Operating on Matrices

ID: 11357

Time Required
20 minutes

Activity Overview

In this activity, students will learn how to add, subtract, and multiply matrices. Students will also learn how to find the determinant and inverse of a matrix. A set of practice problems will reinforce their skills.

Topic: Matrices

- *Operations*
- *Determinant*
- *Inverse*

Teacher Preparation and Notes

- *This activity is created to be a learning tool for how to work with matrices. The activity can be a teacher-led demonstration of the skills, followed by independent practice work by the students, or the students can work independently on the skills in class or at home.*
- *Students should either insert a Calculator page to begin the activity or open the Scratchpad application. If you would like students to have individual histories of each problem, you may want them to use a new Calculator page per problem.*
- *The Practice Problems section can be assigned as classwork or homework.*
- ***To download the solution TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "11357" in the quick search box.***

Associated Materials

- *OperateMatrices_Student.doc*
- *OperateMatrices.tns (not required)*
- *OperateMatrices_Soln.tns*

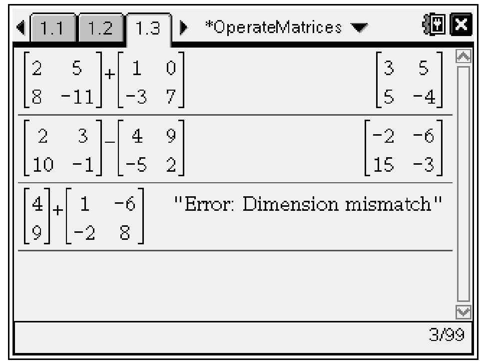
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *All Systems Go (TI-Nspire technology) — 10086*
- *Finding Atomic Weight Using Matrices (TI-Nspire technology) — 9709*
- *Properties of Matrices (TI-Nspire technology) — 8986*
- *Cramer's Rule (TI-Nspire technology) — 8793*

Part 1 – Adding and Subtracting Matrices

Students will use their handhelds to solve addition and subtraction problems. By solving the problems, students should be able to find out a rule for adding and subtracting two matrices. Students should see that the matrices need to have the same dimensions. Elements in corresponding locations should be added or subtracted.

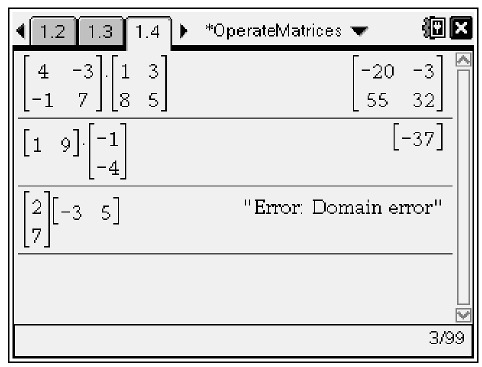


<u>Discussion Questions:</u>

- *How does the handheld add or subtract matrices?*
- *What are the criteria that allow two matrices to be added or subtracted?*

Part 2 – Multiplying Matrices

The goal of this section is for students to understand when matrices can and cannot be multiplied and what the dimensions of the resulting matrix are. Students should observe the patterns when multiplying two matrices. They should see that, in order to multiply two matrices, the number of columns in the first matrix needs to match the number of rows in the second matrix. The resulting matrix will have the same number of rows as the first matrix and number of columns as the second matrix.

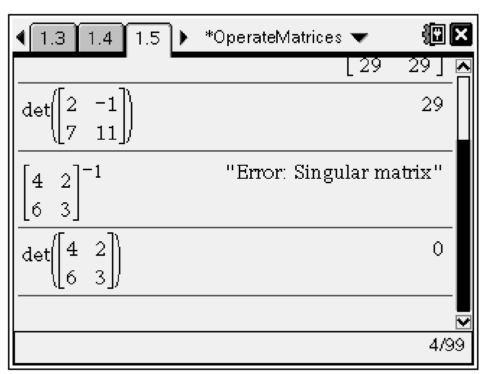


<u>Discussion Questions:</u>

- *Why does the handheld not multiply some matrices?*
- *What are the criteria for when two matrices can be multiplied together?*

Part 3 – Inverses and Determinants of Matrices

In this section, students will find out when a matrix has an inverse and when the determinant can be found. Students should find out that only square matrices have an inverse and a determinant.



<u>Discussion Questions:</u>

- *When does a matrix have an inverse or determinant?*
- *Can you find the determinant of a 2×2 matrix without the handheld? If so, create a rule for finding the determinant of a 2×2 matrix.*

Student Solutions

1. **a.** $\begin{bmatrix} 3 & 5 \\ 5 & -4 \end{bmatrix}$

b. $\begin{bmatrix} -2 & -6 \\ 15 & -3 \end{bmatrix}$

c. does not exist

d. does not exist

2. Matrices can be added or subtracted when their dimensions are the same.

3. The error message only happens when the dimensions of the matrices do not match. The error message explains that the dimensions of the matrices do not match.

4. To add or subtract matrices, elements in corresponding locations should be added or subtracted. The matrices also need to have the same dimensions.

5. **a.** $\begin{bmatrix} -20 & -3 \\ 55 & 32 \end{bmatrix}$

b. $\begin{bmatrix} -37 \end{bmatrix}$

c. does not exist

d. $\begin{bmatrix} 55 & 14 & 60 \\ -68 & -43 & -69 \\ -11 & -35 & -55 \end{bmatrix}$

e. does not exist

6. 2 by 2 • 2 by 2, 1 by 2 • 2 by 1, 3 by 3 • 3 by 3; the 'inside' dimension numbers are the same.

7. The number of columns in the result is the same as the number of columns in the first matrix. The number of rows in the result is the same as the number of rows in the second matrix.

8. If an $a \times b$ matrix is multiplied by a $c \times d$ matrix, then b = c for the result to exist.

9. **a.** inverse does not exist, determinant = 0

b. inverse = $\begin{bmatrix} \frac{5}{13} & -\frac{8}{13} & -\frac{1}{26} \\ \frac{1}{13} & \frac{1}{13} & \frac{5}{26} \\ \frac{2}{13} & \frac{2}{13} & -\frac{3}{26} \end{bmatrix}$, determinant = –26

c. inverse = $\begin{bmatrix} \frac{1}{3} & -\frac{2}{9} \\ \frac{1}{3} & \frac{1}{9} \end{bmatrix}$, determinant = 9

d. inverse and determinant do not exist

e. inverse and determinant do not exist

10. The matrices that have a determinant are square matrices. The matrices that have an inverse are square and the determinant is not equal to zero.

11. If the determinant exists and is not equal to zero, the inverse exists. Also, it appears that the elements in the inverse matrix were divided by the determinant.

12. Yes; the determinant must be exist and be non-zero for an inverse to exist.

Practice Problem Solutions

1. $\begin{bmatrix} 27 & 24 \\ -11 & 13 \end{bmatrix}$

2. $\begin{bmatrix} 1.7 & -0.3 \\ 2.4 & 2.6 \end{bmatrix}$

3. $\begin{bmatrix} -\frac{3}{4} & \frac{33}{4} \\ -2 & -\frac{9}{35} \end{bmatrix}$

4. $\begin{bmatrix} 8.6 & 18 \\ 5 & 31 \end{bmatrix}$

5. $\begin{bmatrix} -14 & 2 \\ 73 & -12 \end{bmatrix}$

6. $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$

7. does not exist

8. $\begin{bmatrix} 18 & -14 & 6 \\ -\frac{9}{8} & \frac{7}{8} & -\frac{3}{8} \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

10. $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$

11. does not exist

12. –3