Dilations "Lesson Bundle"

We are excited that you will be using these interactive investigations to assist your students in exploring and learning about Transformational Geometry. They are designed so that the students investigate and discover the math in under 15 seconds on any one of these TI-Nspire™ learning platforms:

- TI-Nspire™ CX handheld
- TI-Nspire™ CX Teacher or Student Software
- TI-Nspire™ App for iPad®

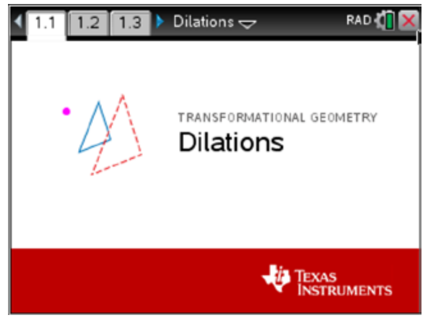
To obtain the optimal results for your students, we make these suggestions:

- Always do the "Tour" activity first – before doing any of the other lessons. The Tour contains information that is needed to understand how to do the other lessons.
- Follow the recommended sequence: Tour first, then Lesson 1, Lesson 2, etc. However, once the Tour is completed, the lessons can be done in any order.
- Transformational Geometry experts recommend investigations of transformations without a grid and without numerical properties first. Then introduce numerical properties such as side lengths, angle measures, coordinates, as needed – with or without a grid.
- These activities are designed as a thorough first exposure to geometric transformations in middle grades and high school.
- Encourage students to PLAY – INVESTIGATE – EXPLORE – DISCOVER using this technology.
- We suggest the Think-Pair-Share classroom technique. First students should do the activity individually and make their own conjectures. Then they share their ideas with a partner or group. Finally, the students share their conjectures with the class.
- We encourage you to use other paper and pencil activities – including compass and straightedge – to reinforce these concepts.
- The final lesson in this sequence must be done with compass and straightedge.
- These lessons are created to investigate one or two concepts at a time. If you wish to investigate the concepts in a different order, you can create your own lessons utilizing this technology. Read the Teacher Tip located at the end of Lessons 1 through 6 in this "Lesson Bundle" to see how to easily create your own investigations using the Options menu.
- To investigate the geometric transformations more deeply, we encourage you to use the other TI-Nspire activities found at the MathNspired.com website. These are located within the Geometry section and the Transformational Geometry subsection.
- The directions are written primarily for the TI-Nspire handheld. If using the computer software, use the mouse to select and move objects. If using the iPad app, tap on the appropriate icons and figures to move objects.

About the Dilations “Lesson Bundle”

In the Dilations “lesson bundle”, students will explore dilations and their properties. Throughout the lessons, students will be encouraged to make observations related to their investigations, leading them to discovering the properties of dilated figures. This lesson bundle includes the following lessons:

- **Lesson 0: Dilations Tour.** Explore the defining properties of dilations and learn how to use these dilations files. ***This must be done first – before doing the other lessons.*** Page 4.
- **Lesson 1: Sides & Angles.** Explore the relationship between measures of corresponding angles and lengths of corresponding sides of dilated triangles. Page 9.
- **Lesson 2: Perimeters & Area**. Explore the relationship between the perimeters and areas of dilated triangles and their ratios. Page 13.
- **Lesson 3: Corresponding Sides**. Explore the relationship between the pairs of corresponding sides (segments) of dilated triangles. Page 18.
- **Lesson 4: P to Vertices Distance**. Investigate the distances from the point of dilation to each of the vertices of dilated triangles. Page 22.
- **Lesson 5: Coordinates.** Explore the relationship between the coordinates of corresponding vertices of triangles dilated about the origin.
Page 26.
- **Lesson 6: Self-Assessment**. Summarize, review, explore and extend ideas about dilations. Page 29.
- **Lesson 7: Compass Construction.** Use a compass and straightedge to dilate a triangle about a given point with the assistance of the tns file. Page 33.



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

Lesson dependent

TI-Nspire document

Dilations.tns

Dilations_Lesson7.tns

Math Objectives

Students will:

- verify experimentally the properties of dilations given by a point of dilation and a scale factor.
- learn to identify and perform dilations.
- understand the effects that dilation has upon a triangle by manipulating the triangle.
- explore the relationship between the corresponding parts of the pre-image and image triangles.
- explain how underlying properties relate to dilation.

Vocabulary:

- Scale Factor
- Point of Dilation
- Corresponding sides
- Dilation
- Area
- Conjecture
- Similarity
- Corresponding angles
- Image/Pre-Image
- Perimeter
- Coincide

TI-Nspire™ Navigator™

- Use Class Capture to monitor student’s use of the TI-Nspire document.

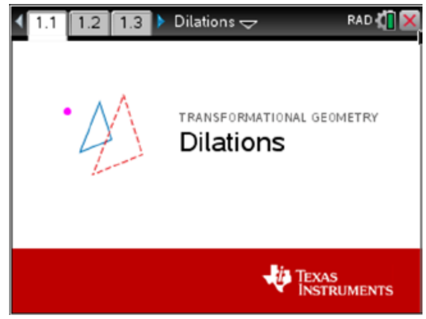
Activity Materials

Compatible TI Technologies : TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Teacher Tip: These lessons are created to investigate one or two concepts at a time. If you wish to investigate other concepts, press the icon or the shortcut key [O] to open the Options menu. Use the [tab] key or the directional arrows (▲ ▼ ◀ ▶) to navigate through the list. Use the space bar [␣] to select or un-select the options. In this way, you are able to create your own investigations.

Lesson 0: Dilations Tour

In this activity, you will investigate the defining properties of the transformation known as a dilation. You will also learn how to easily and quickly maneuver within all the Dilations activities – using shortcut keys or the tab key. **It is important that this is the first lesson that students complete.**



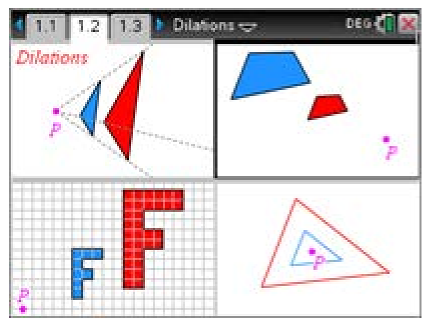
Open the document: *Dilations.tns*

On the handheld, press ctrl ▶ to go to the next page of the lesson and ctrl ◀ to go back to the previous page.

On the iPad®, select a page thumbnail in the page sorter panel.

Move to page 1.2.

PLAY INVESTIGATE EXPLORE DISCOVER



1. What do the 4 parts of the screen have in common?
   Make two conjectures. A **conjecture** is an opinion or conclusion based upon what is observed.
   Quickly discuss with your group.

   <u>**Sample answer(s):**</u>

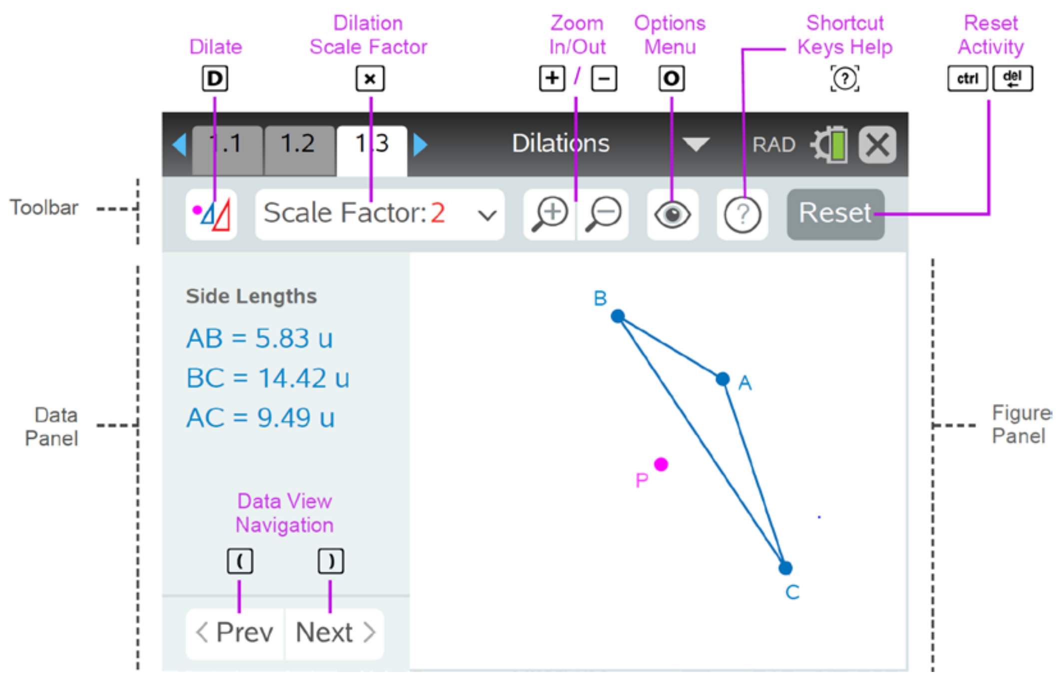
   - Each part of the screen has a pink point P.
   - Each part has a blue figure and a red figure.
   - The blue and red figures appear to be the same shape but a different size (similar).
   - Neither the blue or red figure is always the larger or smaller of the two.

   Teacher Tip: Use the classroom technique of **Think, Pair, and Share**. First students do the activity individually and make their own conjectures. Then, they share their ideas with a partner or group. Finally, the students share their conjectures with the class.

Teacher Tip: Page 1.2 is an interactive page. You may wish to come back to this page **at the end** of the activity – after exercise 9. This page might provide an opportunity to discuss some of the vocabulary associated with a dilation. Encourage students to grab and move objects and discuss with each other what is happening. NOTE: The shortcut keys do **not** work on this page. However, students can click in each section and then click to drag point P. Use this page only if you are using an iPad or if the students are familiar with grabbing and moving a point on the handheld without using the shortcut k.

Move to page 1.3.

Look at the figure below of an overview of the main dilations page and shortcut keys.



O , D , × (multiply key), + , − , ? , ctrl del , (,) (parentheses keys) are examples of **shortcut keys**.

Handheld Tech Tip:

To choose an option or object, use any of the following 3 methods:

- Use the touchpad to move the pointer over the option or object and press the center of the touchpad ( ) to select (**click**) it.
- Use [tab] to move to the next option or object on the screen and use [⇧shift] [tab] to go to the previous option or object.
- Use a **shortcut key** (ex: [A] for vertex A, [D] to Dilate, etc.). Letters [A], [B], [C],… are located at the bottom of the handheld.

iPad Tech Tip: To choose a command or object, tap on the icon or the object.

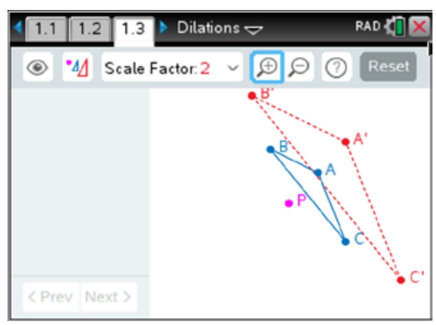
Move to page 1.3.

2. On the handheld, press the tab key ([tab]) multiple times and notice each of the icons and points as they are highlighted. To go in the opposite direction, press [⇧shift] then [tab]. Investigate.

3. **Shortcut keys** provide a fast way to perform actions and/or select objects on the screen on the handheld. A list of all shortcuts can be found in the Shortcut Keys Help menu (click on or press [ctrl] [trig]). **Look at this list now.** Use as needed. Press [enter] or [esc] to close the Shortcut Keys Help menu.

To dilate △ABC about point P with a Scale Factor of 2,

1. Select the Dilate key. (click on or press [D])..
2. Zoom in ([+]) or out ([−]) as needed.
3. Observe what happens on the screen.



$\triangle ABC$ is called the pre-image and $\triangle A'B'C'$ is called the image. $\triangle A'B'C'$ is read “triangle A prime, B prime, C prime”.

Teacher Tip: Make sure students understand and use the proper terminology of dilating the triangle about the point with a scale factor. Also make sure students pronounce $\triangle A'B'C'$ (“triangle A prime, B prime, C prime”) properly.

Encourage students to use precise language, using phrases like “corresponding angles are equal” instead of incorrectly saying “all angles are equal.”

4. To move and grab a vertex, press the letter key that corresponds to the vertex such as A (A), and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to move vertex A. Play and explore to discover ideas and investigate patterns. (*Note: you can also use the* ***tab*** *key or* ***click*** *on the vertex that you want.*) On the iPad, tap on the desired point and then move it.
   Repeat for vertex B (B) and vertex C (C). Observe.
   Discuss with your partner or group: what seems to be true about the pre-image and its image?

   Sample Answer(s): The triangles seem to be the same shape but a different size. The triangles seem to be similar. The red image triangle is always larger than the blue pre-image triangle (for a dilation of scale factor 2).

5. Grab and drag the entire triangle shape by pressing the S key. Use the directional arrows to move the entire shape.
   On the iPad, tap on a side of the triangle (not a vertex) and slide the triangle.
   Investigate and observe. What seems to be true about the pre-image and its image?

   Sample Answer(s): Moving the entire shape does not change the two triangle sizes or shapes, only their location on the screen.

6. Grab and drag point P (P), the point of dilation, in the same way. Investigate and observe.

   Move point P so that it **coincides** with one of the vertices (**P and the vertex are at the same place**).
   Discuss: what do you notice about two of the sides of the triangles?

   Move point P so that it **coincides** with another vertex.
   Discuss: what do you notice about two of the sides of the triangles?

   Sample Answer(s): When the point of dilation coincides with one of the vertices, two of the corresponding sides appear to be parallel. The other two pairs of sides overlap.

7. Reset the page with the current menu settings. Press Reset (ctrl del).

8. Do a similar investigation to become familiar with the shortcut keys but using a different scale factor. If working with a partner or in a group, each person should choose a different scale factor. If working on your own, use a scale factor of ½. To change the scale factor, press Scale Factor: 2 (× (multiply key)). Use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to select the scale factor, then press enter or .
   Dilate △ABC with the scale factor chosen (or D).

   Zoom in (+) or out (−) as needed. Observe. Repeat steps 5 – 7.

Write at least two conjectures about the pre-image triangle and its image.

A ***conjecture*** is an opinion or conclusion based upon what is observed.

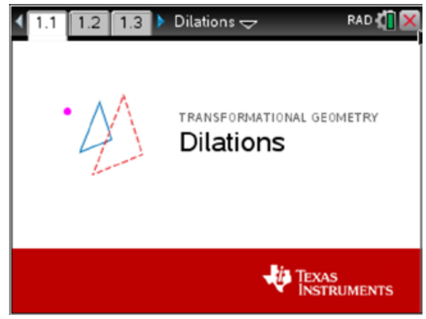
Sample Answer(s):

- The triangles appear to be the same shape but with a different size.
- The triangles appear to be similar to each other.
- The corresponding angles seem to be equal in measure.
- The sides of the image triangle appear to be the scale factor as long as the corresponding sides of the pre-image triangle.
- Note: Do not permit students to say "all angles are equal". They need to be more precise and use the phrase "corresponding angles are equal".
- The points P, A, and A' appear to be collinear.
- If the scale factor is greater than one, the image triangle is larger than its pre-image triangle.
- If the scale factor is less than one, then the image triangle is smaller than its pre-image triangle.
- If the scale factor is one, then the image triangle is congruent to its pre-image and coincides with the image triangle.

Teacher Tip: Ask the students to try to grab and move point A', B', or C'. Have them discuss why this is not possible – because the image depends upon (or is a function of) the pre-image.

Lesson 1: Sides & Angles

In this lesson, you will investigate the relationship between measures of corresponding angles and lengths of corresponding sides of dilated triangles.

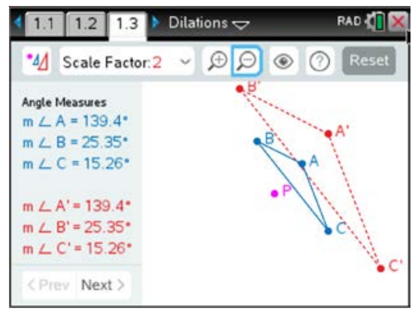


Open the document: *Dilations.tns.*

PLAY INVESTIGATE EXPLORE DISCOVER

Note: It is important that the Dilations Tour be done before any Dilations lessons.

Move to page 1.3.



1. Press menu to open the menu.

   (On the iPad, tap on the wrench icon to open the menu.)
   Press 1 (1: Templates) then 1 (1: Angles & Sides).

   Dilate ΔABC about point P with a Scale Factor of 2 (or D). Zoom in (+) or out (−) as needed. Observe.

   Record the *Original* angle measures (*first measures displayed*) in the first row of the table below.

2. a. Investigate **Angle Measures** by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Record the data in the table below.
   b. Move point P and record the angle measures in the table.
   Sample Answers:

Scale Factor = 2	$m\angle A$	$m\angle B$	$m\angle C$	$m\angle A'$	$m\angle B'$	$m\angle C'$
Original	**139.4°**	**25.35°**	**15.26°**	**139.4°**	**25.35°**	**15.26°**
Figure 1	**119.74°**	**40.03°**	**20.22°**	**119.74°**	**40.03°**	**20.22°**
Figure 2	**101.31°**	**51.34°**	**27.35°**	**101.31°**	**51.34°**	**27.35°**

3. Make a **conjecture** about the angles of a triangle and its image under a dilation about a point. (A ***conjecture*** is an opinion or conclusion based upon what is observed.)

   Sample Answer: Corresponding angles of a pre-image and a dilated image of a triangle are always equal in measure. Moving the point of dilation, point P, does not change the shape of either triangle, nor the measures of the angles.
   ****Do not permit students to make incorrect conjectures such as "All angles are equal". Make sure the proper vocabulary is used – "*corresponding* angles are equal".**

4. Reset the page (Reset or ctrl followed by del).
Repeat the earlier investigation using a different scale factor. If working with a group, each person should choose a different scale factor. If working on your own, use a scale factor of ½.
To change the scale factor, press Scale Factor: 2 (times key (×)) and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to select the scale factor, then press [click] or enter.
Dilate ΔABC with the scale factor chosen (or D).
Zoom in (+) or out (−) as needed.
Create different triangles as before by grabbing and moving vertices and point P.
Record angle measures for three different figures.
Record the scale factor here: **Scale Factor =** ______ and the **Angle Measures** in the table below.
<u>Sample Answer: (with scale factor = ½)</u>

	$m\angle A$	$m\angle B$	$m\angle C$	$m\angle A'$	$m\angle B'$	$m\angle C'$
Figure 1	**71.57°**	**77.47°**	**30.96°**	**71.57°**	**77.47°**	**30.96°**
Figure 2	**71.57°**	**77.47°**	**30.96°**	**71.57°**	**77.47°**	**30.96°**
Figure 3	**66.8°**	**74.05°**	**39.14°**	**66.8°**	**74.05°**	**39.14°**

Does your conjecture from question 3 still apply?
Compare your results to those of your classmates who used different scale factors.

<u>Sample Answers:</u> Yes, regardless of the scale factor, the corresponding angles of a triangle and its dilated image are equal in measure.

5. Reset the page (Reset or ctrl followed by del).
Dilate ΔABC about point P with a Scale Factor of 2 (or D).
Zoom in (+) or out (−) as needed.
Advance to the **Side Lengths** data by pressing Next () right parenthesis key).
Record the *Original* side lengths (*first lengths displayed*) in the first row of the table below.

6. a. Investigate **Side Lengths** by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Record the data for the length of each side in the table below.
Try to make at least one or two lengths in each pre-image a whole number, if possible.
b. Move point P and record the **Side Lengths** in the table.

Sample Answers:

Scale Factor = 2	$\overline{AB}$	$\overline{BC}$	$\overline{AC}$	$\overline{A'B'}$	$\overline{B'C'}$	$\overline{A'C'}$
Original	**5.83 u**	**14.42 u**	**9.49 u**	**11.66 u**	**28.84 u**	**18.97 u**
Figure 1	**5 u**	**12.04 u**	**9.49 u**	**10 u**	**24.08 u**	**18.97 u**
Figure 2	**5 u**	**10.3 u**	**9 u**	**10 u**	**20.59 u**	**18 u**

Teacher Tip: It might be helpful to have a discussion with students about the rounding that is occurring. Because of rounding, the length displayed might not always be exactly twice the length of the pre-image length.

7. Make a conjecture about the side lengths of a triangle and its image under a dilation about a point.

   Sample Answer: Given a scale factor of two, the length of each side of the image triangle is twice the length of the corresponding sides of the pre-image triangle (allowing for rounding).

8. Advance to the **Ratio of Lengths** data by pressing Next ([ )] right parenthesis key). Discuss in your groups what this page is displaying. Create different triangles as before by grabbing and moving vertices and point P. Notice what values are changing and what values are not changing. Does your conjecture from #7 above still apply? Discuss.

   Sample Answer: Yes, the conjecture from #7 is still true. While grabbing and moving vertices, the values for the lengths changed, but the ratios of the lengths of corresponding sides remained at 2.

9. Reset the page ([Reset] or [ctrl] followed by [del]).
   Repeat the earlier investigation using a different scale factor. If working with a group, each person should choose a different scale factor. If working on your own, use a scale factor of ½.

   To change the scale factor, press [Scale Factor: 2 ⌄] ([×]) and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to select the scale factor, then press [click] or [enter].

   Dilate ΔABC with the scale factor chosen ([dilate] or [D]).
   Zoom [zoom in][zoom out] in ([+]) or out ([−]) as needed.
   Advance to the **Side Lengths** data by pressing Next ([ )] right parenthesis key).
   Create different triangles as before by grabbing and moving vertices and point P.
   Try to make at least one or two lengths in each pre-image a whole number, if possible.

   Record the scale factor here: **Scale Factor =** ______ and the **Side Lengths** in the table below.

   Sample answers: (with a scale factor of 3)

	$\overline{AB}$	$\overline{BC}$	$\overline{AC}$	$\overline{A'B'}$	$\overline{B'C'}$	$\overline{A'C'}$
Figure 1	**5.83 u**	**14.42 u**	**9.49 u**	**17.49 u**	**43.27 u**	**28.46 u**
Figure 2	**5 u**	**10.3 u**	**9 u**	**15 u**	**30.89 u**	**27 u**
Figure 3	**1 u**	**7.07 u**	**7 u**	**3 u**	**21.21 u**	**21 u**

Make a conjecture about the side lengths of a triangle and its image under a dilation about a point.

Sample answers: In general, the lengths of the sides of the image triangle are equal to the scale factor times the length of the corresponding sides of the pre-image triangle.
Or the ratio of the length of a side of the image triangle to the length of the corresponding side of the pre-image triangle is equal to the scale factor.

10. Advance to the **Ratio of Lengths** data by pressing Next (right parenthesis key). Discuss in your groups what this page is displaying.
Create different triangles as before by grabbing and moving vertices and point P. Notice what values are changing and what values are not changing. Does your conjecture from #9 above still apply? Discuss.

Sample answers: Yes, the conjecture from #9 still applies. The side lengths are changing but the ratio is not.

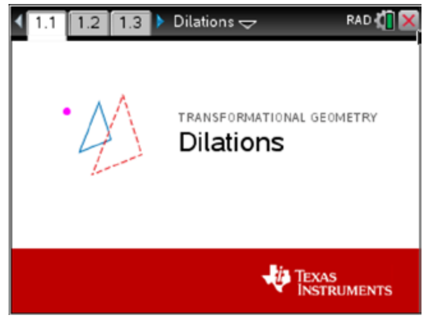
11. Suppose that ΔDEF were dilated about point P with a scale factor of 5.

Answers:

a. If the measure of $\angle D = 20^o$, then the measure of $\angle D' = 20°$.
b. If $DE = 40cm$, then $D'E' = 200cm$.
c. If the measure of $\angle E' = 30^0$, then the measure of $\angle E = 30°$.
d. If $E'F' = 10in$, then $EF = 2in$.

Teacher Tip: These lessons are created to investigate one or two concepts at a time. If you wish to investigate other concepts, press the icon or the shortcut key **O** to open the Options menu. Use the **tab** key or the directional arrows (▲ ▼ ◀ ▶) to navigate through the list. Use the space bar ␣ to select or un-select the options. In this way, you are able to create your own investigations.

Lesson 2: Perimeters & Area



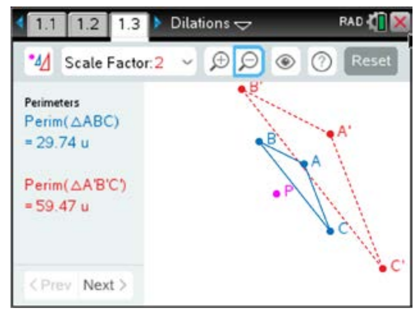
In this lesson, you will investigate the relationship between the perimeters and areas of dilated triangles and their ratios.

Open the document: *Dilations.tns.*

PLAY INVESTIGATE EXPLORE DISCOVER

Note: It is important that the Dilations Tour be done before any Dilations lessons.

Move to page 1.3.



1. a. Press menu to open the menu on the handheld.
   On the iPad, tap on the wrench icon to open the menu.)
   Press 1 (1: Templates) then 2 (2: Perimeters & Areas).
   b. Dilate ΔABC about point P with a Scale Factor of 2
   (or D). Zoom in (+) or out (−) as needed.
   Observe.
   Record the *Original* perimeters (*first perimeters displayed*) in the first row of the table below.
   Discuss in your groups the meaning of the 'perimeter of a triangle.'
2. a. Investigate perimeters by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Try to make one of the perimeters a whole number. Record the data.
   b. Move point P and record the perimeters in the table.

<u>Sample Answers:</u>

Scale Factor = 2	Perimeter (ΔABC)	Perimeter ($\Delta A'B'C'$)
Original	**29.74 u**	**59.47 u**
Figure 1	**25.9 u**	**51.8 u**
Figure 2	**23.01 u**	**46.02 u**

3. Make a **conjecture** about the perimeters of a triangle and its image under a dilation about a point. (A ***conjecture*** is an opinion or conclusion based upon what is observed.)

   <u>Sample answer:</u> The perimeter of a triangle dilated about a point with scale factor 2 is twice the perimeter of the original (pre-image) triangle.

Teacher Tip: It might be helpful to have a discussion with students about the rounding that is occurring. Because of rounding, the perimeter displayed might not always be exactly twice the perimeter of the pre-image.

4. Reset the page (Reset) or ctrl followed by del). Change the scale factor to 3 by pressing Scale Factor: 2 (×) and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to select the Scale Factor 3, then press or enter.

   Dilate ΔABC with the scale factor chosen (or D). Zoom in (+) or out (−) as needed. Advance to the 'Areas' data by pressing Next ()) (right parenthesis key).
   Observe. Record the *Original* areas (*first areas displayed*) in the first row of the following table.

5. a. Investigate areas by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Record the area data. Discuss in your groups the meaning of the 'area' of a triangle.
   b. Move point P and record the areas in the table.

Sample Answer(s): (with scale factor = 3)

	Area (ΔABC)	Area ($\Delta A'B'C'$)
Original	**18 sq. in.**	**162 sq. in.**
Figure 1	**30 sq. in.**	**270 sq. in.**
Figure 2	**21 sq. in.**	**189 sq. in.**

6. Make a conjecture about the areas of a triangle and its image under a dilation about a point.

   Sample answer: The area of a triangle dilated about a point with scale factor 3 is 9 times the area of the original (pre-image) triangle.

7. Reset the page (Reset) or ctrl followed by del). To validate the conjectures, change the scale factor to 4 by pressing Scale Factor: 2 (×) and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to select Scale Factor 4, then press or enter.

   Dilate ΔABC about point P with a Scale Factor of 4 (or D).

   Zoom in (+) or out (−) as needed.
   Advance to the "Ratio of Perimeters" ('Perim $(\Delta A'B'C')$ / Perim (ΔABC)') data by pressing Next ()) twice. Observe. Record the *Original* ratios of the perimeters (*first ratios displayed*) in the table on the next page.

8. a. Investigate the Ratios of Perimeters by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Record the ratios of the perimeters for each triangle in the table.

   b. Move point P and record the ratios of the perimeters in the table.

   c. Advance to the 'Ratio of Areas' ('Area $(\Delta A'B'C')$ / Area (ΔABC)' data by pressing Next (▯). Record the Original ratios of the areas (*first ratios displayed*) in the table on the next page.

   d. Investigate the ratios of the areas by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Move point P as well.

   Record the ratios of the areas for each triangle in the table on the next page.

   Sample answers:

Scale Factor = 4	$\frac{Perim(\Delta A'B'C')}{Perim(\Delta ABC)}$	$\frac{Area(\Delta A'B'C')}{Area(\Delta ABC)}$
Original	**4**	**16**
Figure 1	**4**	**16**
Figure 2	**4**	**16**
Figure 3	**4**	**16**

9. Reset the page (Reset or ctrl followed by del). Repeat the earlier investigation for the ratios of perimeters and areas but using a different scale factor than 2 or 4. If working with a partner or in a group, each person should choose a different scale factor. If working on your own, use a scale factor of 1/2.

   To change the scale factor, press Scale Factor: 2 (×) and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to select the scale factor, then press (click) or enter.

   Dilate ΔABC with the scale factor chosen (or D). Zoom in (+) or out (−) as needed.

   a. Create different triangles as before by grabbing and moving vertices and point P. Record the ratios of the perimeters and the ratios of the areas for three different figures. Use the Next (▯) and Prev (▯) buttons to access the desired data.

   b. Record the scale factor here: **Scale Factor =** ______ and the ratios in the table below.

Sample answers: (scale factor = 1/2)

	$\dfrac{Perim(\Delta A'B'C')}{Perim(\Delta ABC)}$	$\dfrac{Area(\Delta A'B'C')}{Area(\Delta ABC)}$
Figure 1	**0.5= 1/2**	**.25=1/4**
Figure 2	**0.5=1/2**	**.25=1/4**
Figure 3	**0.5=1/2**	**.25=1/4**

If the ratios are expressed as decimals, also write the ratios as their fraction equivalents.
Based upon the entries in the table, write at least two conjectures about what you have observed.

Sample answers:
In general, the perimeter of a triangle dilated about a point with scale factor n is the product of n and the perimeter of the original (pre-image) triangle. The area of a triangle dilated about a point with a scale factor n is the product of n squared and the area of the original (pre-image) triangle.

$$\frac{length\ \ of\ \ image\ \ side}{length\ \ of\ \ pre-image\ \ side} = \frac{perimeter\ \ of\ \ image}{perimeter\ \ of\ \ pre-image} = scale\ \ factor$$

$$\frac{area\ \ of\ \ image}{area\ \ of\ \ pre-image} = (scale\ \ factor)^2$$

Compare your results with your classmates.

10. Advance to the ‘Scale Factor’ data by pressing Next ($\boxed{\rangle}$).
What do you notice on this page? How does this compare with your conjectures?
Discuss in your groups.

Sample answers: This page displays the scale factor and the square of the scale factor.
It supports the conjectures.

11. Suppose that ΔDEF were dilated about point P with a scale factor of 5.

a. $\dfrac{Perim(\Delta D'E'F')}{Perim(\Delta DEF)} = 5$

b. $\dfrac{Area(\Delta D'E'F')}{Area(\Delta DEF)} = 25$

c. $\dfrac{Perim(\Delta DEF)}{Perim(\Delta D'E'F')} = \dfrac{1}{5}$

12. Suppose that ΔDEF were dilated about point P with a scale factor of $\dfrac{1}{3}$.

a. $\dfrac{Perim(\Delta D'E'F')}{Perim(\Delta DEF)} = \dfrac{1}{3}$

b. $\dfrac{Area(\Delta D'E'F')}{Area(\Delta DEF)} = \dfrac{1}{9}$

c. $\dfrac{Perim(\Delta DEF)}{Perim(\Delta D'E'F')} = \dfrac{3}{1}$ *or* 3

13. What is the relationship between the ratios of the perimeters and the scale factor of dilated images? Explain your answer.

<u>**Sample answer:**</u> The ratio of the perimeter of an image under a dilation to the perimeter of its pre-image is the same as the scale factor of the dilation.

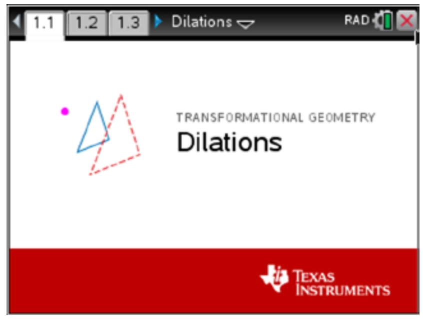
14. What is the relationship between the ratios of the areas and the scale factor of dilated images? Explain your answer.

<u>**Sample answer**</u>: The ratio of the area of an image under a dilation to the area of its pre-image is the same as the scale factor squared.

Teacher Tip: These lessons are created to investigate one or two concepts at a time. If you wish to investigate other concepts, press the icon or the shortcut key **O** to open the Options menu. Use the **tab** key or the directional arrows **(▲ ▼ ◀ ▶)** to navigate through the list. Use the space bar ␣ to select or un-select the options. In this way, you are able to create your own investigations.

Lesson 3: Corresponding Sides

In this lesson, you will investigate the relationship between the pairs of corresponding sides (segments) of dilated triangles.

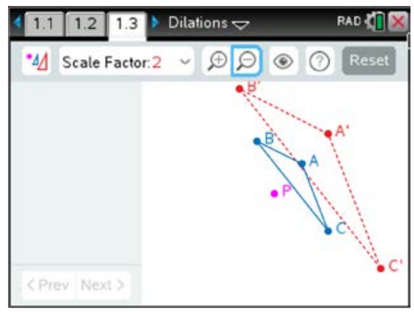


Open the document: *Dilations.tns.*

PLAY INVESTIGATE EXPLORE DISCOVER

Note: It is important that the Dilations Tour be done before any Dilations lessons.

Move to page 1.3.



1. Dilate ΔABC about point P with a Scale Factor of 2 (or D). Zoom in (+) or out (−) as needed. Observe the corresponding segments: $\overline{AB}$ *and* $\overline{A'B'}$, $\overline{BC}$ *and* $\overline{B'C'}$, $\overline{AC}$ *and* $\overline{A'C'}$.

   Visually, what looks to be true about each pair of segments (**not their lengths**)? Record your observations in the Original row in the table below.

2. a. Investigate the corresponding segments by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Record your observations in the table below.

   b. Move point P and record your observations.

   Complete the table.

   <u>Sample Answers:</u>

Scale Factor = 2	$\overline{AB}$ *and* $\overline{A'B'}$	$\overline{BC}$ *and* $\overline{B'C'}$	$\overline{AC}$ *and* $\overline{A'C'}$
Original	**Parallel**	**Parallel**	**Parallel**
Figure 1	**Parallel**	**Parallel**	**Parallel**
Figure 2	**Parallel**	**Parallel**	**Parallel**
Figure 3	**Parallel**	**Parallel**	**Parallel**

   d. Drag A or B until $\overline{AB}$ is horizontal $\leftrightarrow$. What appears to be true about $\overline{A'B'}$?

   <u>Sample Answer</u>: $\overline{A'B'}$ is also horizontal and has a slope of zero.

   e. Drag B or C until $\overline{BC}$ is vertical $\updownarrow$. What appears to be true about $\overline{B'C'}$?

   <u>Sample Answer:</u> $\overline{B'C'}$ is also vertical and has an undefined slope.

Teacher Tip: Make sure there is a discussion with the class about what happens when point P is concurrent with one of the vertices. If so, two pairs of sides overlap. If point P is located on a side (and not a vertex), then one pair of sides overlap. In each case, the other pair(s) of corresponding sides are parallel.

3. Make a **conjecture** about the corresponding sides (segments) of a triangle and its image under a dilation about a point. (A ***conjecture*** is an opinion or conclusion based upon what is observed.)

   Sample Answer: When a triangle is dilated about a point with scale factor of 2, it appears that the corresponding sides of the pre-image and image triangle are either parallel or overlap. (NOTE: have students use proper terminology, the phrase ‘*corresponding* sides’.)

4. Reset the page (Reset or ctrl del). In a similar manner, investigate using a different scale factor. If working with a partner or in a group, each person should choose a different scale factor. If working on your own, use a scale factor of 1/2. To change the scale factor, press Scale Factor: 2 (×) and select the scale factor, then press or enter.
   Dilate ΔABC with the scale factor chosen (or D).
   Zoom in (+) or out (−) as needed. Observe the corresponding segments: $\overline{AB}$ *and* $\overline{A'B'}$, $\overline{BC}$ *and* $\overline{B'C'}$, $\overline{AC}$ *and* $\overline{A'C'}$.

   a. What seems to be true about each pair of these segments (**not their lengths**)?
   Sample Answer: **The corresponding sides appear to be parallel.**

   b. Investigate by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Move point P.

   c. Does your previous conjecture still apply? Compare your results to those of your classmates who used different scale factors.

   Sample Answer: Yes.

5. Does the conjecture always work? If not, list a counter example.
   In groups, determine some ways to prove the conjecture. List them here.

   Sample answer(s):
   The corresponding sides of dilated triangles with any scale factor are either parallel or they overlap when the point of dilation, P, coincides with one of the vertices or if P is on a side. To validate they are parallel, we would need to compare the slopes of corresponding sides. Parallel lines have equal slopes.

6. Press [menu] to open the menu on the handheld. (On the iPad, tap on the wrench icon to open the menu.) Press [1] (1: Templates), then [3] (3: Slopes).

Dilate ΔABC about point P with a Scale Factor of 2 (or [D]). Zoom in ([+]) or out ([−]) as needed. Look at the slopes of the corresponding segments and record the results in the table on the next page in the Original row.

a. Investigate the slopes of corresponding segments by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Record the slopes of corresponding sides of each triangle in the table.

b. Move point P and record the slopes in the table.

Note: m($\overline{AB}$) means 'the slope of segment AB'.

Sample Answers:

Scale Factor = 2	m($\overline{AB}$)	m($\overline{BC}$)	m($\overline{AC}$)	m($\overline{A'B'}$)	m($\overline{B'C'}$)	m($\overline{A'C'}$)
Original	**-0.6**	**-1.5**	**-3**	**-0.6**	**-1.5**	**-3**
Figure 1	**0**	**-1.13**	**-3**	**0**	**-1.13**	**-3**
Figure 2	**0**	**-1.8**	**undef**	**0**	**-1.8**	**undef**
Figure 3	**0.17**	**-1.8**	**10**	**0.17**	**-1.8**	**10**

What is the relationship between slope and parallel segments?

Answer: Parallel segments (lines containing the segments) have equal slopes.

7. Reset the page (Reset or [ctrl] [del]). Repeat the investigation with a different scale factor than 2. If working with a partner or in a group, each person should choose a different scale factor. If working on your own, use a scale factor of 1/2.

To change the scale factor, press Scale Factor: 2 ([×]) and select the scale factor, then press [click] or [enter]. Dilate ΔABC with the scale factor chosen (or [D]).

Zoom in ([+]) or out ([−]) as needed. Create different triangles as before by grabbing and moving vertices and point P. Record the slopes for three different figures.

Record the scale.. factor here: **Scale Factor =** ______ and the slopes in the table below.

Sample Answers: (scale factor = ½)

	m($\overline{AB}$)	m($\overline{BC}$)	m($\overline{AC}$)	m($\overline{A'B'}$)	m($\overline{B'C'}$)	m($\overline{A'C'}$)
Figure 1	**-0.33**	**-1.5**	**-5**	**-0.33**	**-1.5**	**-5**
Figure 2	**0.17**	**-1.12**	**-5**	**0.17**	**-1.12**	**-5**
Figure 3	**0.17**	**-1.6**	**9**	**0.17**	**-1.6**	**9**

Do your previous conjectures still apply? Compare your results to those of your classmates who used different scale factors.

Sample Answers: Yes, the corresponding sides have the same slope and are either parallel or overlap.

> **Teacher Tip:** Encourage students to make sides horizontal and vertical. Also encourage moving point P to a side or vertex.

8. State the conjecture concerning corresponding sides of dilated triangles. Be sure to include all cases. Explain the conjecture.

Sample Answer: Corresponding sides of a triangle dilated about a point with any scale factor are either parallel or they overlap when the point of dilation, P, coincides with one of the vertices, or if P is on a side.

9. Suppose that ΔDEF were dilated about point P with a scale factor of 5.

   a. If the slope of $\overline{DE} = \dfrac{2}{3}$, then the slope of $\overline{D'E'} = 2/3$.

   b. If the slope of $\overline{EF} = -1$, then the slope of $\overline{E'F'} = -1$.

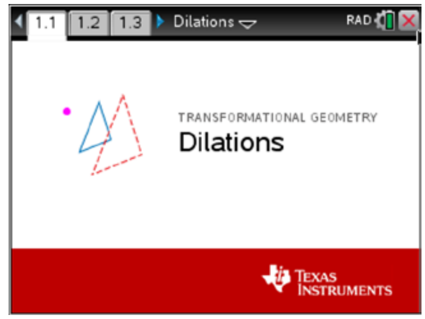
   c. If the slope of $\overline{DF} = 0$, then what two segments should be horizontal? $\overline{DF}$ and $\overline{D'F'}$.

   d. If $\overline{DE}$ is vertical, then the slope of $\overline{D'E'}$ **is undefined (no slope).**

> **Teacher Tip:** These lessons are created to investigate one or two concepts at a time. If you wish to investigate other concepts, press the icon or the shortcut key **O** to open the Options menu. Use the **tab** key or the directional arrows (▲ ▼ ◀ ▶) to navigate through the list. Use the space bar ␣ to select or un-select the options. In this way, you are able to create your own investigations.

Lesson 4: P to Vertices Distance

In this lesson, you will investigate the distances from the point of dilation to each of the vertices of dilated triangles.

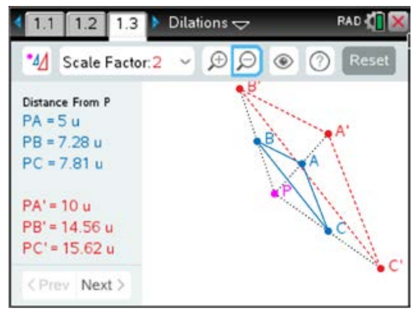


Open the document: *Dilations.tns.*

PLAY INVESTIGATE EXPLORE DISCOVER

Note: It is important that the Dilations Tour be done before any Dilations lessons.

Move to page 1.3.



1. Press [menu] to open the menu on the handheld. (On the iPad, tap on the wrench icon to open the menu.)
   Press [1] (1: Templates) then [4] (4: Dist P to Vertices).
   Dilate ΔABC about point P with a Scale Factor of 2 (or [D]). Zoom in ([+]) or out ([-]) as needed.
   Observe the distances from the point of dilation, P, to each vertex. Record the *Original* distances (*first distances displayed*) in the first row of the table below.

2. a. Investigate the distances from P to the vertices by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles.

   Try to make at least one of the distances a whole number. Record the data in the table below.

   b. Move point P and record the distances in the table.

   Record the distances in the table below:

   <u>**Sample Answer:**</u>

Scale Factor = 2	P to A	P to B	P to C	P to A'	P to B'	P to C'
Original	**5**	**7.28**	**7.81**	**10**	**14.56**	**15.62**
Figure 1	**5**	**5.83**	**7.81**	**10**	**11.66**	**15.62**
Figure 2	**7.07**	**5.83**	**7.81**	**14.14**	**11.66**	**15.62**
Figure 3	**7.07**	**5.83**	**5.1**	**14.14**	**11.66**	**10.2**

3. Make a **conjecture** about the distances from P to the vertices of a triangle and its image under a dilation about a point. (A ***conjecture*** is an opinion or conclusion based upon what is observed.)

Sample Answer: If a triangle is dilated about a point with scale factor equal to two, the distance from P to a vertex of the image triangle is twice the distance from P to the corresponding vertex of the pre-image triangle.

> **Teacher Tip:** It might be helpful to have a discussion with students about the rounding that is occurring. Because of rounding, the distances might not be exactly twice the original distances.

4. Reset the page (Reset or ctrl del).

Repeat the investigation using a different scale factor. If working with a partner or in a group, each person should choose a different scale factor. If working on your own, use a scale factor of 1/2.

To change the scale factor, press Scale Factor: 2 (×) and select the scale factor, then press or enter.

Dilate ΔABC with the scale factor chosen (or D). Zoom in (+) or out (−) as needed. Create different triangles as before by grabbing and moving vertices and point P.

Record the scale factor here: **Scale Factor =** _______and the distances in the table below.

Sample Answer: (scale factor = ½)

	P to A	P to B	P to C	P to A'	P to B'	P to C'
Figure 1	**5**	**7.28**	**7.81**	**2.5**	**3.64**	**3.91**
Figure 2	**5**	**5.39**	**6.4**	**2.5**	**2.69**	**3.2**
Figure 3	**7.07**	**6**	**7.21**	**3.54**	**3**	**3.61**

Does your previous conjecture still apply? Compare your results to those of your classmates who used different scale factors.

Sample Answer: Yes, the previous conjecture still applies, the distance from P to a vertex of the image triangle is the scale factor times the distance from P to the corresponding vertex of the pre-image triangle.

5. Make a conjecture about the distances between point P and each vertex of a triangle and its image under a dilation about a point.

Sample Answer: The distance from the point of dilation, P, to a vertex on the image is equal to the scale factor multiplied by the distance from point P to its corresponding vertex on the pre-image. If the scale factor is equal to one, the image triangle is on top of the pre-image triangle.

6. Reset the page (Reset or ctrl del). Change the Scale Factor to 3.

   To change the scale factor, press Scale Factor: 2 (×) and select Scale Factor 3, then press (click) or enter.

   Dilate ΔABC with the scale factor chosen (or D). Zoom in (+) or out (−) as needed. Advance to the 'ratios' data by pressing Next ( )) (right parenthesis key).
   Observe the distances in red and blue and look at the ratios. Record the *Original* ratios of the distances (*first ratios displayed*) in the first row of the table below.

   a. Investigate the ratios of the distances by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Record the ratios in the table.

   b. Move point P and record the ratios in the table.

   .

   Complete the table.

   Sample answers:

Scale Factor = 3	PA' : PA	PB' : PB	PC' : PC
Original	**3**	**3**	**3**
Figure 1	**3**	**3**	**3**
Figure 2	**3**	**3**	**3**
Figure 3	**3**	**3**	**3**

7. Based upon the data in the previous table, what conclusion can be made about the ratio of the distance of point P to each vertex of the image triangle to the distance of point P to each corresponding vertex of the pre-image triangle?

   Sample answer: In each case the ratio appears to be equal to the scale factor (with an allowance for rounding error).

8. Reset the page (Reset or ctrl del).
   Repeat the investigation using a different scale factor than 3. If working in a group, each person should choose a different scale factor. If working on your own, use a scale factor of 1/2.

   To change the scale factor, press Scale Factor: 2 (×) and select the scale factor, then press (click) or enter.

   Dilate ΔABC with the scale factor chosen (or D). Zoom in (+) or out (−) as needed. Advance to the 'ratios' data by pressing Next ( )) (right parenthesis key).
   Create different triangles as before by grabbing and moving vertices and point P.
   Record the scale factor here: **Scale Factor =** _______and the ratios in the table below.

Sample Answers: (scale factor = ½)

	PA' : PA	PB' : PB	PC' : PC
Figure 1	**0.5**	**0.5**	**0.5**
Figure 2	**0.5**	**0.5**	**0.5**
Figure 3	**0.5**	**0.5**	**0.5**

Do your previous conjectures still apply? Compare your results to those of your classmates who used different scale factors.

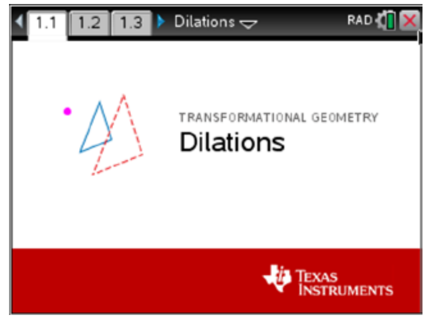
Sample Answer: Yes, the previous conjectures still apply, the distance from P to a vertex of the image triangle is the scale factor times the distance from P to the corresponding vertex of the pre-image triangle.

9. Suppose that ΔDEF were dilated about point P with a scale factor of 5.
   a. If PD = 10, then PD' = **50** and DD' = **40.**
   b. If PE = 15, then PE' = **75** and EE' = **60.**
   c. If PF' = 20, then PF = **4** and FF' = **16**.

10. Suppose that ΔDEF were dilated about point P with a scale factor of $\frac{1}{3}$.
    a. If PD = 12, then PD' = **4** and DD' = **8**.
    b. If PE = 15, then PE' = **5** and EE' =**10**.
    c. If PF' = 21, then PF = **63** and FF' =**42**.

Teacher Tip: These lessons are created to investigate one or two concepts at a time. If you wish to investigate other concepts, press the icon or the shortcut key O to open the Options menu. Use the tab key or the directional arrows (▲ ▼ ◀ ▶) to navigate through the list. Use the space bar ␣ to select or un-select the options. In this way, you are able to create your own investigations.

Lesson 5: Coordinates



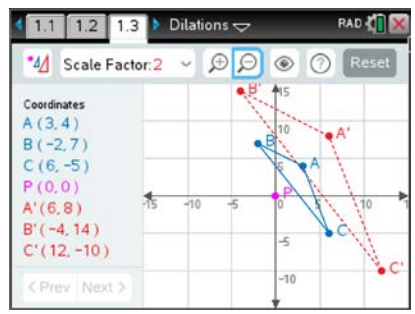
In this lesson, you will investigate the relationship between the coordinates of corresponding vertices of triangles dilated about the origin.

Open the document: *Dilations.tns.*

PLAY INVESTIGATE EXPLORE DISCOVER

Note: It is important that the Dilations Tour be done before any Dilations lessons.

Move to page 1.3.



1. a. Press [menu] to open the menu on the handheld. (On the iPad, tap on the wrench icon to open the menu.) Press [1] (1: Templates) then [5] (5: Grid & Coordinates). Grab point P ([P]) and move it to the origin, if necessary.

   b. Dilate ΔABC about point P with a Scale Factor of 2 (or [D]). Zoom in ([+]) or out ([−]) as needed.

   c. Observe the coordinates and look for patterns.

   d. Record these *Original* coordinates (*first coordinates displayed*) in the first row of the table below.

2. a. Investigate the coordinates of corresponding vertices by grabbing and moving each of the three vertices of ΔABC to create different shaped triangles. Record the data.

   b. Repeat the above step by creating new triangles for Figures 1-3 in the table below.

   c. Move point P and record the coordinates for the vertices in the row labeled 'Figure 4.'

   What are the coordinates of point P? ______________

   Record the coordinates of the vertices listed in the table below.

   Sample answers:

   What are the coordinates of point P? **(1,1)**

Scale Factor = 2	A	B	C	A'	B'	C'
Original	**(3,4)**	**(-2,7)**	**(6,-5)**	**(6,8)**	**(-4,14)**	**(12,-10)**
Figure 1	**(4,-2)**	**(-1,2)**	**(1,-3)**	**(8,-4)**	**(-2,4)**	**(2,-6)**
Figure 2	**(3,-2)**	**(0,2)**	**(-1,-3)**	**(6,-4)**	**(0,4)**	**(-2,-6)**
Figure 3	**(4,-2)**	**(0,1)**	**(0,-4)**	**(8,-4)**	**(0,2)**	**(0,-8)**
Figure 4 (move P)	**(4,-2)**	**(0,1)**	**(0,-4)**	**(7,-5)**	**(-1,1)**	**(-1,-9)**

3. Make a **conjecture** about the coordinates of the vertices of a triangle and its image under a dilation about the origin. (A ***conjecture*** is an opinion or conclusion based upon what is observed.)

Sample answer: If a triangle is dilated about the origin with a scale factor of 2, the coordinates of each of the vertices of the image triangle are found by multiplying the corresponding vertices of the pre-image triangle by two.
Note: Make sure students are aware that this only works if the point of dilation is at the origin.

4. Reset the page (Reset or ctrl del). Grab point P (P) and move it to the origin, if necessary. Repeat the earlier investigation using a different scale factor. If working with a partner or in a group, each person should choose a different scale factor. If working on your own, use a scale factor of ½.

To change the scale factor, press Scale Factor: 2 (×) and select the scale factor, then press or enter. Dilate ΔABC with the scale factor chosen (or D).
Zoom in (+) or out (−) as needed.

a. Create different triangles as before by grabbing and moving the vertices **only**.
Record coordinates for three different Figures.
b. Move point P and record the coordinates in the row labeled 'Figure 4.'

What are the coordinates of point P? _____________
Record the scale factor here: **Scale Factor =** ____ and the coordinates listed in the table below.

Sample answers: (scale factor = ½)
What are the coordinates of point P? **(-2,1)**

	A	B	C	A'	B'	C'
Figure 1	**(3,-1)**	**(0,3)**	**(1,-3)**	**(1.5,-0.5)**	**(0,1.5)**	**(0.5,-1.5)**
Figure 2	**(4,-2)**	**(-1,3)**	**(0,-4)**	**(2,-1)**	**(-0.5,1.5)**	**(0,-2)**
Figure 3	**(4,-4)**	**(2,4)**	**(0,-2)**	**(2,-2)**	**(1,2)**	**(0,-1)**
Figure 4 (move P)	**(4,-4)**	**(2,4)**	**(0,-2)**	**(1,-1.5)**	**(0,2.5)**	**(-1,-0.5)**

c. Does your conjecture from question 3 still apply? Compare your results to those of your classmates who used different scale factors.

Sample answer: Yes. If a triangle is dilated about the origin, the coordinates of each of the vertices of the image triangle are found by multiplying the corresponding vertices of the pre-image triangle by the scale factor.

d. Generalize your conjecture.

Sample answer: If a triangle is dilated about the origin with a scale factor of k, then the coordinates of each of the vertices of the image triangle are found by multiplying the coordinates of the corresponding vertices of the pre-image triangle by k.
Example: $A:(x,y) \rightarrow A':(k\cdot x, k\cdot y)$

5. Suppose that ΔDEF were dilated about point P with a scale factor of 5.

a. If point P is at the origin and vertex D has coordinates (20, – 30), then the coordinates of D’ are **(100, -150).**

b. If point P is at the origin and vertex E has coordinates (– 5, 10), then the coordinates of E’ are **(-25, 50)**.

c. If point P is at the origin and vertex F’ has coordinates (10, 3), then the coordinates of F are **(2, 3/5)**.

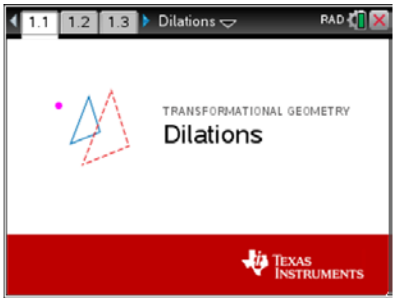
d. If point P has coordinates (1, 1) and vertex D has coordinates (3, – 1), then the coordinates of D’ are ___________.

Two answers are acceptable for d: 1. Cannot be done because the point of dilation is not the origin. OR 2. The correct answer is (11, -9).

Teacher Tip: These lessons are created to investigate one or two concepts at a time. If you wish to investigate other concepts, press the icon or the shortcut key **O** to open the Options menu. Use the **tab** key or the directional arrows **(▲ ▼ ◀ ▶)** to navigate through the list. Use the space bar ␣ to select or un-select the options. In this way, you are able to create your own investigations.

Lesson 6: Self-Assessment

In this lesson, you will be given the opportunity to summarize, review, explore and extend ideas about Dilations.

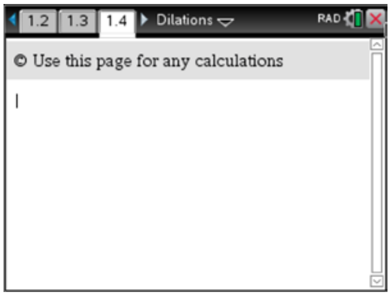


Open the document: *Dilations.tns.*

PLAY   INVESTIGATE   EXPLORE   DISCOVER

Note: It is important that the Dilations Tour be done before any Dilations lessons.

Move to page 1.4.

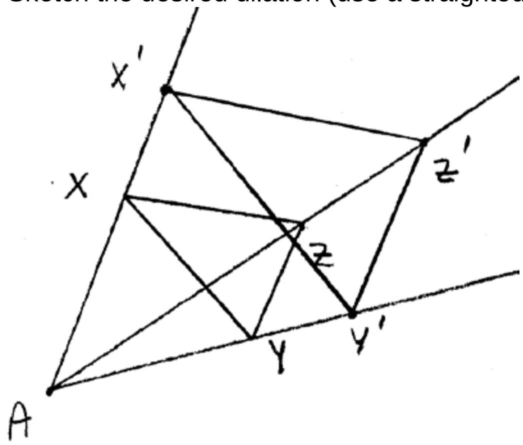


This activity will be a self-assessment of the ideas explored in earlier lessons.

First, use the area below question 1 to make a sketch where ΔXYZ has been dilated about point A with a scale factor of 1.5.

Use the calculator application on page 1.4 as needed for any calculations.

1. Sketch the desired dilation (use a straightedge).



2. If $m\angle X = 20^o$, then $m\angle X' = 20^o$
3. If $YZ = 8$ cm, then $Y'Z' = 12$ cm
4. If $X'Z' = 30$ in, then $XZ = 20$ in.
5. If the perimeter of ΔXYZ is 60 cm, then the perimeter of $\Delta X'Y'Z' = 90$ cm.

6. Calculate the following ratios. Write your answers as decimals rounded to three decimal places and as fractions.

   a. $\dfrac{perimeter(\Delta X'Y'Z')}{perimeter(\Delta XYZ)} = 1.5$ = 3/2

   b. $\dfrac{area(\Delta X'Y'Z')}{area(\Delta XYZ)} = 2.25$ = 9/4

   c. $\dfrac{perimeter(\Delta XYZ)}{perimeter(\Delta X'Y'Z')} = 2/3 \approx 0.667$

7. If the area of ΔXYZ = 72 in^2, then the area of $\Delta X'Y'Z' = 162$ in^2

8. What is true about the segments $\overline{XZ}$ *and* $\overline{X'Z'}$?

   Parallel (or could overlap)

9. The slope of $\overline{XY}$ is $-\dfrac{3}{4}$. List another segment and its slope.

   $\overline{X'Y'}$ has a slope of -3/4

10. If $AX = 10cm,$ then $AX' = 15$ *and* $XX' = 5$ cm.

11. Calculate the ratios. Write your answers as decimals rounded to three decimal places and as fractions.

a. $\dfrac{AX'}{AX} = \dfrac{3}{2} = 1.5$

b. $\dfrac{AY}{AY'} = \dfrac{2}{3} \approx 0.667$

c. $\dfrac{perimeter(\Delta X'Y'Z')}{perimeter(\Delta XYZ)} = \dfrac{3}{2} = 1.5$

d. $\dfrac{area(\Delta XYZ)}{area(\Delta X'Y'Z')} = \dfrac{4}{9} \approx 0.444$

e. $\dfrac{XZ}{X'Z'} = \dfrac{2}{3} \approx 0.667$

f. $\dfrac{area(\Delta X'Y'Z')}{area(\Delta XYZ)} = \dfrac{9}{4} = 2.25$

g. $\dfrac{m\angle X}{m\angle X'} = 1$

h. $\dfrac{m\angle Z'}{m\angle Z} = 1$

12. If point A is at the origin, answer the following questions.
    a. If the coordinates of X are $(6,-12)$, then the coordinates of X' are **(9, -18).**
    b. If the coordinates of Z' are $(6,-12)$, then the coordinates of Z are **(4, -8).**
    c. If the coordinates of Y are $(-7,11)$, then the coordinates of Y' are **(-10.5, 16.5).**
    d. If the coordinates of X' are $(-18,24)$, then the coordinates of X are **(-12, 16).**

13. If point A were to coincide with point X:
    <u>Answer:</u>
    a. Which pairs of sides will overlap? $\overline{XY}$ *and* $\overline{X'Y'}$**;** $\overline{XZ}$ *and* $\overline{X'Z'}$**.**
    b. What is the other pair of sides and what is true about these sides? $\overline{YZ}$ **is parallel to** $\overline{Y'Z'}$**.**
    c. What is true about point X'? **It also coincides with A and X.**

14. Check answers to the questions above: **<u>Move</u> to page 1.3** (ctrl ◄).
    Press menu to open the menu on the handheld. (On the iPad, tap on the wrench icon to open
    the menu.) Press 1 (1: Templates) then 7 (7: Most Options Grid).
    Change the Scale Factor (×) to 1.5.
    Next Dilate the triangle about point P with a scale factor of 1.5 (or D).
    Use the features on this page to test your answers, make corrections, and validate what you have learned.

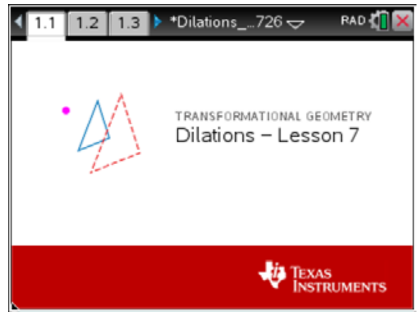
15. List the properties that have been discovered about dilating a triangle about a point with a scale factor. Make sketches and illustrate with examples as necessary.

Sample Answer(s):

- The triangles appear to be the same shape but with a different size.
- The triangles appear to be similar to each other.
- The corresponding angles seem to be equal in measure.
- The sides of the image triangle appear to be the scale factor as long as the corresponding sides of the pre-image triangle.
    - Note: Do not permit students to say "all angles are equal". They need to be more precise and use the phrase "corresponding angles are equal".
- The points P, X, and X' appear to be collinear. Also P, Y, Y' and P, Z, and Z'.
- If the scale factor is greater than one, the image triangle is larger than its pre-image triangle.
- If the scale factor is less than one, then the image triangle is smaller than its pre-image triangle
- The ratio of the perimeters of the image to the pre-image is the same as the scale factor. The ratio of the areas of the image to the pre-image is the scale factor squared.
- If the triangle is dilated about the origin, the coordinates of the image are found by multiplying the corresponding coordinates of the pre-image by the scale factor.

Lesson 7: Compass Construction

In this lesson, you will dilate a triangle about a given point using a compass and straightedge.



Open the document: *Dilations_Lesson7.tns.*

PLAY INVESTIGATE EXPLORE DISCOVER

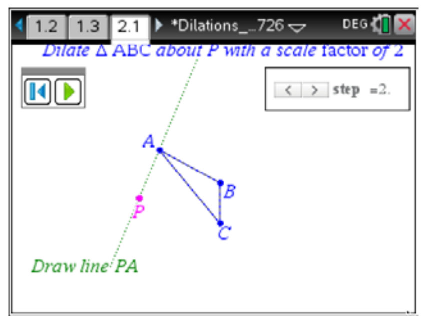
Note: It is important that the Dilations Tour be done before any Dilations lessons.

NOTE: the shortcut keys do NOT work in this file, Dilations_Lesson7.tns.

Move to page 1.2 to 2.1.

Read the directions on page 1.3, and then move to page 2.1.

1. Move the cursor to the slider in the top right corner and click on the right arrow to see the first step. Then press the right arrow on the touchpad to advance a step and press the left arrow to go back a step. Follow along with a compass and straightedge on the drawing below.



Do not erase any of the construction lines and show all work on this paper.

Teacher Tip: It is important to have a discussion with students of what is involved with a Euclid construction, namely a compass and straightedge.

By construction, dilate ΔABC about point P with a scale factor of 2.
Result; $\Delta A'B'C'$