Coefficient of Friction (Skid Pad)

STEM Lesson for TI-Nspire™ Technology

Objective: Calculate the coefficient of friction from a sample set of data. Explain how maximum speed relates to the coefficient of friction. Explain how maximum speed relates to the radius of a curve.

About the Lesson: Because racing can happen on various surfaces, teams must be prepared to choose the best tires for the surface on which they are racing. Knowing the coefficient of friction between different tires or tire coatings and surfaces will allow teams to choose the best tire for a particular surface and race. A higher coefficient of friction will allow for a higher speed without sliding. This keeps you from plowing into walls as you turn corners. In a professional skid pad test, a test-driver drives at a constant speed around a circular track. The driver gradually increases speed until the car reaches its limits of control, the point just before it flies off the track (under-steer) or spins out (over-steer). You'll do the same thing on your 1:10 scale track.

Materials: RC Car (optional)
Skid Pad track with a lane width of 5 feet
Student Worksheets

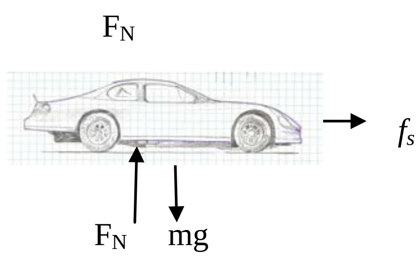
Procedure:

1. Layout a skid pad track: Circles with an inner diameter of 20 ft, a diameter of 30 ft, a diameter of 40 ft and a diameter of 50 ft.
2. Do some practice laps to determine if the surface is so slippery that you need to add weight so the car drives without slipping. If necessary for traction, add weight. Make sure you measure and record the exact weight and use that for every trial.
3. Drive the car at its limits of control around the inner circular track at least ten times without stopping. You should be ALMOST flying off the track on each lap. You want to be at the limits of control.
4. Record the total elapsed time after every lap.
5. Repeat the trial without changing anything at least two more times.
6. Now move to the next circle and repeat steps 1-5. Continue this process for all circles.

Analysis:

F_N - The Normal Force : The force exerted on the car by the ground
m - mass of the object
g - gravity
f_s - The static frictional force exerted on the tires
a_c - the direction of the centripetal acceleration which results in a centripetal force pointed toward the center of the circle.
μ_s - coefficient of static friction

Below is a simple force-diagram of a car rounding a flat curve (flat curve as opposed to a banked turn).



Newton's 2nd Law states that the net force on an object is equal to its mass times acceleration.

First, sum the forces in the x direction, in other words find the net force. Since there is acceleration, all the forces going in the direction of acceleration minus all the forces in the opposite direction equals the centripetal force. In this case there is only one force in the direction of acceleration and it is the force due to static friction.

$$ma_c = f_s$$

Second, sum the forces in the y direction. Since there is no acceleration, the net force is zero. Therefore, all the forces going up equal all the forces pointing down.

$$F_N = mg$$

The relationship between the friction and normal force is given by the equation below.

$$f_{s\,max} = \mu_s F_N$$

1. Substitute the first two equations into the third equation. Show your work below.

Centripetal acceleration is related to the speed and radius of the circle in which the car is traveling as stated below.

$$a_c = \frac{v^2}{r}$$

2. Substitute this definition in for centripetal acceleration. Show your work below.

Since velocity is distance divided by time, the velocity of a circle is the circumference divided by time.

$$v = \frac{2\pi r}{T}$$

3. Now substitute the definition for velocity into the latest equation. Show your work below.

4. What is your independent variable?

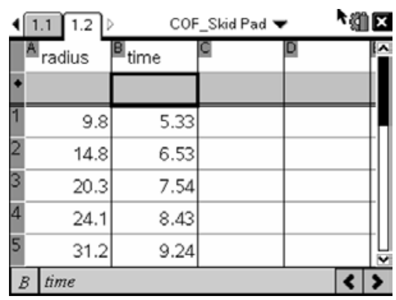
5. What is your dependent variable?

6. Based on your answers above, arrange this equation in slope-intercept form (hint: one of your variables will be squared). Show your work below.

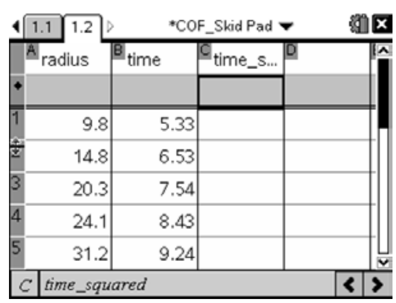
In this instance T^2 is plotted on the y-axis and r is plotted on the x-axis. This implies that $2\pi/\mu_s g$ is equivalent to your slope. Keep in mind we are trying to determine the coefficient of static friction between the tires and the road based on the data collected. If we can find the slope through graphing, then we can set it equal to the slope in this equation and solve for the coefficient of static friction.

7. On your handheld, go to My Documents and open the file named *COF_Skid Pad.tns*.

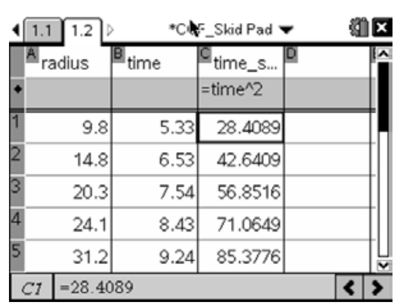
8. Use ctrl ▸ to move to page **1.2**. Enter your radius and period data into the appropriate columns. Make sure to start in the white box beside the number 1.



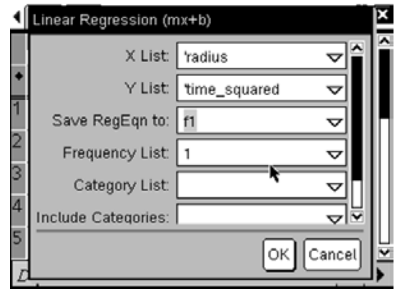
9. Since our equation has time squared we need to square the period data. Move to the top of column C and type **time_squared**. Press enter.



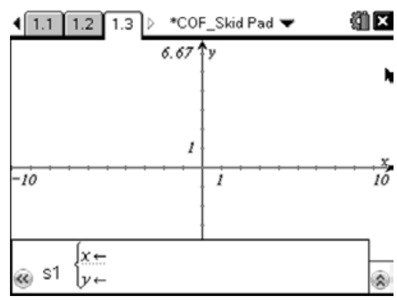
10. In the shaded cell type **time** then press x^2. This should automatically fill the column with each time squared.



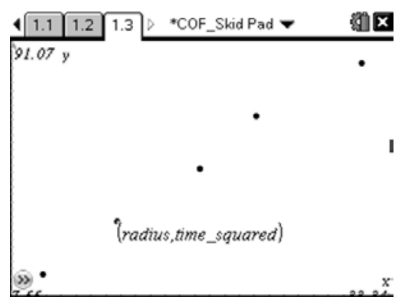
11. Do a linear regression analysis on your data. Press menu then choose **Statistics > Stat Calculations > Linear Regression (mx+b)**. The regression template will pop up. Remember to press tab to switch boxes. Click the arrow at the right of each box to access the drop down menus. Choose *radius* from the drop down menu for the X List and *time_squared* for the Y list. Tab through the other options until you get to 1st Result Column. Click beside the letter and change it to 'e' if it isn't already. Click OK.



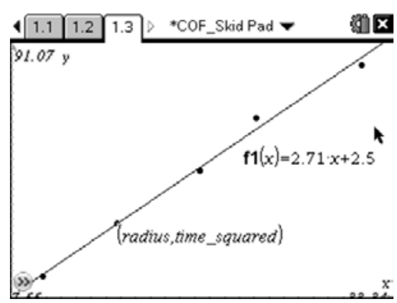
12. Move to page **1.3**. Press menu then choose **Graph Type > Scatter Plot**. Notice the function bar at the bottom of the screen changes from f1 to s1.



13. Press var and choose *radius*. Press ▼ then var and choose *time_squared*. Press enter. Now adjust your window to view the data by pressing menu then choosing **Window/Zoom > Zoom – Data**.



14. Now graph the regression line. Press menu then choose **Graph Type > Function**. Press ▲ to see your function in f1 then press enter. You should see your line on top of your data points.



15. What is the equation of your function?

16. What number in your equation represents the slope of the data?

This slope is equal to the slope represented in the original equation:

$$m = \frac{4\pi^2}{\mu_s g}$$

17. Use your slope for *m*, 32 ft/s² for *g*, and solve for μ_s. Show all of your work.

When the coefficient of friction is 0, there is no friction. The larger the coefficient of friction, the more force you have to apply to get the object to move. When the coefficient of friction is equal to one, that implies that the horizontal force applied must equal to the weight of the object.

18. Based on this information, does your answer in question 17 make sense?

19. What does your value of the coefficient of friction mean in terms of force?

20. With a constant coefficient of friction, what happens to the maximum speed of the car as the radius increases? Explain your reasoning.

21. On page **1.3**, you plotted T^2 vs. *radius*. Speed is determined from time and distance. Sketch what a plot of *speed* vs. *radius* would look like.

The coefficient of friction tells you how "sticky" a surface is. For example walking on ice is difficult because you have no traction. That "traction" is friction.

22. When it rains outside, what happens to the coefficient of friction between tires and the road?

23. What happens to how fast people should drive to stay safe when the road is wet?:

24. As you increase the coefficient of friction, what happens to the magnitude of your maximum speed? Explain your reasoning.