NUMB3RS Activity: A Bit of Basic Blackjack
Episode: "Double Down"

Topic: Probability involving sampling without replacement and dependent trials.

Grade Level: 8 - 12

Objective: Compute the probability of winning in several blackjack situations

Time: 20 - 30 minutes

Materials: Several standard decks of cards**,** TI-83/84 Plus calculator

Introduction

In "Double Down," three students are tied up in a money-laundering scheme involving money won in the card game blackjack. In this activity, we discuss a simplified version of blackjack and analyze the probability of winning in various situations. Because cards are drawn from the deck and not replaced, the probabilities are based on the concepts of sampling without replacement and dependent trials. (Note: aspects of betting are not considered in this activity.)

Discuss with Students

Blackjack is essentially a card game between two people – the player and the dealer. (There may be other players at the table, but each player is only playing against the dealer.) To win, a player's hand must have a value closer to 21 than the dealer's hand, without going over 21.

Before starting this activity, you will want to review the rules of blackjack with your students. Although some students may be familiar with the game, there are different variations of the game, and it is important that all students understand the rules used in this activity.

Each player is dealt two cards, and can take as many additional cards as he or she wishes. The dealer's hand must be played according to certain rules, with no choices involved. Aces are worth either 1 or 11 (at the player's discretion); tens, jacks, queens, and kings each worth 10; and all other cards are worth their face value. (Note that the suits of the cards do not matter in blackjack.) The term "blackjack" means that you get a value of 21 with only two cards – an ace worth 11 and a 10, J, Q, or K worth 10.

Simplified Rules of Blackjack

1. There is a dealer and from one to seven players at a table.

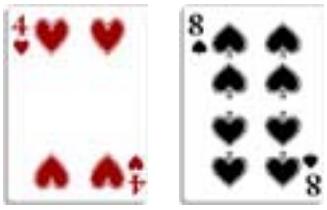
2. A deck of cards is shuffled and cut (several decks of cards may be used). The dealer deals two cards to each player and two cards to himself – one face up and one face down.

3. Each player in turn may request one or more additional cards, with the goal of attaining a total value as close to 21 as possible without going over. A player immediately loses if he or she goes over 21.

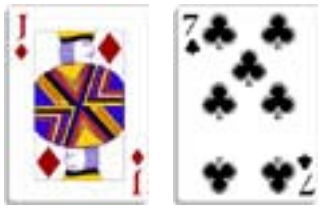
4. When all players are satisfied with their current totals, the dealer may take additional cards according to the following rules:
- the dealer must "hit" (take another card) if the current total is 16 or less, and
- the dealer must "stand" (not take another card) if the current total is 17 or more.

5. Any player with a total greater than the dealer's total wins. If the dealer goes over 21, all players with a total of 21 or less win.

To clarify the rules, you could play some sample games with all of the cards face-up. For example, suppose a player and the dealer have the cards shown below.



Player's cards



Dealer's cards

The player has 12 and the dealer has 17. Because the dealer must stand on 17, the player will win if he or she hits and receives one or more cards whose total is 6, 7, 8, or 9.

Probability and Dependent Events

In the activity, students will be asked to find the probability of being dealt a blackjack with the first two cards of a deck. You could review the needed probability concepts by finding the probability of being dealt two aces in a row.

Remember that probability is defined as $\frac{\text{number of successes}}{\text{number of possible outcomes}}$ when all outcomes are equally likely (as they are in these problems). To find the probability of two dependent events, multiply the probability of the first event and the probability of the second event given the first event.

In a standard deck of cards there are 52 cards, and 4 of the cards are aces. So, $P(\text{ace}) = \frac{4}{52}$.	When a card is dealt, it is not replaced in the deck – there are now 3 aces left in the remaining 51 cards. So, $P(\text{ace, after ace}) = \frac{3}{51}$.	Multiply to find the probability of being dealt two aces in a row: $P(\text{ace and ace}) = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = \frac{1}{221} \approx 0.0045$.

Student page answers: 1. *Answers vary but will be around* $\frac{2}{50}$ *or* $\frac{3}{50}$. **2a.** *52 × 51 = 2,652* **2b.** *2 × 4 × 16 = 128* **2c.** $\frac{2 \times 4 \times 16}{52 \times 51} \approx 0.0483$ **3a.** *Answers vary but will be around* $\frac{2}{50}$ *or* $\frac{3}{50}$. **3b.** $\frac{2 \times 8 \times 32}{104 \times 103} \approx 0.0478$; $\frac{2 \times 12 \times 48}{156 \times 155} \approx 0.0476$ **3c.** *The probability is slightly less for 2 and 3 decks, but the probabilities are close to each other. Students might have expected to see a greater difference in the probabilities when more decks of cards are used.* **4.** *If the dealer shows a 7, he could have a total of 18 with an ace, a total of 17 with a 10, J, Q, or K, or a total of 16 with a 9. If he has less than 16, he could get additional cards so his total exceeds 16. With a 6 showing, the dealer could have a total of 17 with an ace, but would be forced to hit with any other card. His chances of going over 21 are much higher.* **5.** 0.11456

Name: _____ Date: _____

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In “Double Down,” three students are tied up in a money laundering scheme involving money won playing the card game blackjack. Blackjack is essentially a game between two people – the player and the dealer. To win, a player’s hand must have a value closer to 21 than the dealer’s hand, without going over 21. The dealer’s hand must be played according to certain rules, with no choices involved. You (the player) are dealt two cards, and can take as many additional cards as you wish. Aces are worth either 1 or 11 (at your discretion), tens, jacks, queens, and kings are each worth 10, and all other cards are worth their face values. (The suits of the cards do not matter.)

In the problems below, assume that you are the only player and only a single deck of cards is being used (unless indicated otherwise).

Finding the Probability of Being Dealt a Blackjack

The term “blackjack” means that you get a value of 21 with only two cards (an ace and a card with that is worth 10). What is the probability of being dealt a blackjack with the first two cards of a single deck of cards?

1. First, explore the experimental probability of being dealt a blackjack with the first two cards.

a. Simulate this problem with a deck of cards. Shuffle the cards and deal the top two cards from the deck. Check to see if they form a blackjack. Replace the cards in the deck and shuffle. Again, deal the top two cards and check to see if they form a blackjack. Repeat this 50 times. Then find the experimental probability of being dealt a blackjack with the first two cards of a single deck.

b. If you have a TI-83/84 Plus calculator with the **Prob Sim** App, you can simulate this problem using the Draw Cards option. (The **Prob Sim** App can be downloaded for free at **http://education.ti.com/educationportal/sites/US/productDetail/us_prob_sim_83_84.html**.) To draw two cards without replacement, start the application, and press SET to change the settings as follows: Decks = 1; Replace = No; Deck Size = 52. Draw two cards and record whether they form a blackjack. Then press CLEAR and YES to simulate shuffling the deck. Repeat 50 times and compare your results with those from part 1a.

2. Now find the theoretical probability of being dealt a blackjack with the first two cards.

a. When you are dealt two cards, the order of the cards matters because only one card faces up – for example, the hands {Q, 8} and {8, Q} are different. How many different pairs of cards can you be dealt? (Hint: Think of the number choices for the first and second cards.)

b. If a pair is a blackjack, then you were either dealt an ace followed by a 10, J, Q, or K, **or** a 10, J, Q, or K followed by an ace. How many different blackjack pairs are there? (Remember, there are four cards of each of the 13 denominations in the deck.)

c. What is the theoretical probability of being dealt a blackjack?

3. In some blackjack games, several decks of cards are used. Do you think the probability of being dealt a blackjack on the first two cards will increase, decrease, or stay the same if more than one deck of cards is used? Answer the following questions to find out.

a. Using several decks of cards or the **Prob Sim** App on your calculator, simulate this problem for two and three decks of cards. Again, perform 50 trials in each case. What is the experimental probability?

b. Calculate the theoretical probability of being dealt a blackjack using both two and three decks of cards.

c. Were the probabilities of being dealt a blackjack from 1, 2, and 3 decks of cards what you expected?

Basic Blackjack Strategy and Probability

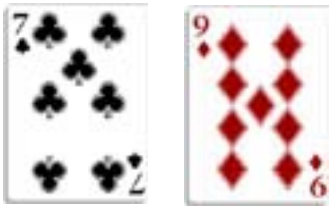
In the actual game of blackjack, you must make your decision whether to hit or stand after seeing only one of the dealer's two cards (the other card is face-down on the table). If your total is "high" (17 or more) you would want to stand, and if your total is "low" (11 or less) you would want to hit. The difficult decisions arise when your total is between 12 and 16. Many books and Internet sites publish tables indicating the better strategy in all cases.

4. Look at the table below.

Player Total	Dealer Up Card	
	6	7
16	Stand	Hit

Give an intuitive reason why a player with a total of 16 should stand if the dealer is showing a 6 but hit if the dealer is showing a 7.

5. **Challenge** Suppose you start another hand with 1 'fresh' deck of cards. Your first two cards total 16 without using an ace, and you are reasonably sure that the dealer's two cards total 20.



Your cards

Dealer's cards

What is the probability you will win this hand by hitting and obtaining one or more cards worth a total value of 5?

Hint: There are 15 different ways to draw cards with a total value of 5. Find the probability for each of these card combinations, and add the probabilities. Remember that four cards have already been dealt, so there are 48 cards remaining in the deck. Also remember that there are four cards of each denomination.

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Blackjack is generally accepted to have evolved out of French games such as "chemin de fer" and "French Ferme." It originated in French casinos around the 1700s and is derived from a family of card games that include Baccarat, Vingt-Et-Un, and Seven and a Half. Vingt-Et-Un is better known to Americans as "21" and to Australians as "Pontoon."

For the Student

- What is the limiting probability of obtaining a blackjack with the first two cards when *n* decks of cards are used?

- Suppose you start another hand with 1 'fresh' deck of cards. Your first two cards total 15 without an ace, but you are reasonably sure that the dealer's cards total 20. What is the probability you will win this hand by hitting and obtaining one or more cards worth 6? (Remember that you may have already used one 5 in making your total of 15.)

- Suppose you have a total of 12 on your first two cards. Give some reasons why you should hit if the dealer's up card is 3, but should stand if it is 4.

- Bringing Down the House: The Inside Story of Six MIT Students Who Took Vegas for Millions by Ben Mezrich is an exciting book about of how a group of students made money playing blackjack in Las Vegas. It is also the subject of the video "Breaking Vegas," that has been shown on the History Channel. (The author Ben Mezrich and the MIT card counting team can be seen in the background of the "Double Down" episode.)

Additional Resources

- Packel, Edward, *The Mathematics of Games and Gambling*, Mathematical Association of America, 1981

- "Mathematics of Blackjack" **http://www.bjmath.com/main.htm**

Related Topic

Students can also analyze the probabilities of winning in many other casino games such as poker, roulette, or craps.

Tom Butts, UT Dallas