Centroid
Guided Investigation

Teacher Notes & Answers

7 8 9 **10** **11** 12

TI-Nspire™ Investigation Student 50 min

Introduction

The centroid is one of many 'centres' for a triangle and is one of the easiest to calculate in coordinate geometry. In this investigation it will be computed two different ways. The centroid also represents the centre of mass for a triangle of uniform density. The centroid is constructed using three median lines. A median joins a vertex to the midpoint on the opposite side.

Scan the QR code or use the link to watch a video to help set up the diagram on your calculator and to see how to check your answers. While the video relates to the circumcentre, the skills are very similar.

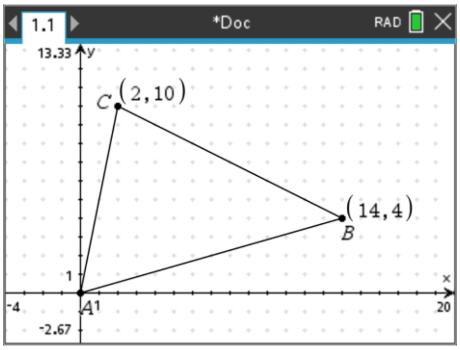
https://bit.ly/Circumcentre

Geometry

Open a New TI-Nspire Document and insert a **Graphs Application**.

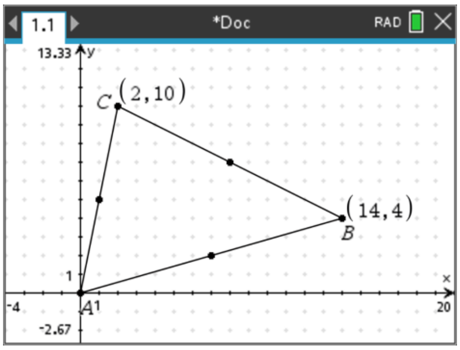
Draw a triangle with vertices:

A:(0, 0) B:(14, 4) C:(2, 10)



Use the midpoint tool to place midpoints on each of the sides: AB, BC and CA.

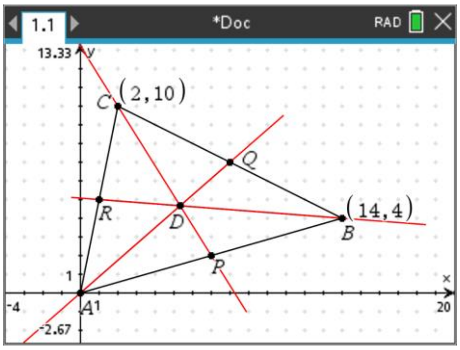
menu > **Geometry** > **Construction** > **Midpoint**



Construct lines from each vertex to the opposite midpoint.

menu > **Geometry** > **Points & Lines** > **Line**

Note: Points have been labelled to provide references for the following questions. Colour has been used to highlight the constructed lines over the original triangle.



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Question: 1.

Determine the coordinates of point Q.

Answer: $\left(\frac{x_1+x_2}{2},\frac{y_2+y_1}{2}\right)=\left(\frac{14+2}{2},\frac{4+10}{2}\right)=(8,7)$

Question: 2.

Determine the equation of the median: AQ where Q is the midpoint of BC.

Answer: $y=\frac{7}{8}x$

Question: 3.

Determine the coordinates of point P.

Answer: $\left(\frac{x_1+x_2}{2},\frac{y_2+y_1}{2}\right)=\left(\frac{14+0}{2},\frac{4+0}{2}\right)=(7,2)$

Question: 4.

Determine the equation of the median: CP where P is the midpoint of AB.

Answer: $y=\left(\frac{10-2}{2-7}\right)(x-2)+10$ which simplifies to: $y=-\frac{8}{5}x+\frac{66}{5}$

Question: 5.

Determine the coordinates of point R.

Answer: $\left(\frac{x_1+x_2}{2},\frac{y_2+y_1}{2}\right)=\left(\frac{2+0}{2},\frac{10+0}{2}\right)=(1,5)$

Question: 6.

Determine the equation of the median: BR where R is the midpoint of AC.

Answer: $y=\left(\frac{5-4}{1-14}\right)(x-14)+4$ which simplifies to: $y=-\frac{1}{13}x+\frac{66}{13}$

Question: 7.

Use simultaneous equations to determine the point of intersection for the AQ and CP.

Answer: Equations: $y=\frac{7}{8}x$ & $y=-\frac{8}{5}x+\frac{66}{5}$

$$\frac{7}{8}x=-\frac{8}{5}x+\frac{66}{5}$$
$$\frac{99}{4}x=\frac{66}{5}$$
$$x=\frac{16}{3}$$

$$y=\frac{7}{8}\left(\frac{16}{3}\right)$$
$$y=\frac{14}{3}$$

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Question: 8.

Verify the point of intersection (centroid) using the point of intersection for BR and AQ.

Answer: Equations: $y=\frac{7}{8}x \;\; \& \;\; y=-\frac{1}{13}x+\frac{66}{13}$

$$\frac{7}{8}x=-\frac{1}{13}x+\frac{66}{13}$$
$$\frac{99}{104}x=\frac{66}{13}$$
$$x=\frac{16}{3}$$

$$y=\frac{7}{8}\left(\frac{16}{3}\right)$$
$$y=\frac{14}{3}$$

Question: 9.

Where A_x represents the abscissa of point A, determine the value of: $\frac{A_x+B_x+C_x}{3}$, comment on the result.

Answer: $\frac{A_x+B_x+C_x}{3}=\frac{0+14+2}{3}=\frac{16}{3}$ This is the abscissa of the centroid!

Question: 10.

Where A_y represents the ordinate of point A, determine the value of: $\frac{A_y+B_y+C_y}{3}$, comment on the result.

Answer: $\frac{A_y+B_y+C_y}{3}=\frac{0+4+10}{3}=\frac{14}{3}$ This is the ordinate of the centroid!

Question: 11.

Calculate the ratio of the lengths: AD:DQ.

Answer: $AD=\sqrt{\left(\frac{16}{3}\right)^2+\left(\frac{14}{3}\right)^2}=\frac{2\sqrt{113}}{3}$ and $DQ=\sqrt{\left(\frac{16}{3}-8\right)^2+\left(\frac{14}{3}-7\right)^2}=\frac{\sqrt{113}}{3}$

Ratio: 2:1

Question: 12.

Calculate the ratio of the lengths: CD:DP and comment on your findings.

Answer: $CD=\sqrt{\left(\frac{16}{3}-2\right)^2+\left(\frac{14}{3}-10\right)^2}=\frac{2\sqrt{89}}{3}$ and $DP=\sqrt{\left(\frac{16}{3}-7\right)^2+\left(\frac{14}{3}-2\right)^2}=\frac{\sqrt{89}}{3}$

Ratio: 1:2 Comment: The centroid divides the two Cevians in the same ratio.

Question: 13.

Calculate the ratio of the lengths: BD:DR and comment on your findings.

Answer: $BD=\sqrt{\left(\frac{16}{3}-14\right)^2+\left(\frac{14}{3}-4\right)^2}=\frac{2\sqrt{170}}{3}$ and $DR=\sqrt{\left(\frac{16}{3}-1\right)^2+\left(\frac{14}{3}-5\right)^2}=\frac{\sqrt{170}}{3}$

Ratio: 1:2 Comment: The centroid divides all the Cevians in the same ratio.

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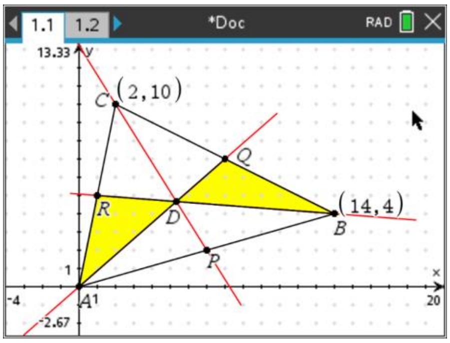
Extension

Use the shapes tool in the Geometry menu to draw two triangles:

Δ ADR and Δ BDQ

Use the measurement tool in the Geometry menu to determine the area of each of these triangles.

Note: Watch the tip very carefully when measuring the area to ensure you measure the correct area(s).
Try hovering the over segment BD when the area tool is active.



Measure and compare the areas of triangles:

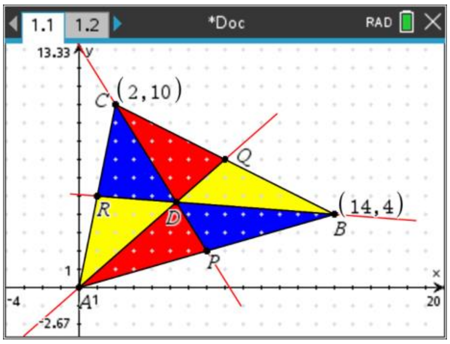
Δ CDQ and Δ ADP

Then:

Δ CDR and Δ BDP

Consider the area of triangles:

Δ CAP and Δ CBP



Question: 14.

Explain your results and explain why the centroid is the centre of mass for a triangle of uniform thickness and density.

Answer:

Consider Δ CDQ & Δ ADP. ∠CDQ = ∠ADP (Vertically opposite)

CD ÷ DP = AD ÷ DQ (Established in calculations)

∴ CD x DQ = AD x DP

Area Δ ADP = ½ AD.DP.sin(∠ADP)

Area Δ CDQ = ½ CD.DQ.sin(∠CDQ)

∴ Area ΔCDQ = Area ΔADP

The same process applies for each pair of triangles:

∴ Area Δ ADR = Area Δ BDQ

∴ Area Δ CDR = Area Δ BDR

Consider Δ CDB. Since BQ = QC (midpoint) then Area Δ BDQ = Area Δ CDQ

Similarly for Area Δ ADR and Area Δ CDR

∴ Areas: Δ ADR = Δ CDR = Δ CDQ = Δ BDQ = Δ BDP = Δ APD

As all the areas are equal, so too such as Δ ADC = Δ BDC, we see that the triangle is balanced around the centroid.

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