What's My Rule?

Activity Overview

This activity encourages students to gain experience with the language of the Cartesian coordinate system. Each of the problems shows two points, Z and W. Point Z can be dragged, and point W moves in response. In describing the rule that governs the location of point W, students will most likely use language involving coordinates. Depending on the background of the students, these problems can also allow students to use the language of symmetry or to make connections with complex numbers.

Concepts

- *Cartesian coordinate system*
- *Symmetry*

Teacher Preparation and Notes

- *This activity is appropriate for Algebra 1 students. However, it can also be used with students in Geometry or Algebra 2/Precalculus. It is best used as an introductory activity, but can also be used as review.*
- *This activity is designed to have students explore* ***individually and in pairs****. However, an alternate approach would be to use the activity in a whole-class format.*
- *Before beginning the activity, introduce or review the necessary language or terminology you want students to use in their rule descriptions. At the Algebra 1 level, this is the language of the Cartesian plane: coordinates, origin, axes, quadrants. For higher levels, the language of symmetry and the algebra of complex numbers may also be used.*
- *You may wish to demonstrate dragging point Z in Problem 1 (page 1.2). Explain to students that point W is only movable by its (mystery) relationship to Z.*
- *Solutions for each of the different settings (Algebra 1, Geometry, and Algebra 2/ Precalculus) are provided in the following pages.*
- *The screenshots on pages 3-5 demonstrate expected student results.*

Associated Materials

- *WhatsMyRule.tns*

For Algebra 1 students, posed in terms of coordinates, the task is:

If the location of point *Z* is given by the coordinates (*a*, *b*), what are the coordinates of point *W*?

As a result of this activity, students should be able to:

- recognize that the *x*-coordinate represents the distance of a point from the *y*-axis, or the horizontal distance, and the *y*-coordinate represents the distance from the *x*-axis, or the height of the point
- identify a point and its reflection in each of the quadrants
- recognize that the points on a horizontal line have the same *y*-coordinate and that the points on a vertical line have the same x-coordinate

Examples of scaffolding inquiry questions that may be asked as students explore:

- In what quadrant is *W* if *Z* is in the first quadrant? second? third? fourth?
- What happens if *Z* is on the *x*-axis? *y*-axis? the origin?
- Is it possible to get *W* in the first quadrant? second? third? fourth? If so, where must *Z* be located in each case?
- Is it possible for *W* to be on the positive *x*-axis? negative *x*-axis? positive *y*-axis? negative *y*-axis? the origin? If so, where must *Z* be located in each case?
- Is it possible for *Z* and *W* to be in the same quadrant?
- Is it possible for *Z* and *W* to coincide?
- If you move *Z* horizontally to the right (or along some other described path and direction), how does *W* move?

For Geometry students, posed in terms of transformations, the task is:

Describe a geometric transformation (i.e., translation, rotation, reflection, dilation, projection), that can be used on point *Z* to obtain point *W*. Can you use more than one transformation to describe the relationship? Explain.

Posed in this way, you may wish to hide the displayed coordinates of the two points.

For Algebra 2/Precalculus students, posed in terms of complex numbers, the task is:

The complex number $Z = a + bi$ has coordinates (*a*, *b*) in the complex plane. Write the location of *W* as a function of the complex number *Z*.

Students may identify the correct rule stated in very different ways.

Compare and discuss!

Solutions

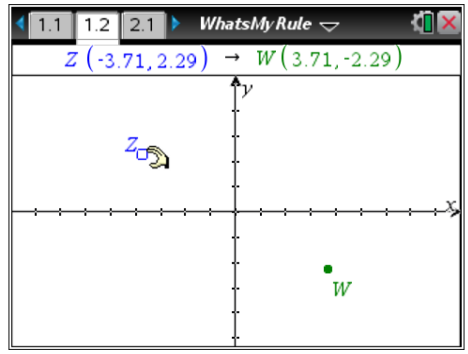
The solution to each problem is indicated in three ways – in terms of coordinates, geometric transformation, and complex numbers.

Problem 1

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are (–*a*, –*b*).

Geometric Transformation: Reflect *Z* through the origin to get the image *W* (equivalently, rotate 180° about the origin).

Complex Numbers: If $Z = a + bi$, then $W = -a - bi$. Equivalently, $W = -Z$.



TI-Nspire Navigator Opportunity: ***Class Capture, Quick Poll, and Live Presenter***

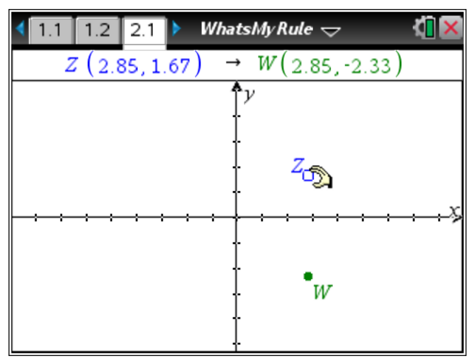
See Note 1 at the end of this lesson.

Problem 2

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are (*a*, *b* – 4).

Geometric Transformation: Translate *Z* four units down to get the image *W*.

Complex Numbers: If $Z = a + bi$, then $W = a + (b - 4)i$. Equivalently, $W = Z - 4i$.

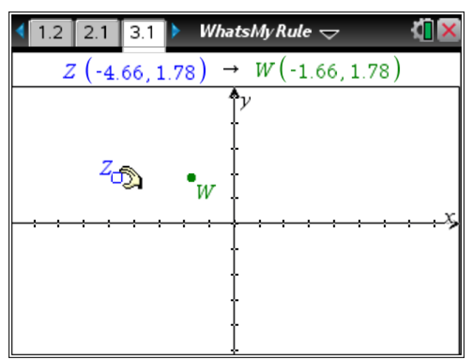


Problem 3

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are (*a* + 3, *b*).

Geometric Transformation: Translate *Z* three units to the right to get the image *W*.

Complex Numbers: If $Z = a + bi$, then $W = (a + 3) + bi$. Equivalently, $W = Z + 3$.

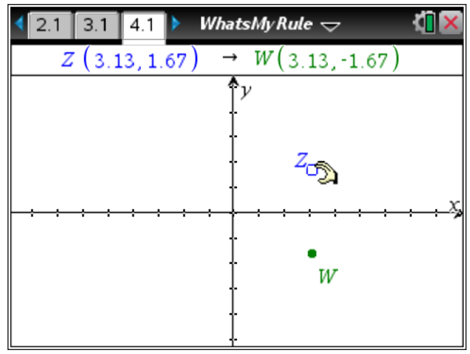


Problem 4

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are (*a*, –*b*).

Geometric Transformation: Reflect *Z* over the *x*-axis to get the image *W*.

Complex Numbers: If $Z = a + bi$, then $W = a - bi$. Equivalently, $W = \overline{Z}$ (its conjugate).

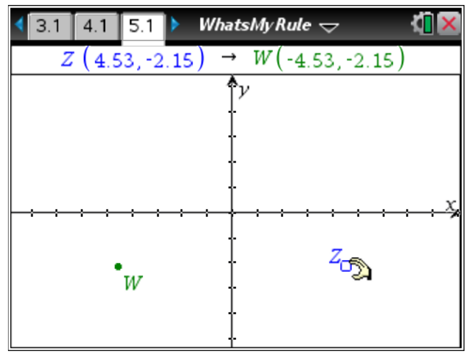


Problem 5

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are (–*a*, *b*).

Geometric Transformation: Reflect *Z* over the *y*-axis to get the image *W*.

Complex Numbers: If $Z = a + bi$, then $W = -a + bi$. Equivalently, $W = -\overline{Z}$.

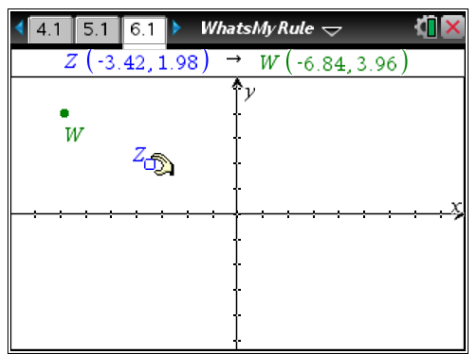


Problem 6

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are (2*a*, 2*b*).

Geometric Transformation: Dilate *Z* by a factor of 2 from the origin to get the image *W*.

Complex Numbers: If $Z = a + bi$, then $W = 2a + 2bi$. Equivalently, $W = 2Z$.

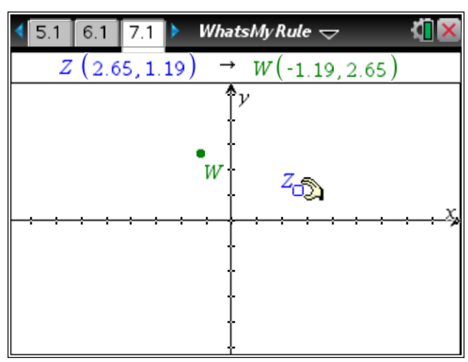


Problem 7

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are (–*b*, *a*).

Geometric Transformation: Rotate *Z* about the origin 90° counterclockwise to get the image *W*.

Complex Numbers: If $Z = a + bi$, then $W = -b + ai$. Equivalently, $W = iZ$.

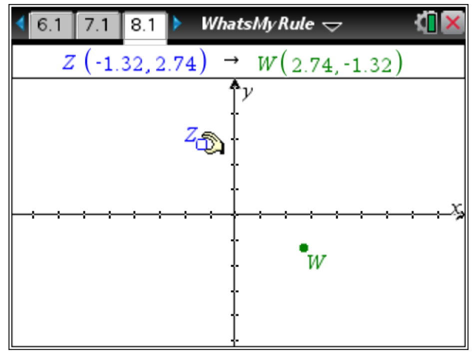


Problem 8

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are (*b*, *a*).

Geometric Transformation: Reflect *Z* across the line *y* =*x* to get the image *W*.

Complex Numbers: If *Z* = *a* + *bi*, then *W* = *b* + *ai*. Equivalently, $W = i\overline{Z}$.

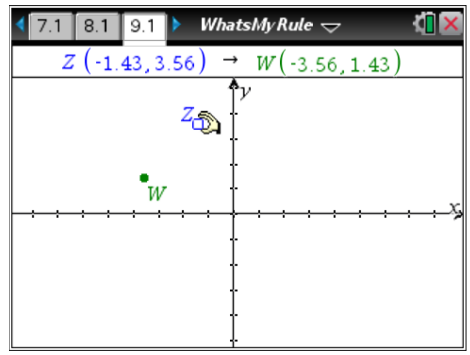


Problem 9

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are (–*b*, –*a*).

Geometric Transformation: Reflect *Z* across the line *y* =–*x* to get the image *W*.

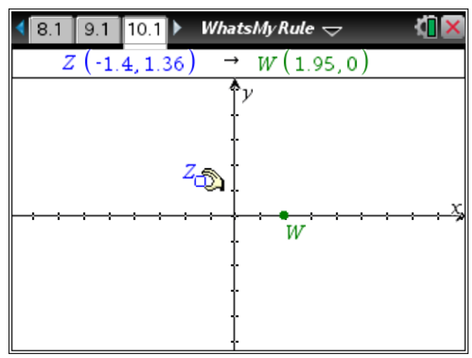
Complex Numbers: If *Z* = *a* + *bi*, then *W* = –*b* – *ai*. Equivalently, $W = -i\overline{Z}$.



Problem 10

If students are having difficulty with this problem, have them consider the type of triangle that is formed by points *Z* and *W* and the origin. If they can get a mental picture of the isosceles triangle and know how to find the distance between two points in a plane, they should be able to then determine the answer.

Coordinates: If the coordinates of *Z* are (*a*, *b*), then the coordinates of *W* are ($\sqrt{a^2+b^2}$, 0).



Geometric Transformation: Point *W* is the same distance from the origin along the positive *x*-axis as point *Z* is from the origin.

Complex Numbers: If *Z* = *a* + *bi*, then *W* = $\sqrt{a^2+b^2}$. Equivalently, *W* = $|Z|$.

TI-Nspire Navigator Opportunity

Note 1
Problem 1: *Class Capture, Quick Poll, and Live Presenter*
Throughout the lesson:

- Use Class Capture to let students compare locations and coordinates of points *Z* and *W*.
- Use Quick Poll to allow students to share and discuss their responses.
- Use Live Presenter to allow students to explain and defend their reasoning.

