

## Chapter 4

### Applications

#### This chapter includes:

- ◆ Linear motion introduction
- ◆ Vertical motion
- ◆ Horizontal motion
- ◆ Velocity
- ◆ Sliding ladder
- ◆ Circular motion
- ◆ Row and run
- ◆ Newton's method
- ◆ Curve fitting
- ◆ Growth and decay models
- ◆ Finding Max/Min when you cannot differentiate

*The scripts in this chapter require the installation of the programs in the Appendix to run correctly. You can download these programs from <http://www.ti.com/calc/docs/92scripting.htm> and then load them to your calculator using TI-Graph Link.*

### Linear motion introduction

*Scripting for the TI-92 and TI-92 Plus: Precalculus and Calculus Applications* includes several scripts (Vertical Motion, Horizontal Motion, Average Velocity, and Sliding Ladder) that help students learn about linear motion. Begin by reading this introduction to these linear motion scripts.

The TI-92 provides a rich set of tools for helping students understand the motion of an object specified by one or by two position functions. Students can use symbolics to compute limits, derivatives, and integrals. They can also graph position functions and view the motion with a primitive, but useful, animation facility.

Begin a study of motion by considering linear motion. Use position functions to specify the motion of objects that move along straight line paths. A position function specifies the location of an object for each time in a given interval.

### Notes

- ◆ Let  $s(t)$  meters denote the position of an object at time  $t$  seconds. Students have trouble relating the graph of  $y = s(t)$  to corresponding motion of the object. The scripts that follow make a point of graphing the position function and asking students to produce a window suitable for viewing the object's motion. Students learn that the range of  $y = s(t)$

determines one dimension of the parametric graphing window. A difficulty is that many calculus students do not understand range. This is a good place to work on this troublesome concept.

- ◆ Students can use **ZoomFit** to automatically configure the parametric graphing window, but if a proper foundation is not laid, they may not understand the relationship between the position function and the motion.
- ◆ It is interesting that most calculus books/courses use velocity as an important example incarnation of the derivative while most physics instructors say that few of their students understand velocity (or acceleration). [Reference McDermott from the University of Washington.]
- ◆ The linear motion scripts can be used at various times throughout a course rather than in the order presented here.
- ◆ It is helpful for students to learn to record useful information from these scripts, such as window information, arrow diagrams, point traces, and verbal descriptions of position and speed. Using scripts, students can consider many examples and learn to make useful observations and generalizations.

## Vertical motion

### Purpose

To help students understand motion.

The script determines the range of the position function, set-up the graph window, and performs the animation.

### Prerequisites

Introduction to parametric equations

### Problem

Examine the motion of a ball thrown into the air with an initial velocity of 72 ft/s. Discuss the position of the ball as well as its velocity. The position function is given by  $s(t) = -16t^2 + 72t$  ft at time  $t$  seconds.

### Script: vmotion

What you will change:  $s(t), a, b$

```

C:BeginScr()
C:SetMode("Graph","Function")
C:NewPrb()
C:Define s(t) = -16t^2 + 72t
C:Define a=0
C:Define b=5
C:SetWind(a,b,-5,5)
C:Graph s(t),t
C:ZoomFit
C:Define c = ymin
C:Define d = ymax
: View the motion
C:animate()
C:SetpWind(a,b,0,2,c,d)
C:Define xt1(t) = 1
C:Define yt1(t) = s(t)

```

```

C:Define tstep = 0.3
C:Style 1,"Square"
C:DispG
C:EndScr()

```

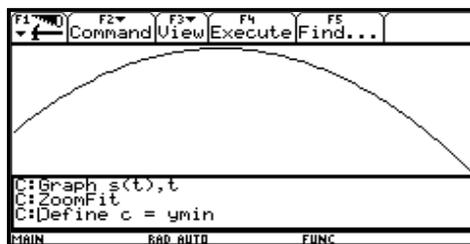
### Student Activity

Provide these instructions to your students:

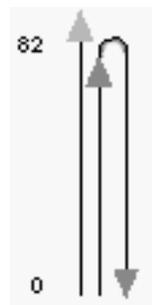
Use the script to work through the following guided solution to the problem.

### Guided Solution

1. Graph the position function and determine its range.
  - a. First, graph the position function over the time interval in which the ball is in the air. To find when the ball hits the ground, solve the equation  $s(t) = 0$  for  $t$ .



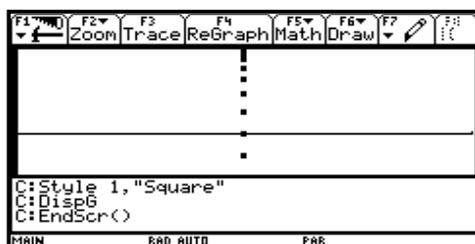
- b. Use the range of the graph of  $y = s(t)$ , to set the range for the motion of the ball. Use parametric mode and **setpwind**( $a,b,0,2,c,d$ ). Be able to explain where the numbers come from. (See **SetpWind** in the Appendix.) The script above will help you carry out the animation so you can see the motion of the ball. In general, you can find the range by looking at **ymin** and **ymax** in the Window Editor after using **ZoomFit**.
2. From the graph of the position function, draw an arrow diagram of the motion of the object.



An arrow diagram for the motion of the ball appears above. It simply shows that the ball starts up from position 0, changes direction at  $y = 82$  ft. and falls back to the ground. For many position functions, the arrow diagram will be more interesting. (Give students practice drawing the arrow diagram given the graph of the position function. Students can use the animation produced by the script to check their arrow diagrams.)

3. From the graph of the position function, draw a point trace of the object's motion.

The point trace produced by the TI-92 is shown on the next page. You should be able to obtain this yourself and use the TI-92 to check your work. Remember that the points mark off the position over uniform time intervals so you can see the changes in the speed and acceleration of the object.



Since this object doubles back over itself, it is useful to superimpose the point trace on top of the arrow diagram. Give it a try now.

4. Set up the TI-92 to do a simulation of the motion. Specify the parametric window information for the motion and run the animation. Use the result from the animation to check your point plot when you are not given a correct answer like you were above.
5. Answer these questions:
  - ◆ Where does the object move fastest? Slowest? How do you know? If you extend  $y_{\max}$  to 90, you can see the answer more easily when you trace.
  - ◆ The object moves slowly at positions near  $y = 82$  ft; moves fastest near  $y = 0$ . The farther apart successive points, the faster the object moves. Why?
  - ◆ How high does the ball travel? How do you know?
  - ◆ When does it reach its highest point? How do you know?
  - ◆ When does the ball strike the ground?
6. Study the vertical motion given by the following position functions over the indicated time intervals. You should graph of the position function over the interval specified and then produce a point trace and an arrow diagram by hand, and write descriptions of the motion. Use the script to check your arrow diagram and point trace.
  - a.  $y(t) = 5t$  over  $[0,4]$  s
  - b.  $y(t) = -t^4 + 2t^2$  while  $y(t) \geq 0$ .

### Note

A point trace qualitatively shows both the average velocity (average rate of change of position) and average acceleration (average rate of change of velocity, sort of) of the moving object. Have students make a point trace from a graph of the position function and check the results with the script.

For complicated motion it is often helpful to combine arrow diagrams and point traces, because the point traces produced by the TI-92 double back, making it difficult to see the desired velocity and acceleration information.

## Horizontal motion

### Purpose

To help students understand motion.

The script determines the range of the position function, sets up the graph window, and performs an animation that depicts the motion.

## Problem

Examine the motion of an object moving along a horizontal line with position function given by  $x(t) = s(t) = \sin(t)$ . Consider the motion for  $t$  in the closed interval  $[0, 2\pi]$  seconds. (We will discuss the position of the object as well as its velocity.)

## Script: hmotion

Given  $s(t)$  and  $[a, b]$ , specify window information and draw a motion trace of the corresponding horizontal motion.

```
C:BeginScr()
C:SetMode("Graph", "Function")
C:NewPrb()
  :Change s(t), a, b, as needed.
C:Define s(t) = sin(t)
C:Define a=0
C:Define b=2*π
C:SetWind(a, b, -5, 5)
C:Graph s(t), t
C:ZoomFit
C:Define c = ymin
C:Define d = ymax
  :View the motion
C:animate()
C:SetpWind(a, b, c, d, 0, 2)
C:ClearAll()
C:Define xt1(t) = s(t)
C:Define yt1(t) = 1
C:Style 1, "Square"
  :Adjust the space between points.
C:.2>tstep
C:DispG
C:EndScr()
```

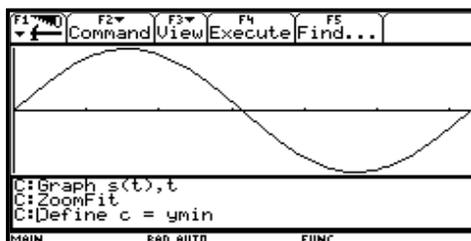
## Student Activity

Provide these instructions to your students:

Use the script to work through the following guided solution to the problem.

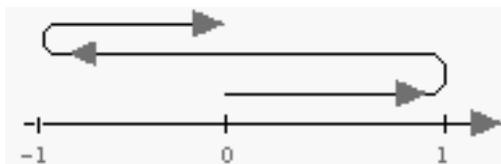
## Guided Solution

- Graph the position function and determine its range.



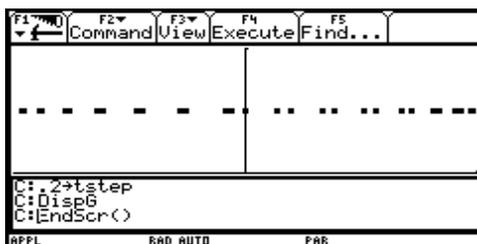
The function in this example is well known, so students should know its range without using **ZoomFit**. For general functions, students can find the range by looking at **ymin** and **ymax** in the Window Editor after using **ZoomFit**.

- From the graph of the position function, draw an arrow diagram of the motion of the object.



3. From the graph of the position function, draw a point trace of the object's motion.

Here is the point trace produced by the TI-92 in the next part. You should be able to obtain this yourself and use the TI-92 to check your work. Remember that the points mark off the position over uniform time intervals, so you can see the changes in the speed and acceleration of the object.



4. Since this object doubles back over itself, it is useful to superimpose the point trace on top of the arrow diagram. Give it a try now.

Set-up the TI-92 to do a simulation of the motion. Specify the parametric window information for the motion.

Use **SetpWind**(0,  $2\pi$ , -1, 1, 1.1, 0, 2) and relevant parts of the script above. Note that **xmin = -1.1** and **xmax = 2.2** are derived from the range of the position function by adding a little more.

5. Answer these questions:
- ◆ Where does the object move fastest? Slowest? How do you know?
  - ◆ The object moves slowly at positions near  $x = -1$  and  $x = 1$ ; moves fastest near  $x = 0$ . The farther apart successive points, the faster the object moves. Why?
  - ◆ How far to the right does the object go? How do you know?
  - ◆ When does the object reach its rightmost position?
  - ◆ For 6 through 11 below, given  $s(t)$  and  $[a,b]$ , specify window information and draw a motion trace of the corresponding horizontal motion. Answer each of the questions in the *Guided Solution* above.
6.  $s(t) = 3t$  for  $t$  in  $[0,5]$
7.  $s(t) = \sin(2t)$  for  $t$  in  $[0,4\pi]$
8.  $s(t) = 7$  for  $t$  in  $[0,5]$
9.  $s(t) = t^3$  for  $t$  in  $[-1,1]$
10.  $s(t) = |t|$  for  $t$  in  $[-1,1]$
11.  $s(t) = t^2$  for  $t$  in  $[-1,1]$

12. Compare the motion of the objects in #10 and #11. Discuss both position and velocity. If you need to see the motions simultaneously, enter the following on the Home screen or perform equivalent calculator operations.

```
Define xt1(t) = abs(t)
Define yt1(t) = 1
Style 1,"Square"
Define xt2(t) = t^2
Define yt2(t) = 0.5
Style 2,"Square"
SetGraph("Graph Order","SIMUL")
DispG
```

13. What is meant when you are asked to *examine the motion of an object*? What does the word *motion* mean to you?

### Notes

- ◆ Students have more trouble with horizontal motion than with vertical motion, so it is better to do vertical motion first.
- ◆ At this point it might be good to ask students to answer the questions in #5 using only the graph of the position function.

## Velocity

### Purpose

To introduce the average velocity function as a function derived from a position function.

To help students understand velocity by analyzing average velocity functions using approximate-and-guess and by using limits.

### Prerequisites

Functions, limits, previous visual linear motion topics

### Problem

Determine the velocity at time  $t = 2$  s of an object with position function  $s(t) = t^2$  meters. Depending on your physics and mathematics backgrounds, you may not know what velocity is. The script below contains a definition for velocity of an object moving along a straight line.

### Script: vel

```
C:BeginScr()
C:SetMode("Graph","Function")
C:NewPrb()
C:Define s(t) = t^2
C:Define a=2
C:Define av(h) = (s(a+h)-s(a))/h
C:av(h)
:What happens when h=0?
C:av(0)
C:av({0.5,0.1,0.01,0.001})
C:limit(av(h),h,0)
C:EndScr()
```

### Student Activities

Provide these instructions to your students:

1. Work through the script above and answer the following questions.
  - a. Obtain a formula for the average value,  $av(h)$ , of the velocity of the object over the interval  $[2, 2+h]$ .
  - b. Explain why  $av(0)$  is undefined.

```

F1 [ ] F2 [ ] F3 [ ] F4 [ ] F5 [ ]
Command View Execute Find...
Define s(t)=t^2 Done
Define a=2 Done
Define av(h)=(s(a+h)-s(a))/h Done
av(h) h+4
av(0) undef
:What happens when h=0?
C:av(0)
C:av(.5,.1,.01,.001)
    
```

- c. Make a table that shows how  $av(h)$  behaves for  $h$  near 0.

$h$	$av(h)$
.5	4.5
.1	4.1
.01	4.01
.001	4.001

- d. Based on the data in the table, guess what happens to  $av(h)$  as  $h$  approaches 0.
  - e. Use the TI-92 to compute the limit as  $h \rightarrow 0$ . What actually happens to  $av(h)$  as  $h$  approaches 0?

```

F1 [ ] F2 [ ] F3 [ ] F4 [ ] F5 [ ]
Command View Execute Find...
av(h) h+4
av(0) undef
av(.5 .1 .01 .001)
{4.5 4.1 4.01 4.001}
lim av(h)
h->0 4
C:av(.5,.1,.01,.001)
C:Limit(av(h),h,0)
C:EndScr()
    
```

- f. How might one define the velocity of this object at time  $t = 2$  seconds? Use an important calculus concept.

For problems 2 through 5 investigate the velocity of each object at the time specified. Answer each of the questions in #1 (a through e) for these problems.

2.  $s(t) = t^3, a = 2$ .
3.  $s(t) = \sin(t), a = 0$ .
4.  $s(t) = \text{abs}(t - 2), a = 2$ .
5.  $s(t) = 16t^2 - 8t, a = 4$ .

### Note

This average velocity script could also be used when learning about derivatives.

## Sliding Ladder

### Purpose

To demonstrate how to use a script to create a simple animation.

### Prerequisites

Parametric representation of linear motion

### Problem

Suppose a 14 foot ladder leans against a wall. Its bottom is *slowly* pulled away from the wall at a constant rate of 1 foot per minute. Obtain and analyze a model of the height of the ladder as a function of the time in minutes. Assume the bottom of the ladder is against the wall at time  $t = 0$  minutes.

### Script: ladder

```
C:BeginScr()
C:Animate()
C:NewPrb()
C:Define s(t)=√(14^2-t^2)
C:setpwind(0,14,-1,14,-1,15)
C:Define xt1(t)=t
C:Define yt1(t)=0
C:Style 1,"Square"
C:Define xt2(t)=0
C:Define yt2(t)=s(t)
C:Style 2,"Square"
:Adjust space between points.
C:.5→tstep
C:DispG
:Watch the ladder fall.
C:for i,0,14,.5: line xt1(i),yt1(i),xt2(i),yt2(i): endfor
:You could add another line command that erases the one
:just drawn. See the TI-92 Guide for a command that erases
:lines.
C:EndScr()
```



### Student Activity

*Provide these instructions to your students:*

This is an active note-taking activity. Your job is to write a complete solution to the problem posed. Include any graphs and tables that help in understanding this problem and its solution. You may either use the script below or perform indicated operations from the Home screen.

First, note that, if time  $t = 0$  when the bottom of the ladder is next to the wall, the distance of the ladder from the wall is given by  $x(t) = t$  feet where  $t$  is measured in minutes.

1. A thought experiment: Before using a TI-92, think about the ladder sliding down the wall as the bottom of the ladder is pulled out at a constant rate. Describe the motion of the top of the ladder. Mention both position and speed.
2. Simulate the motion of the bottom and the top of the ladder.
  - a. Get into parametric mode and set up the graphing window as follows:
 

**tmin** = 0    **tmax** = 14

**xmin** = -1    **xmax** = 14

**ymin** = -1    **ymax** = 15
  - b. Use Parametric mode to simulate the motion of the bottom of the ladder.
  - c. Use Parametric mode to simulate the motion of the top of the ladder. **Hint:** You might need Pythagorean Theorem.
  - d. Use **Simultaneous** mode (Graph Window, press **[F1]**, **9:Format, Graph Order, Seq**) to see the top and bottom of the ladder move simultaneously.
  - e. Now describe the motion of the top of the ladder.
3. Use calculus to find out how fast the top of the ladder is falling when it strikes the ground.
4. Reality check: Is it possible to pull the bottom all of the way out at a constant rate? Explain. (Does the speed of the top predicted by this model of the ladder exceed the speed of light?)
5. Here is another way to find  $dy/dt$  in terms of  $dx/dt$ :
  - a. Use the Pythagorean Theorem to write an equation relating  $x$  and  $y$ . You should have already done this in **#2**.
  - b. Note that both  $x$  and  $y$  are functions of  $t$ . Differentiate both sides of the equation with respect to  $t$  and solve for  $dy/dt$  in terms of  $dx/dt$ .
  - c. If  $dx/dt = 1$  foot/min., do you get the same expression for  $dy/dt$  that you used in **#3**? You have just solved your first *related rates* problem in this course.

## Notes

- ◆ This activity can be done with a complete script, without a script, or with an incomplete script that leaves out information the instructor wants students to develop themselves.
- ◆ Students may be able to see the motion better with the **for** loop that draws lines removed. See script **ladder2**.

## Circular motion

### Purpose

To introduce students to the notion of motion in the plane specified by two position functions.

## Prerequisites

Parametric representations, algebraic representations of circles

## Problem

Use parametric representation of a circle to visualize circular motion.

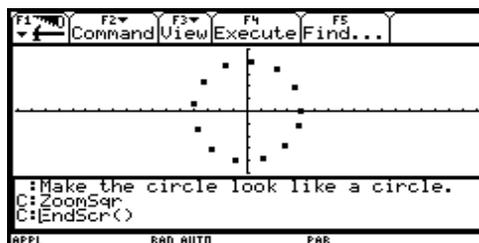
## Script: circular

```
C:BeginScr()
C:Animate()
C:NewPrb()
C:Define xt1(t) = 4cos(t)
C:Define yt1(t) = 4sin(t)
C:Style 1, "Square"
C:0.5 → tstep
C:SetpWind(0,2π,-5,5,-5,5)
C:DispG
:Make the circle look like a circle.
C:ZoomSqr
C:EndScr()
```

## Student Activities

Provide these instructions to your students:

1. Execute the script or enter it from the Home screen.



2. Describe the motion of the object. Discuss the position, speed, and acceleration. (These are two-dimensional ideas, so you must be sensitive to your students' backgrounds. I allow very intuitive language since most of my students don't know much about vectors.)
3. Make the object move the other way around the circle.
4. Find an equation of the circle in terms of  $x$  and  $y$ .
5. Compute  $dy/dx$  at the point  $(2, \sqrt{12})$ .
  - a. From the equation in #4.
  - b. From  $yt1'(t)/xt1'(t)$ , where  $t = \pi/3$ .
6. Add another moving object.

```
C:Define xt2(t) = 2*cos(t)
C:Define yt2(t) = 4*sin(t)
C:Style, 2, "Square"
```

7. View the motion of the two objects. You may want to slow down the animation by setting **tstep = .1**.
  - a. Do the paths of the two objects intersect?
  - b. Do the two objects collide? Prove this analytically.

### Note

For some classes it will be helpful to ask students to make tables with columns labeled as  $t$ ,  $xt1(t)$ , and  $yt1(t)$ . Have them plot several points so they see the relationship between the position functions and the points showing up in the point plots.

## Row and run

### Purpose

To demonstrate how a traditional word problem can be extended to include one or more parameters. Computer algebra makes it reasonable to solve the problem several times for different values of the parameter and to solve the problem with the parameter.

### Prerequisites

Derivative, maximum/minimum problems.

### Problem

Jobe is in a row boat at point A, which is 3 km directly off shore from point B. Jobe wants to row to point C, yet to be determined, and run to point D, which is 8 km south of point B. Time is of the utmost importance, so Jobe needs to get from point A to point D in the least possible time. Assume that Jobe rows at 6 km/hr and runs at 8 km/hr.

Here is a sketch of the physical layout.

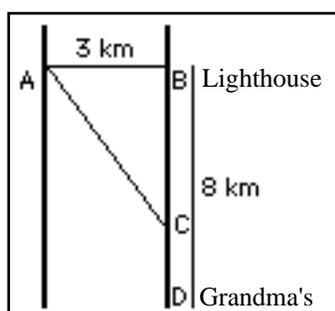


Figure 1

### Script: rowrun

```

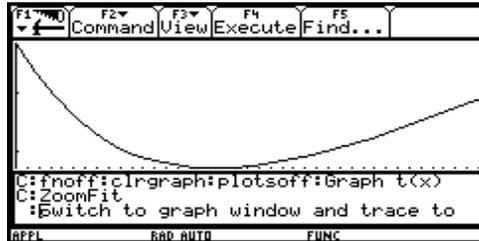
C:BeginScr()
C:SetMode("Graph","Function")
C:NewPrb()
C:DelVar dt
:Row and run problem with
:1.Perpendicular distance from boat to lighthouse = 3 m
:2.Rowing speed = 6 km/hr
:3.Running speed = 8 km/hr
C:Define t(x)=sqrt(x^2+9)/6+(8-x)/8
C:Define dt(x)=d(t(x),x)
C:approx(zeros(dt(x),x))

```

```

:What is the solution to the problem?
C:SetWind(0,8,-5,5)
C:Graph t(x)
C:ZoomFit

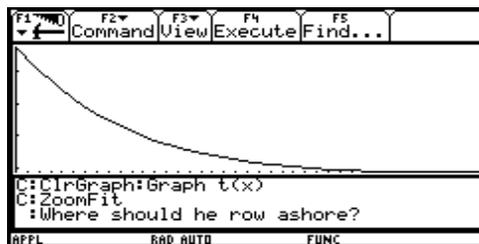
```



```

:Switch to graph window and trace to
:check solution. ESC to return.
C:Trace
:Vary the running speed,rs.
:Row and run problem with
:1.Perpendicular distance from boat to lighthouse = 3 m
:2.Rowing speed = 6 km/hr
:3.Running speed = 6.5 km/hr
C:Define t(x)=√(x^2+9)/6+(8-x)/6.5
C:Define dt(x)=d(t(x),x)
C:approx(zeros(dt(x),x))
:What is the solution for rs = 6.5 km/s?
C:SetWind(0,8,-5,5)
C:ClrGraph:Graph t(x)
C:ZoomFit

```



```

:Where should he row ashore?
: Row and run problem with
:1.Perpendicular distance from boat to lighthouse = 3 m
:2.Rowing speed = 6 km/hr
:3.Running speed = rs km/hr
C:Define t(x)=√*(x^2+9)/6+(8-x)/rs
C:Define dt(x)=d(t(x),x)
C:approx(zeros(dt(x),x))
:Draw graph of x(rs) for rs in [6,10]
:You will have to do some of this
:by hand.
C:DelVar t,dt
C:EndScr()

```

### Student Activity

*Provide these instructions to your students:*

Solve the Problem and the Problem with Parameter. Use your calculator as needed.

**Guided Solution**

1. Let  $x$  denote the distance from the lighthouse B to position C where Jobe lands the row boat.
2. Carefully articulate the goal of this problem. In order to achieve this goal, what function of  $x$  must you obtain?
3. Compute the total time for  $x = 8$  and for  $x = 0$ .
4. What are the key relationships you must use to solve this problem?  
**Hint:** One of them relates speed, distance, and time.
5. Solve the Problem.
  - a. Obtain the total time traveled as a function of  $x$ , the distance between B and D.
  - b. Graph the total time traveled function over its implied domain.
  - c. Graphically approximate the choice of  $x$  that corresponds to the minimum time.
  - d. Use calculus to find the exact value of  $x$  that produces the minimum time.

**Problem with a parameter**

Adjust this problem by varying the running speed throughout the interval  $[6,10]$  km/s.

**Guided Solution**

Let  $rs$  denote the running speed.

1. Solve the problem for  $rs = 6, 6.5, 7, 7.5, 8, 8.5, 9, 9.5, 10$  km/s.
2. Make a table of your results.
3. Solve the new problem for each value of  $rs$  in the closed interval  $[6,10]$ .
4. How does  $x$  corresponding to the minimum total time depend upon  $rs$ ?
5. Sketch a graph of the minimum time as a function of  $rs$  in the interval  $[6,10]$  meters?

**Newton's method**

Newton's method is an important method for approximating zeros of functions. This is a method used by some older calculators. Basically, it uses the zeros of a sequence of tangent lines to approximate the zeros of the function.

This section will help students understand:

- ◆ What one can do with Newton's method.
- ◆ How to derive the Newton's method formula.
- ◆ How to use it (to approximate zeros of functions; to approx  $\sqrt{5}$ ).
- ◆ What can go wrong.

**Purpose**

To understand how to approximate using Newton's method. This is one of many parts of the approximation strand that flows through reform calculus. There are problems that can neither be solved by exact methods nor directly by the TI-92, but which can be solved using Newton's method.

## Prerequisites

Derivative and tangent line, zero of a function.  
(In particular, finding a zero of  $f(x) = \sin(x^2) - 1/2$ .)

## Problem

Approximate a zero of a function using Newton's method.

## Script: newt1

```
C:Define f(x) = sin(x^2) - 1/2
C:Define df(x) = d(f(x),x)
C:Define NextTerm(x) = x - f(x)/df(x)
C:1.0 → a
C:NextTerm(a) → a
:Switch to the Home screen and
:Enter as needed
```

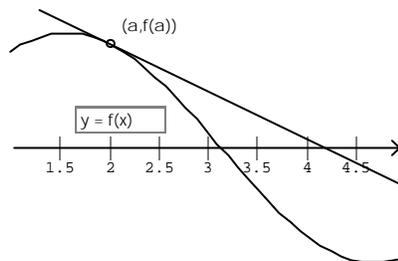
Repeat the previous command as needed (until the approximation of the zero doesn't change much).

## Student Activities

Provide these instructions to your students:

Use the information and script to help you write a complete, understandable set of notes on Newton's Method that achieves the four goals described in the **Purpose** section.

The figure below displays the graph of  $f$  with a tangent line at the point  $(a, f(a))$ . The number  $a$  serves as the current approximation for a zero of  $f$ . Your task is to develop a formula for the next approximation of a zero of  $f$ .



1. Find an equation for this tangent line.
2. Use this equation to find a formula for the next approximation for the zero of  $f$  that's near  $a$ . (The next approximation is the  $x$ -intercept of the tangent line.)
3. On the figure above, draw a vertical line segment from the  $x$ -intercept of the tangent line to the graph of  $f$ . Call the point on the graph  $(b, f(b))$ , where  $b$  is the next approximation of the target zero.
4. Sketch the tangent to the graph of  $y = f(x)$  at the point  $(b, f(b))$ .
5. Does the new tangent produce an improved approximation of the desired zero of  $f$ ?
6. Repeat steps 3 through 5 to get successive approximations of the zero. Continue until you are within 0.1 units of the zero. In script **newt2**, you must manually repeat the three commands between REPEAT and END REPEAT.

**Example**

Use Newton's Method to solve  $\sin(x^2) - 1/2 = 0$  for  $x$ , where  $x$  is in  $[0,2]$ . Using Newton's method you can only obtain an approximate solution. Make a table of successive approximations.

**Additional Problems**

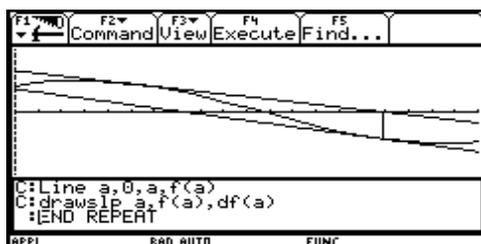
1. Use Newton's Method to approximate the next positive solution of the equation in the previous example. **Hint:** In the script below (or the one above), use a larger initial approximation.
2. Show that if  $h > 0$ , applying Newton's method to  $f(x) = \sqrt{|x|}$  leads to  $\text{NextTerm}(h) = -h$  and  $\text{NextTerm}(-h) = h$ . Draw a picture to show what is going on. Use the **newt2** script. What happens if you pick  $h < 0$ ?
3. Apply Newton's method to  $f(x) = x^{1/3}$  with  $x_0 = 1$ , and calculate  $x_1, x_2, x_3, x_4$ . Find a formula for  $|x_n|$ . What happens to  $|x_n|$  as  $n \rightarrow \infty$ ? Draw a picture to show what is going on.
4. Apply Newton's Method to  $f(x) = x^2 - 5$  to approximate  $\sqrt{5}$ .

**Script newt2: Graphically shows how Newton's method works.**

```

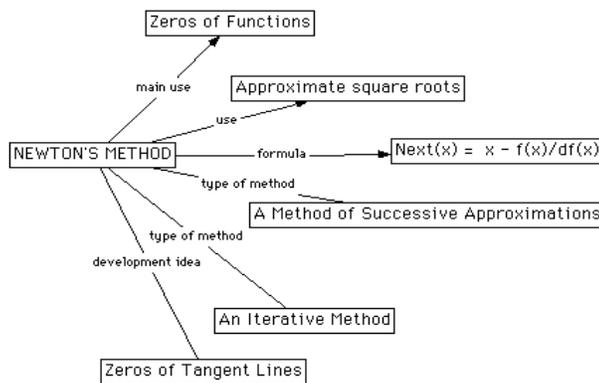
C:BeginScr()
C:SetMode("Graph","Function")
C:NewPrb()
C:DelVar df
C:Define f(x)=sin(x)
C:Define df(x)=d(f(x),x)
C:Define a=2.0
:Choosing window parameters is a bit
: tricky.
C:Setwind(1,5,-2,2)
C:Graph f(x)
C:Drawslp a,f(a),df(a)
:REPEAT as needed.
C:Define a=a-f(a)/(df(a)) : a
C:Line a,0,a,f(a)
C:Drawslp a,f(a),df(a)
:END REPEAT
C:a
C:f(a)
C:EndScr()

```



Screen after first time through REPEAT block

## Concept Map



## Curve fitting

### Purpose

This script shows students an important method for obtaining a functional model of a world situation.

### Prerequisites

Function and basic curve fitting idea

### Problem

Determine what sort of a function might model the stopping of a car. The table below shows measurements taken as a car stopped at a stop sign near my house. The first row contains time measured in seconds while the second row contains the corresponding distance from the stop sign in seconds.

t s	0	1	2	3	4	5	6
s(t) ft	100	45	15	6	2	0	0

### Script: curvfit

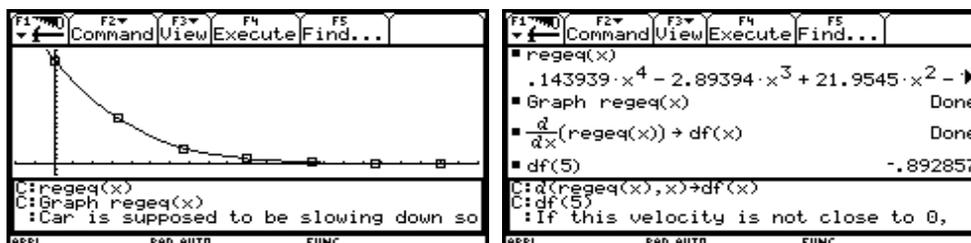
```

C:BeginScr()
C:SetMode("Graph","Function")
C:NewPrb()
C:DelVar 11,12
  : Enter the data: x's first, then y's
C:{0,1,2,3,4,5,6}>11
C:{100,45,15,6,2,0,0}>12
C:NewPlot 1,1,11,12
C:ZoomData
C:quartReg 11,12
C:regeq(x)
C:Graph regeq(x)
  :Car is supposed to be slowing down so velocity should be 0 when it stops
  :at t = 5 seconds.
C:d(regeq(x),x)>df(x)
C:df(5)
  :If this velocity is not close to 0, try another type of curvfit.
C:EndScr()
  
```

## Student Activities

Provide these instructions to your students:

1. Examine the table of times and distances. Which variable is the independent variable? Why?
2. Try different types of curves to determine which fit is best. Your TI-92 supports **LinReg**, **QuadReg**, **CubicReg**, **QuartReg**, **PowerReg**, and **ExpReg**. The TI-92 also supports **MedMed**, **SinReg** and **Logistic**. After you plot the data, try each type that you believe will fit the data fairly well.



3. Sketch the graph of the derivative of  $df(x)$ . You may need to change **ymin**. Based on the problem, what characteristics should you see? Explain.

## Growth and decay models

### Purpose

To show how to use TI-92 calculators to obtain the parameters in an exponential growth/decay model.

### Prerequisites

Exponential functions

### Problem

A certain small European city had a population of 10 and a population of 101 one year later. If the same circumstances remain in effect, what will the population be in another year and a half (that is, 2 1/2 years after the population was 10)?

### Script: growth (TI-92 Plus only)

```
C:BeginScr()
C:newprb()
:PROBLEM A certain small European city had a population of 10 and a
:population of 101 1 year later. If the same circumstances remain in
:effect, what will the population be in another year and a half?
:We know that unlimited growth can be modeled by a function of the form of
:f below.
C:Define f(x)=c*e^(k*x)
:Use the two points of information to determine the constants c and k.
C:solve(f(0)=10 and f(1)=101,{c,k})
C:f(x)
C:ans(1)|ans(2)->f(x)
:Use our model to estimate f(2.5).
C:f(x)
C:f(2.5)
C:EndScr()
```

## Student Activities

Provide these instructions to your students:

1. Look up several growth and decay problems in your textbook that can be solved using this script (except for minor changes).
2. Find several growth and decay problems that cannot be solved by this script.
3. What distinguishes those that can be solved?
4. Write a script to solve a different kind of exponential growth problem from your textbook.

## Note

These activities are examples of problem classification/abstraction problems. Activities of this type may enhance a student's ability to learn traditional mathematics by helping focus on a higher level of abstraction.

## Finding Max/Min when you cannot differentiate

### Purpose

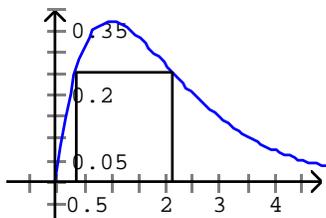
To demonstrate a straightforward method for locating a maximum value without writing down a function.

### Prerequisites

Graphs, basic geometric problem solving skills

### Problem

Find the maximum area of a rectangle inscribed under the graph  $y = x \cdot e^{-x}$  and above the  $x$ -axis.



### Hint:

To determine where the rectangle intersects the graph of  $y = f(x)$  use **solve(f(x) = 2.5, x)**.

### Script

```
NewPrb()
Define f(x)=x*e^(-x)
1.5→a
zeros(f(x)-f(a),x) → z
(z[2]-z[1])*f(a)
```

## Student Activities

Provide these instructions to your students:

1. Write three **Line** commands that will draw the rectangle in the graph shown under the Problem. Let **z1** and **z2** denote the solutions found above from the solve command above.

2. Note that all possible areas correspond to rectangle heights to the left of the maximum point on the graph. Write instructions that will find the maximum value of  $f$  and the  $x$ -value where it is attained.
3. Devise and apply a search strategy that will locate the approximate maximum area of the inscribed rectangles. Be sure to state any assumptions you make. Use the script above for TI-92 commands you may need. Enter the commands from the Home screen.