EXPLORATIONS™

Activity 9

Objectives

- Discover the relationships between position, velocity, and acceleration
- Connect mathematical relationships to real-world phenomena

Materials

- TI-84 Plus / TI-83 Plus
- A spring with a mass attached

Back and Forth—Analysis of Spring Motion

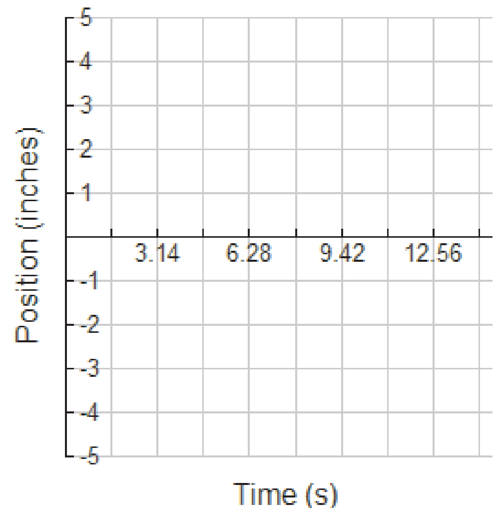
Introduction

The motion of a mass on a spring that moves back and forth on a frictionless path is simple harmonic motion because the force exerted by the spring on the mass is proportional to its position. Mathematically, the equation is represented by $F = {}^{-}kx$ where F is the force on the spring, x is the position of the spring from its unstretched position, and k is the spring constant. Objects in simple harmonic motion are sinusoidal. In this activity, you will explore the relationship between position, velocity, and acceleration for sinusoidal objects.

Exploration

A spring is pulled 4 inches from its unstretched position and released. Its period is 2π, which means it takes 2π seconds to make one complete cycle. In one complete cycle, the spring moves from a position of 4 inches through its unstretched position of zero inches to a position of ⁻4 inches and back to a position of 4 inches. The spring repeats this motion.

1. Make a sketch of the position versus time graph for the spring on the axes shown.

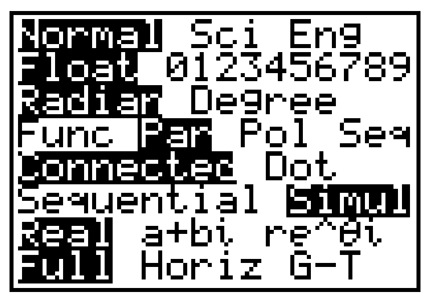


2. Write an equation that models this motion in the form $y = A\cos(Bx)$. Graph it on your handheld using window settings matching those shown on your sketch. Does the graph match up with your sketch? If not, make sure that your mode is set to radians.

3. What does the A represent in this equation?

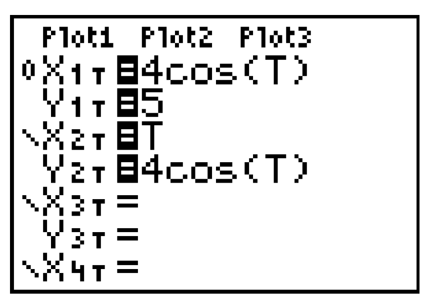
4. What does the B represent in this equation?

5. To simulate the motion of the mass oscillating on the spring, set your graphing handheld to **Par** (parametric) and **Simul** (simultaneous) mode.



6. In the first set of equations of the **Y=** editor, enter the equation for the mass. The x-equation represents the motion of the mass, and the y-equation sets the simulation at $y = 5$ so that the graphing handheld will show the equation of the mass moving as a function of time. The second set of equations represents the position versus time for the motion of the mass on the spring.

   Move the cursor onto the symbol to the left of X_{1T}, and press ENTER until the graph style shows a circle. See the screen shown.



7. Set the viewing window to show the spring through 2 complete cycles with a Tstep of 0.1. Record your window settings below.

Xmin = Xmax =

Ymin = Ymax =

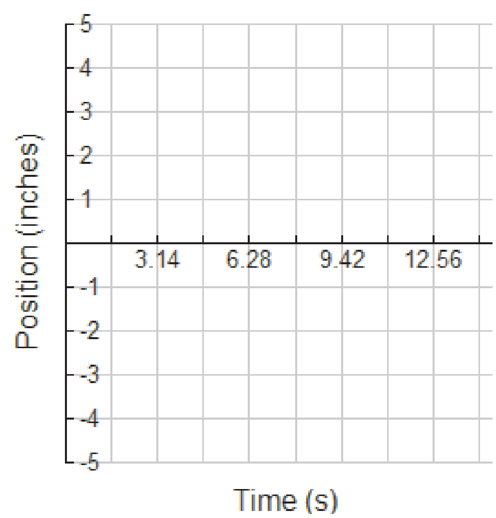
8. Graph the two sets of equations. Notice that the graph style has changed to an open circle for (X_{1T}, Y_{1T}). Record your observations below. You may press the [ENTER] key at any time to pause the motion and press [ENTER] again to resume the motion. Be sure to discuss how the motion of the mass relates to the position versus time graph. If you want to watch the equations graph again, select **1:ClrDraw** from the **DRAW Menu**.

9. What do you notice about the graph of (X_{2T}, Y_{2T}) as the mass moves to the left?

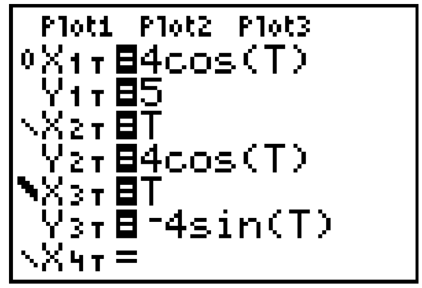
10. What do you notice about the graph of (X_{2T}, Y_{2T}) as the mass moves to the right?

11. When does the motion of the mass seem to move fastest, and when does it seem to move slowest? Does the mass seem to momentarily stop at some point? If so, where?

12. *Velocity* is speed in a particular direction. When the mass moves to the left, the velocity is negative and when the mass moves to the right, the velocity is positive. From your observations, predict what the velocity versus time plot should look like. Sketch your prediction along with the position versus time graph in the space provided.



13. Because velocity is the derivative of position with respect to time, enter the velocity equation in Y_{3T}, and graph it as a function of time. You may want to change the graph style for the velocity equations so that you can easily identify the velocity graph from the position graph. How does it compare with your prediction? Discuss any differences.



14. Describe what the velocity versus time plot looks like when the mass moves to the right and when the mass moves to the left.

15. What does negative velocity look like on the position graph, and what does it look like on the velocity graph?

16. Answer the following questions.

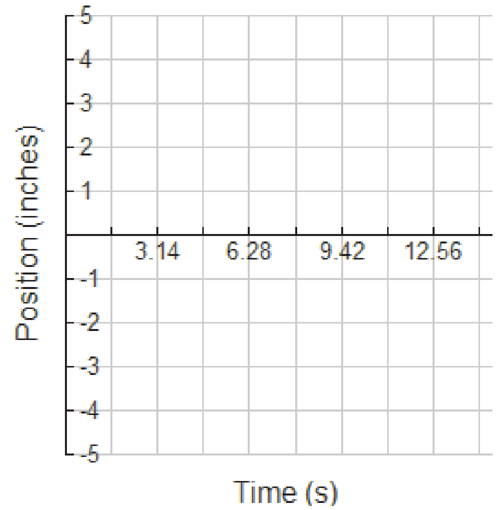
a. When is the velocity equal to zero?

b. What does the position versus time graph look like at this time?

c. What is the mass doing at this time?

d. Using your knowledge of derivatives, why does this make sense?

17. *Acceleration* is the rate of change of velocity with respect to time. In other words, it expresses how much the velocity changes per time interval. So when the velocity is increasing, the acceleration is positive, and when the velocity is decreasing, the acceleration is negative. Using the graph of velocity versus time, sketch a prediction of the graph of acceleration versus time along with the velocity graph.



18. Now, enter your acceleration equation into the **Y=** editor in the fourth set of parametric equations. How does it compare with your prediction?

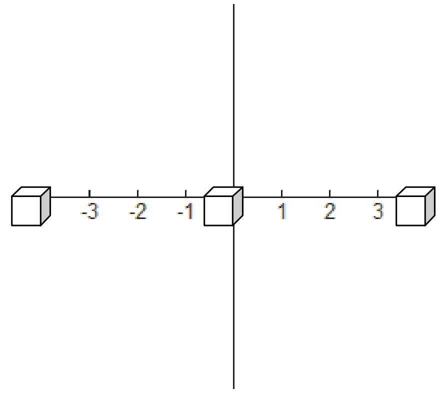
19. Examine the position and acceleration graphs. What do you notice about the graphs?

20. Acceleration is directly proportional to the net force acting on the mass on the spring. In this case, the only force acting is the force of the spring. The force always pulls on the spring to take it back to its unstretched length or equilibrium position. Why does it make sense that when the position of the mass is to the right of the origin (positive), the acceleration acts towards the left (negative)?

21. What is the mass doing when the acceleration is a maximum or minimum?

22. *Speed* is the absolute value of velocity, so when the velocity is ⁻4 inches/second, the speed is 4 inches/second. At what position along its path is the mass moving fastest?

23. Use the diagram of the mass at various locations, and identify the following on the diagram:



a. When is the speed at a maximum?

b. When is the speed zero?

c. When is the acceleration at a maximum?

d. When is the acceleration at a minimum?

e. When is the acceleration positive?

f. When is the acceleration negative?

24. For the following, determine whether the mass speeds up or slows down.

a. The velocity is positive, and the acceleration is positive.

b. The velocity is negative, and the acceleration is negative.

c. The velocity is positive, and the acceleration is negative.

d. The velocity is negative, and the acceleration is positive.

25. What conclusion can you draw from your answers to Question **24**?

26. How would the position equation change if the spring is changed to a new spring with mass that has a period of π?

27. How would the velocity and acceleration graphs compare with the position graph for this new spring?

28. Summarize four ideas presented in this activity.