when given in standard form, $y = ax^2 + bx + c$. As a result of this activity, students will be able to graph a parabola in standard form by determining the vertex and then using the *y*-intercept and a symmetric point to plot the curve. This activity is appropriate for Algebra 1 and Algebra 2 students. If used in an Algebra 1 class, more teacher involvement might be required.

This is an investigation activity and as such, students should be encouraged to explore different values for the coefficients and to develop hypotheses based on these observations. Keep students focused on developing hypotheses for finding the vertex and *y*-intercept of a parabola written in standard ($y = ax^2 + bx + c$) form. The last section of the Student Worksheet, Discuss and Extend What You Have Learned, revisits questions about maximum and minimum values, which students studied in Activity **3**.

This is an initial activity on this form of the quadratic function. The concept of symmetry is used but is not discussed. If this might create a problem for the students, a short discussion after the section labeled Quick Check of What You Have Learned would be appropriate.

It is not necessary that students study the vertex form before doing this activity. The first section was given as a quick review and to set the stage. Students can be equally successful with this activity whether or not the vertex form has been studied.

The section on the effect of changes in *C* on the graph is fairly straightforward and will probably need little to no explanation.

The activity can be used either as a single student investigation or as a group or even whole class study. No matter which instructional method is used, students need to perform all of the investigations on their own. If students work in groups, they should discuss the relevant mathematical concepts in each section. If groups are not used, the teacher should be sure to check that students have met the instructional objectives after each investigation and before moving to the Student Worksheet. It is important to bring closure to the section, Investigating *A* and *B* Together.

Students are expected to deduce the $\frac{-b}{2a}$ relationship in this section, but you should clarify this relationship to make sure that all students have understood it.

Be sure to have your students de-activate the Transformation Graphing App when the session is completed. You also might remind your students to remove the **GridOn**.

Answers

For many problems answers may vary. Possible answers are included below.

A Look Back At Activity 3

1. (3, 5); up
2. It appears narrow. The second time it appears wider.
3. No, it has no effect on the vertex.
4. ($^-3$, 5)
5. x-coordinate = $\frac{-b}{2a}$

Investigating the Effects of A, B, *and* C

5. Answers will vary. Students should predict that it will change its vertical position.

Investigating the Effect of Changes in C

2. The parabola moves upward.
3. The parabola moves down. The vertex and y-intercept would become $^-2$.
4. The effect of changes in C on the graph is the same.

Questions for Discussion

1. The vertex and y-intercept are the same.
2. As long as $B = 0$, the relationship holds. If $B \neq 0$, the vertex and y-intercept are not the same.
3. The y-intercept
4. 3; $^-1$; $^-0.5$
5. No

Investigating A and B Together: Questions for Discussion

1. If it is on the negative side of the *y*-axis it moves both left and down.
2. If it is on the positive side of the *y*-axis it moves both right and down.
3. No the *y*-intercept remained the same. Changes in *A* and *B* do not affect the *y*-intercept.
4.

Value of *B*	-2	-1	0	1	2	4
Value of *A*	1	1	1	1	1	1
***x*-coordinate of the vertex**	1	0.5	0	-0.5	-1	-2

5. In the table the *x*-coordinate of the vertex appears to be the negative of $\frac{1}{2}B$; using that relationship, the *x*-coordinates are 2 and -1.5.
6. It appears to get narrower; No.
7.

Value of *B*	-2	-1	0	1	2	4
Value of *A*	2	2	2	2	2	2
***x*-coordinate of the vertex**	0.5	0.25	0	-0.25	-0.5	-1

8.

Value of *B*	-2	-1	0	1	2	4
***x*-coordinate when *A* = 1**	1	0.5	0	-0.5	-1	-2
***x*-coordinate when *A* = 2**	0.5	0.25	0	-0.25	-0.5	-1

9. It makes the *x*-coordinate half of what it was.
10. *x*-coordinate = $\frac{-b}{2a}$
11.

Value of *B*	1	2	3	4	6	10
Value of *A*	2	5	1	-2	.5	-4
What your expression says the *x*-coordinate should be	-0.25	-0.2	-1.5	1	-6	1.25
Actual *x*-coordinate of the vertex	-0.25	-0.2	-1.5	1	-6	1.25

12. Yes

Quick Check of What You Have Learned

	Equation	*A*	*B*	*C*	*x*-coordinate of the vertex	*y*-intercept
1.	$y = x^2 + 3x - 1$	1	3	-1	-1.5	-1
2.	$y = 2x^2 + 3x - 1$	2	3	-1	-0.75	-1
3.	$y = {}^-2x^2 + 4x + 5$	-2	4	5	1	5
4.	$y = x^2 - 2x + 3$	0.5	-2	3	2	3
5.	$y = {}^-3x^2 - 6x - 10$	-3	-6	-10	-1	-10

Discuss and Extend What You Have Learned

1. Substitute the *x*-coordinate in the equation and solve for *y*.
2. (-2, -5); 3
3. When $A < 0$, the parabola flips. If $A = 0$, you do not have a parabola as there would be no second degree term.

Putting It All Together

	Vertex	*y*-intercept	Symmetry point
1.	(1, 3)	(0, 5)	(2, 5)
2.	(1, -4)	(0, 0)	(2, 0)
3.	(-3, -2)	(0, -3)	(-6, -3)
4.	(-4, 2)	(0, -1)	(-8, -1)
5.	(4, 2)	(0, 1)	(8, 1)

Student Worksheet

1. (-2, -5); -1
2. (-2, -11); -3
3. (1, 2.5); 3
4. (-0.5, 7); 6
5. 29
6. The parabola opens up and thus has only a minimum.
7. Absolute maximum if $A < 0$. Absolute minimum if $A > 0$.
8. If $B = 0$. When $B = 0$ the vertex is the *y*-intercept and thus there will not be a symmetric point relating to the *y*-intercept.