The Region Between Two Curves

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Abstract: This activity is an application of integration. Students use calculus to find the area of a region and the volumes of solids generated by the region. They use the symbolic capacity of their calculator and calculus to determine the exact answers.

NCTM Principles and Standards:
Algebra standards
a) analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
b) use symbolic algebra to represent and explain mathematical relationships;
c) judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
d) draw reasonable conclusions about a situation being modeled.

Geometry standards:
a) Analyze characteristics and properties of two- and three-dimensional geometric shapes and mathematical about geometric relationships
b) draw and construct representations of two- three-dimensional geometric objects using a variety of tools;
c) visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections;

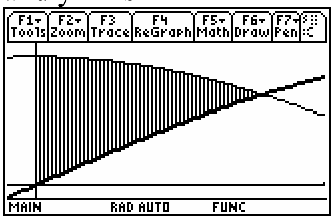
Measurement standards: understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;
Problem Solving Standard: build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.
Reasoning and Proof Standard
a) recognize reasoning and proof as fundamental aspects of mathematics;
b) make and investigate mathematical conjectures;
c) develop and evaluate mathematical arguments and proofs;
d) select and use various types of reasoning and methods of proof.

Representation Standard : use representations to model and interpret physical, social, and phenomena.

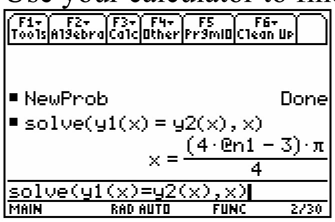
Key topic: Applications of Definite Integrals- determining the area, volume and perimeter of a region

Degree of Difficulty: moderate to advanced
Needed Materials: TI-89 calculator

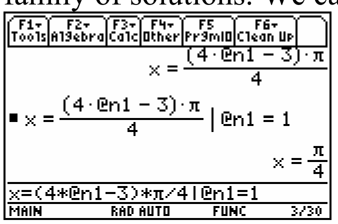
Situation: Consider the region in the first quadrant enclosed by the graphs of y1 = cos x and y2 = sin x



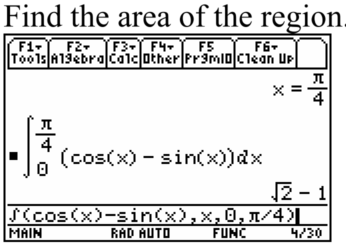
Use your calculator to find where the two graphs intersect:



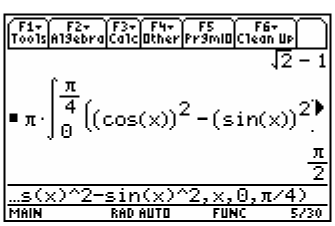
The calculator uses the symbol @n1 to indicate an arbitrary integer as it represents the family of solutions. We can find the value in the first quadrant by setting @n1 =1:



Find the area of the region.

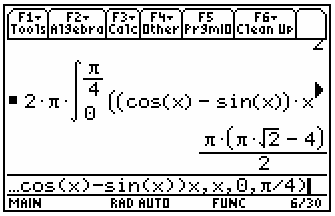


Find the volume of the region as it is rotated about the around x-axis by using the washer method. The outside radius of the region is cos x, the inside radius is sin x, and the thickness is delta x.



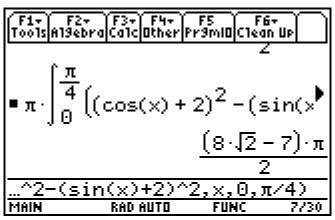
Note: It is important to use the difference of squares of the functions rather than the square of the difference.

Find the volume of the region as it is rotated about the around y-axis by using the shell method. The height of each shell is cos x-sin x, the radius of each shell is x, and the thickness of each shell is delta x.

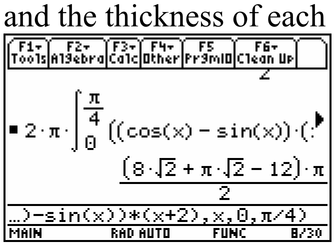


One can also revolve the region around other axes:

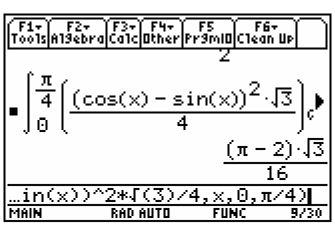
Find the volume of the region as it is rotated about the around the line y = -2 by using the washer method. The outside radius of the region is cos x + 2,the inside radius is sin x + 2, and the thickness is delta x.



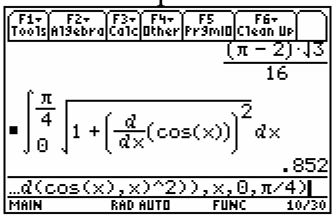
Find the volume of the region as it is rotated about the around the line x = -2 by using the shell method. The height of each shell is cos x - sin x, the radius of each shell is x + 2, and the thickness of each shell is delta x.

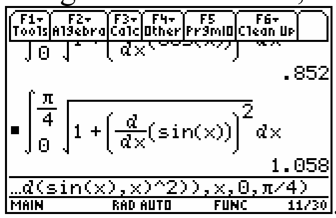


We can also find the volume of solids with known cross sections. Consider the solid whose base is our region and whose cross sections perpendicular to the x-axis are equilateral triangles.



What is the perimeter of the region? To find this, use the arc length formula:





. The other length of the triangular region is 1, so the perimeter is .852 + 1.058 + 1 = 2.91