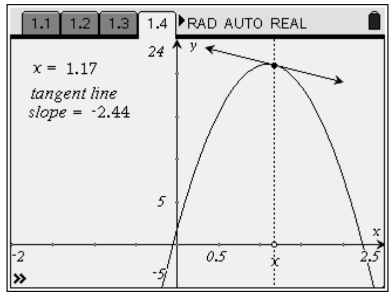
Math Objectives
Students will be able to:

- Visualize the graph of a function's derivative by considering the slope of the graph of the original function.
- Relate a real-world function's derivative to the velocity of an object in flight.
- Relate increasing/decreasing behavior of the function to the sign of its derivative.

Vocabulary
- slope of a tangent line
- derivative at a point
- derivative function
- velocity

About the Lesson
- This lesson is a follow-up lesson to the activity *Derivative Function*.
- Students will explore two graphs that represent the flights of water rockets. They will relate the derivative of the function to the velocity of the rocket.

Related Lessons
- Before this lesson: Derivative Functions
- After this lesson: Exponentially Fast Derivative



TI-Nspire™ Handheld and Computer Software Technology

TI-Nspire Technical Skills:
- Open a document
- Move from one page to another
- Click slider arrows to change values
- Grab and drag a point

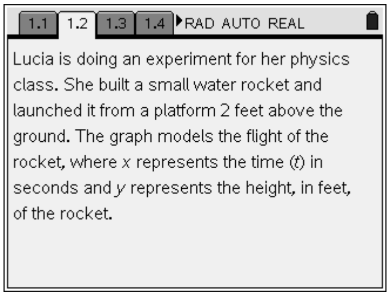
Tech Tip:
- ***Download** the TI-Nspire document to your computer and to your TI-Nspire handhelds.*

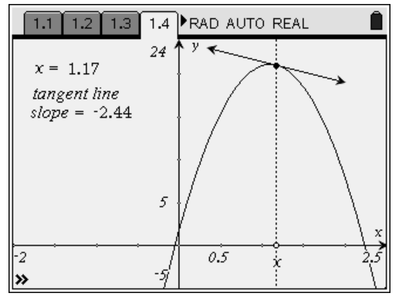
Lesson Materials:
Student Activity
Derivatives_in_Flight_Student.PDF
Derivatives_in_Flight_Student.DOC

TI-Nspire document
Derivatives_in_Flight.tns

Discussion Points and Possible Answers:

<u>TI-Nspire Pages 1.2–1.7</u>





1. On page 1.4, grab the white point labeled x on the x-axis and move it to see the slope $f'(t)$ of the tangent line change as you move along the graph. The slope of the tangent line at each point represents the **velocity** of the rocket at that moment.

Teacher Tip: *Students must make the connection between the (subtle) distinction in notation here:*

- *f'(t) is the slope of the tangent line at x = t*
- *f(t) is the y-value of the point on the graph with x-coordinate a*

It is important to point out that only the first quadrant of the graph represents the flight of the rocket. Negative values for time and height do not fit into the real-world model.

a. What is the value of $f'(t)$ when $t = 0$? What does this value represent?

$f'(0) = 35$

This value represents the starting velocity of the rocket.

b. At approximately what value(s) of t is the derivative $f'(t) = 0$? What happens to the flight of the rocket when the derivative is zero?

$f'(t) = 0$ at $t \approx 1.09$

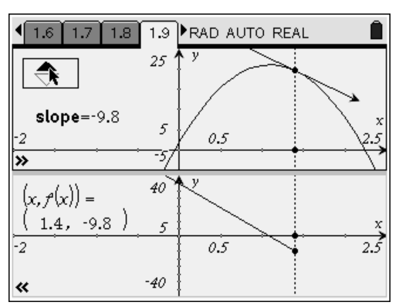
When the derivative is zero, the rocket has reached its maximum height and is starting to fall to the ground.

c. What is the approximate velocity of the rocket when it hits the ground?

$f'(t) = –37$ at $t = 2.25$ when the graph intersects the x-axis and the rocket has zero height.

<u>TI-Nspire Pages 1.8–1.12</u>

Tech Tip: *If students are using the handheld, after each click of either of the arrows for the slider, the graph on the bottom will take a few moments to display. Tell them to allow the graph to appear before clicking the arrow again OR click the arrow just a few times and let the graph appear.*



2. If the value of the derivative $f'(x)$ is plotted as the y-coordinate for each value x, the ordered pairs $(x, f'(x))$ trace out the graph of a new function, $y = f'(x)$, the *derivative function*. Use the up arrow on page 1.9 to change the value of x in the top window and see the graph of the derivative traced out in the bottom window.

Teacher Tip: *This is an opportunity to bring in the language of independent and dependent variables. Students are free to move the value x (the independent variable for the function) but are now considering the dependent value of the slope as it varies.*

   a. What do you notice about the flight of the rocket when $f'(x) > 0$? When $f'(x) < 0$?

When the derivative is positive, the rocket is ascending on its path. When the derivative is negative, the rocket is descending on its path.

   b. What happens to the rocket when $f'(x)$ changes from positive to negative?

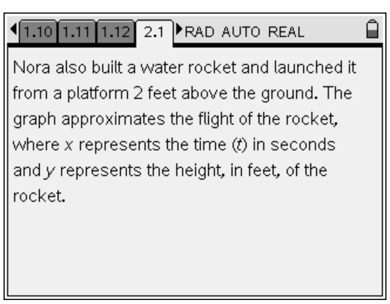
When the derivative changes from positive to negative, the rocket changes direction and heads to the ground.

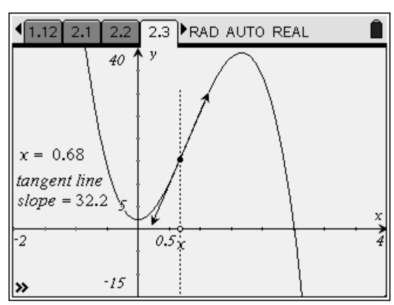
   c. In relation to the flight of the rocket, what determines whether the graph of the derivative is negative or positive?

The direction of the rocket

Teacher Tip: *Students must make the connection that only the* direction *of the object determines whether the derivative is positive or negative. The* speed *of the object is what determines the exact value of the derivative. Remind students that velocity gives both the speed and direction of the object.*

TI-Nspire Pages 2.1–2.6





3. On page 2.3, grab the white point labeled x on the x-axis and move it to see the slope of the tangent line change as you move along the graph.

   a. What is the velocity of the rocket when it is launched? When it hits the ground?

$f'(0) = 0$; $f'(2.53) = -105$

Teacher Tip: *Discuss with students possible reasons for why the slope is zero when t = 0.*

b. How many seconds had passed when the rocket reached its maximum height?

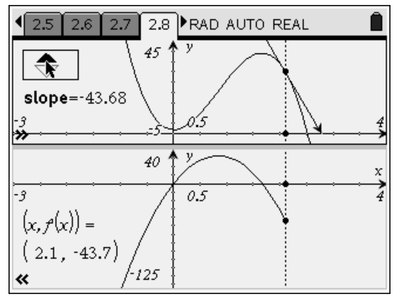
$f'(t) = 0$ when $t \approx 1.67$

c. Describe the flight of Nora's rocket.

Sample response: Nora's rocket looks like it struggled at the beginning, started out slow, but then gained speed.

TI-Nspire Pages 2.7–2.10

Teacher Tip: *Students should focus on the graph of the derivative for positive values of x. In relation to the real-world problem of the rocket, values before the rocket is launched do not exist.*



4. Use the up arrow on page 2.8 to change the value of *x* in the top window and plot the graph of the derivative function $f'(x)$.

   a. What can you say about the flight of the rocket when $f'(x)$ is positive, negative, and zero?

   The rocket is ascending when $f'(x)$ is positive, descending when $f'(x)$ is negative, and changing directions when $f'(x)$ is zero.

   b. Compare the flight of Nora's rocket to the flight of Lucia's rocket. Which rocket went faster and higher? Which was going faster when it hit the ground?

   Sample responses: Nora's rocket went about twice as high as Lucia's rocket. Nora's rocket was going much faster when it hit the ground. The velocity of Lucia's rocket decreased at a steady rate, while Nora's rocket started out slower and sped up briefly as it climbed. The derivative function for Lucia's rocket is a straight line, while the derivative function for Nora's rocket is a curve.

Wrap Up:

Upon completion of the discussion, the teacher should ensure that students understand:

- How to determine the velocity of a function for various times or *x*-values.
- How to relate the value of the derivative to the original graph in relation to the real-world scenario.
- What determines the sign of the derivative.