EXPLORATIONS

Activity 3

Watching Your Weight

Objectives

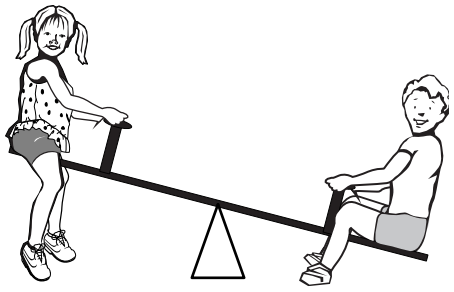
- To find the *y* value of a function, given the *x* value
- To use technology to find a best fit line
- To use technology to plot a set of ordered pairs

Materials

- TI-83 Plus
- Bathroom scale, kitchen scale, or small scale
- Block(s) equal in height to the height of the scale
- Wooden plank 2 – 4 cm thick by 25 – 30 cm wide and 120 – 140 cm long (or meter stick or ruler), one per group
- Textbooks that weigh at least 10 – 14 kilograms or 25 – 30 pounds (or a bathroom size paper cup and at least 50 pennies), one set of weights per group
- Meter stick or tape measure, one per group

Introduction

There is a toy that children used to play on called a seesaw. It is shown in the diagram to the right. Many playgrounds have removed them for safety reasons. The seesaw was a board that was hinged on a bar. When one child pushed off the ground and went up, the child on the opposite end went down. Children loved to go up and down on the seesaw. If the children were of unequal weight, it became a problem, since the heavier child would weigh down the lighter child. This problem could be solved if one of the two children moved closer to the center of the board.

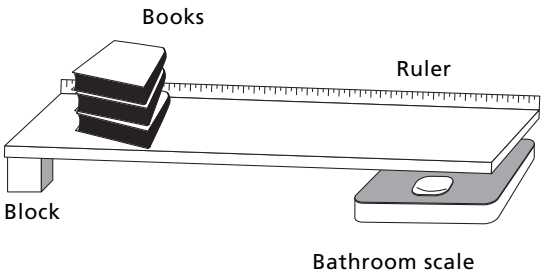


Problem

How does moving a weight along a board affect the downward force on the board? How might a heavier child at one end, or a lighter child at the other end, balance a seesaw when their weights are different?

Collecting the data

To solve this problem, set up a board on a scale and add some books as shown in the diagram to the right. The board represents a portion of the seesaw. The block represents the triangular fulcrum in the diagram of the seesaw shown previously. The books are similar to the child who is sliding back and forth, trying to balance the lighter child.



The block is used as the triangular fulcrum so that you can measure the total weight as the books slide up and down the seesaw.

1. Each group of students should obtain a scale, a wooden plank, and blocks from your teacher. Position the scale and the block(s) far enough apart so that the wooden plank is supported on one end by the scale and the other end by the block(s). Starting at the edge of the scale, place marks at 10-centimeter increments on the wooden plank. See the diagram above.

Note: *If you have a small scale that measures in grams, you will use this setup with a meter stick. If you have a kitchen scale that measures in grams, you will use this setup with a ruler. If your scale measures in pounds, use 1 pound = 0.4536 kilograms to convert to kilograms. If your scale measures in ounces, use 1 ounce = 28.35 grams to convert to grams.*

2. If you are using a wooden plank, obtain textbooks that weigh at least 10 – 14 kilograms (or 25 – 30 pounds) to use for the weight. (If you are using a small scale, use a paper cup with pennies or marbles for the weight.)

3. Place the books on the wooden plank (or cup of pennies on the meter stick or ruler) at the edge of the scale. Record the weight shown on the scale for 0 centimeters in the table on the **Data Collection and Analysis** page.

4. Move the books a distance of 10 centimeters away from the scale. (If you are using a meter stick, move the cup a distance of 3 centimeters away from the scale. If you are using a ruler, move the cup a distance of 1 centimeter away from the scale.) Record the weight shown on the scale in the table on the **Data Collection and Analysis** page.

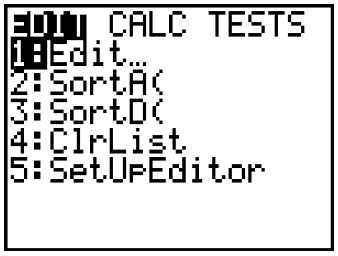
5. Move the books a distance of 20 centimeters from the scale. (If you are using a meter stick, move the cup a distance of 6 centimeters away from the scale. If you are using a ruler, move the cup a distance of 2 centimeters away from the scale.) Record the weight shown on the scale in the table on the **Data Collection and Analysis** page. Continue to move the books away from the scale in 10-centimeter increments and record the weights. (Continue to move the cup in either 3-centimeter or 1-centimeter increments away from the scale.)

Setting up the TI-83 Plus

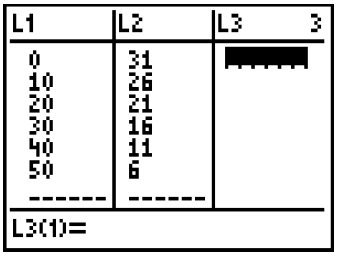
Before starting your data collection, make sure that the TI-83 Plus has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

Entering the data in the TI-83 Plus

1. Press [STAT] and select **1:Edit** by pressing [ENTER].

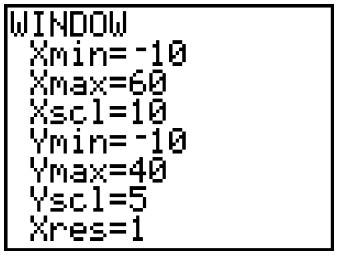


2. Enter the distance from the scale in **L1**.
3. Enter the weight shown on the scale in **L2**.



Setting up the window

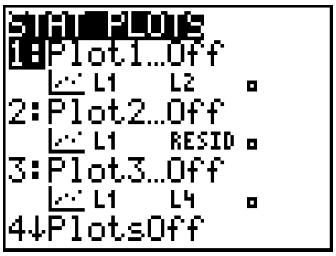
1. Press [WINDOW] to set up the proper scale for the axes.
2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.



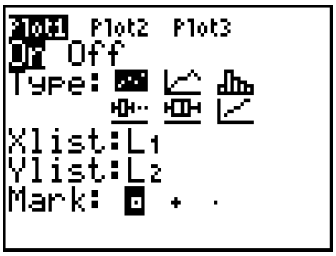
3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. Set the **Xscl** to **10**. (Set **Xscl** to **3** if you are using a small scale or **1** if you are using a kitchen scale.)
4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.
5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **5**. (Set **Yscl** to **0.25** if you are using a small scale or a kitchen scale.)

Graphing the data: Setting up a scatter plot

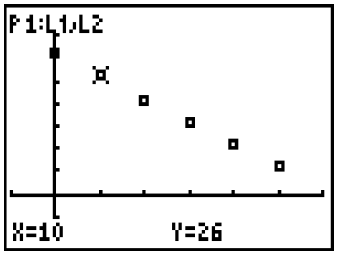
1. Press 2nd [STAT PLOT] and select **1:Plot1** by pressing ENTER.



2. Set up the plot as shown by pressing ENTER ▼ ENTER ▼ 2nd [L1] ENTER 2nd [L2] ENTER ENTER.

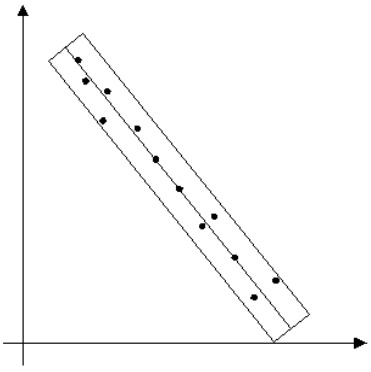


3. Press GRAPH TRACE to see the plot. Discuss the *x* and *y* values.



You can analyze and make predictions using the data that you collected. In order to make predictions, you must describe the data using a mathematical model. Data analysis is not an exact science, and several different methods may be used to find mathematical models. Your data may not fit any model exactly; however, the challenge is to search for a model that best fits the data. The data that you have collected should appear linear; therefore, you will find a *line of best fit* or a *trend line*. You will use two different methods to find a line of best fit or a trend line. The first method is visual and the second method uses the *least-squares line* feature of the TI-83 Plus.

In the first method, use a crude but natural method for finding a line of best fit or a trend line: visually estimate the trend line. Follow the guidelines below to find the trend line.



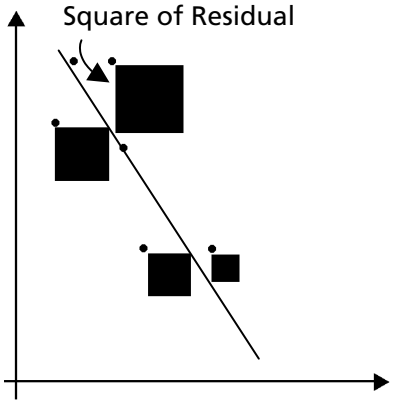
1. Find the smallest rectangle that contains all the points and shows the direction of the points.

2. Find a line that contains as many of the points as possible.

3. Find a line that divides the points equally above and below the line.

The points above or below the line should not be concentrated at one end.

In the second method, use the TI-83 Plus to find a line of best fit. The method of finding the line of best fit employed by the TI-83 Plus uses the formula that minimizes the sum of the squares of the residuals. A *residual* is the distance between the data point and the point on the line. Look at the diagram to the right that shows the squares of the residuals. The formula used by the TI-83 Plus finds the equation where the sum of the squares of the residuals is as small as possible.



Analyzing the data

Finding a trend line (method 1)

The data appears to be linear; therefore, you can determine an equation for a trend line for the data. The equation for a line is $Y = MX + B$, where M is the *slope* and B is the y-intercept.

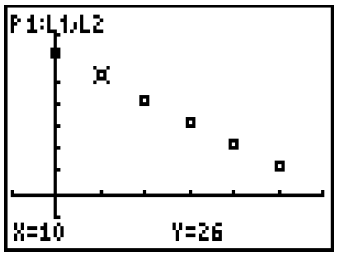
The *slope* of the line is defined as the rise, (change in y) divided by the run, (change in x).

$$\text{Slope} = m$$

$$m = \frac{\text{rise}}{\text{run}}$$

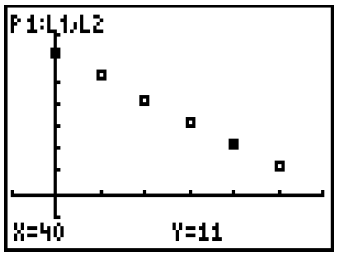
$$= \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

1. Find the *slope* of the line. Press [TRACE] [▶] to move the cursor to a point on the plot. Record the x and y values shown at the bottom of the screen.



X1 = _____ **Y1 =** _____

2. Press [▶] as many times as you need to find a second point on the plot. Record the x and y values shown at the bottom of the screen.



X2 = _____ **Y2 =** _____

Use the following steps to calculate the *slope* of the line and to store the *slope* to **M** in the TI-83 Plus.

3. Press [2nd] [QUIT] to return to the Home screen. Press [CLEAR] to clear the Home screen.

a. Press [(] and enter the value for **Y2**.

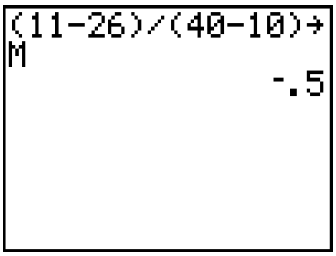
b. Press [−] and enter the value for **Y1**.

c. Press [)] [÷] [(] and enter the value for **X2**.

d. Press [−] and enter the value for **X1**.

4. Press [)] [STO▸] [ALPHA] [M].

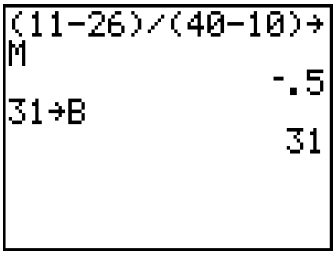
5. Press [ENTER] to calculate the *slope* and store the *slope* to **M** in the TI-83 Plus.



6. The *y*-intercept of a line is the point at which the line crosses the *y*-axis. The *y*-intercept of the trend line is the first value in **L2**. Record the *y*-intercept of your line.

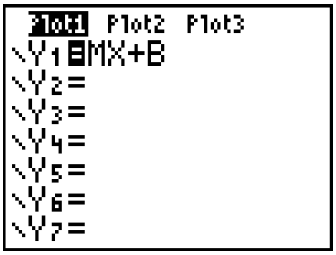
B = _____

7. Store the *y*-intercept to **B** in the TI-83 Plus. Press [STO▸] [ALPHA] [B] [ENTER] to store this value to **B**.



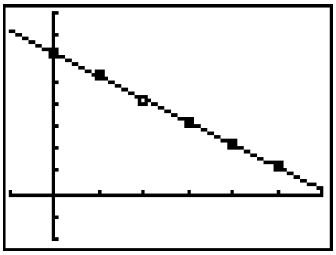
8. Enter the *slope*-intercept form of a linear equation in **Y1**.

9. Press [Y=] [ALPHA] [M] [X,T,Θ,n] [+] [ALPHA] [B] to place the equation $Y = MX + B$ in the Y= menu.



10. Press [GRAPH] to see the graph of the trend line. Record the equation on the line below.

Equation: _____

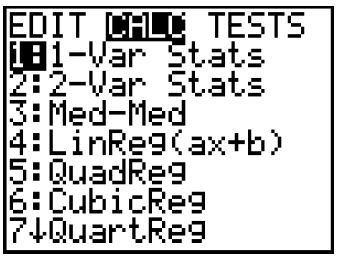


Use your equation to answer questions 1 through 6 on the **Data Collection and Analysis** page.

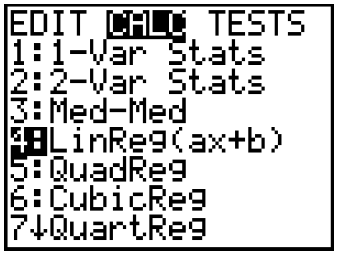
Finding a best fit line (method 2)

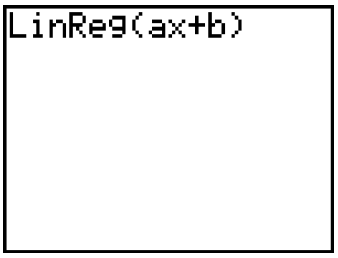
You can use the TI-83 Plus to find the true line of best fit for the data.

1. Find a linear regression equation for the data. Press [STAT] and move the cursor to the **CALC** menu.

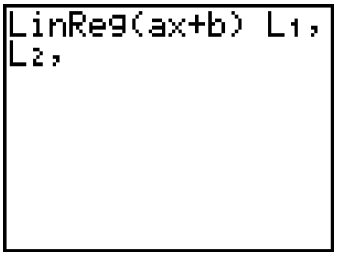


2. Select **4:LinReg(ax + b)** and press [ENTER].

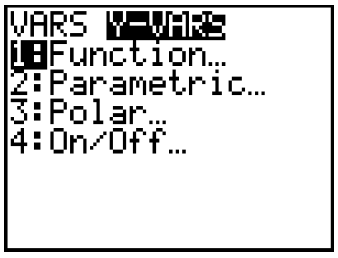




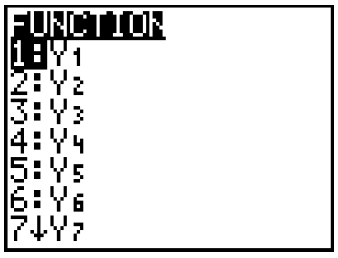
3. Enter **L1**, **L2**, and **Y2**. Press [2nd] [L1] [,] [2nd] [L2] [,].



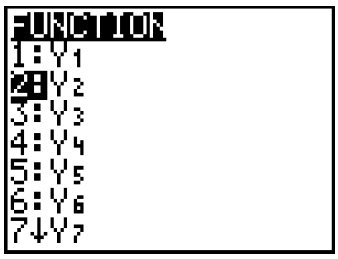
4. Press [VARS] and move the cursor to the **Y-VARS** menu.

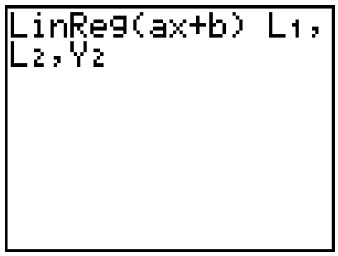


5. Select **1:Function** by pressing [ENTER].

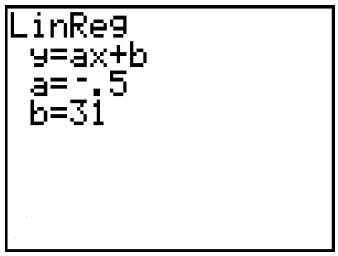


6. Select **2:Y2** and press ENTER.

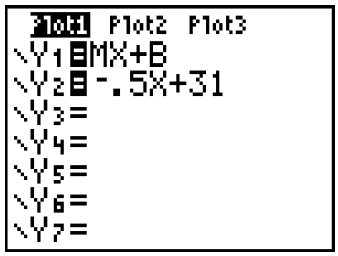




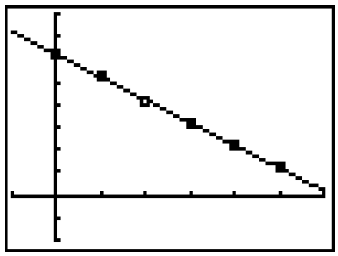
7. Press ENTER to calculate the equation for the best fit line. The function is pasted in **Y2**.



8. Press Y= to see the function.



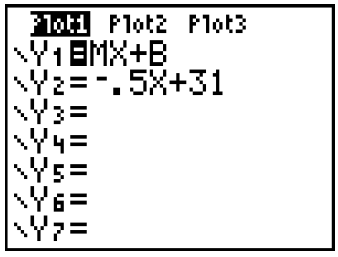
9. Press GRAPH to see the graph of the best fit line.



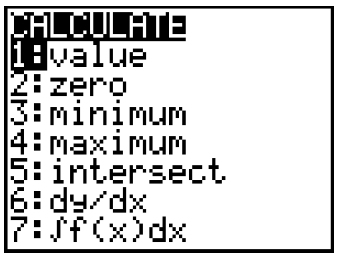
Answer questions 7 through 10 on the **Data Collection and Analysis** page.

Finding the weight at different distances

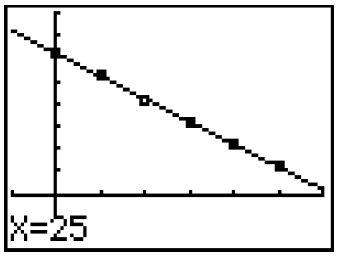
Note: *Turn OFF the equation in* **Y2**. *Press* Y= ▼ ◀ ENTER. *The equal sign should look like that for* **Y2** *as shown.*



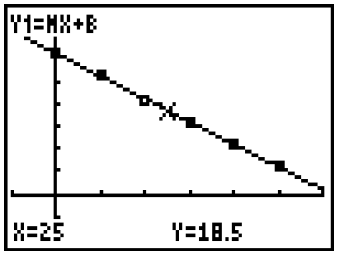
1. Find the weight of the objects at a distance of 25 centimeters from the scale. Press 2nd [CALC] and select **1:value** by pressing ENTER.



2. Enter **2 5** and press ENTER.

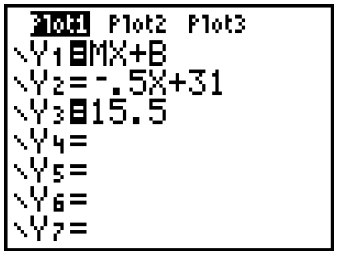


The *y* value is the desired weight.

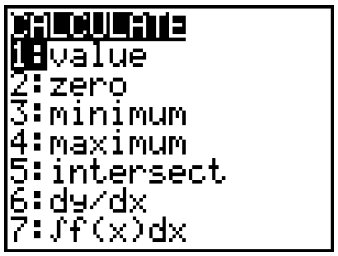


Finding the intersection

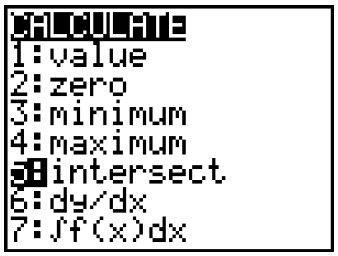
Note: *Make sure* **Y2** *remains OFF. Press* Y= *and then press* ▼ *until you are in the first position for* **Y3**. *Type one-half the weight of the books in* **Y3**. *In this example one-half the weight of the books is 15.5, so enter* **1 5** . **5**.



1. Press 2nd [CALC].

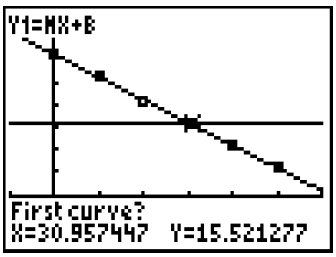


2. Select **5:intersect** and press ENTER.



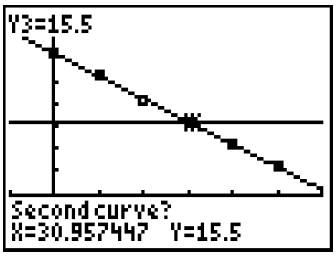
3. The calculator will prompt you for the *First curve*. Make sure the cursor is flashing on the regression line and then press ENTER.

Note: *Use ▲ and ▼ to select the function. Once you have selected the regression line press ENTER.*

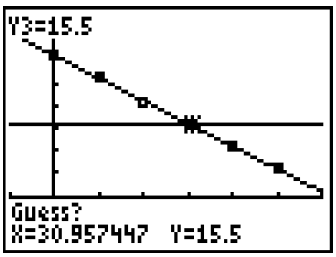


4. The calculator will prompt you for the *Second curve*. Make sure the cursor is flashing on the second line and then press ENTER.

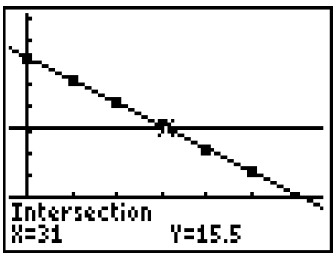
Note: *Use ▲ and ▼ to select the function. Once you have selected the second line press ENTER.*



5. The calculator will prompt you to *Guess*. Try to estimate the coordinates of the point of intersection.



6. Find the exact point of intersection by pressing ENTER. The *x* value is the desired distance.



Data Collection and Analysis

Name _____

Date _____

Activity 3: Watching Your Weight

Collecting the data

Record your data in the appropriate table below.

Distance from scale (in)	Weight shown on scale (lbs)
0	
10	
20	
30	
40	
50	
60	
70	
80	

Distance from scale (cm)	Weight shown on scale (kg)
0	
25.4	
50.8	
76.2	
101.6	
127	
152.4	
177.8	
203.2	

Analyzing the data

Use your equation from number 9 in the **Analyzing the data: Finding a trend line** section to answer questions 1 through 6.

1. The *slope* of the line is _____ .

2. Explain what the *slope* represents.

_____

_____

3. The *y*-intercept of the line is _____ .

4. Explain what the *y*-intercept represents.

_____

_____

5. Find the weight if the objects that you are using are placed at a distance of 25 centimeters from the scale. (If you used a small scale, find the weight of the objects at a distance of 13 centimeters from the scale.)

_____

6. At what distance are the objects if the weight shown on the scale is one-half the original weight of the objects? _____

Use the regression equation that you found in number 4 in the **Analyzing the data: Finding a best fit line** section to answer questions 7 through 10.

7. The *slope* of the regression line is _____ .

8. The *y*-intercept of the regression line is _____ .

9. Find the weight if the objects that you are using are placed at a distance of 25 centimeters from the scale. (If you used a small scale, find the weight of the objects at a distance of 13 centimeters from the scale.)

_____

10. At what distance are the objects if the weight shown on the scale is one-half the original weight of the objects?

_____

11. How do the values that you found in questions 1, 3, 5, and 6, using your model, compare with the values that you found in questions 7 through 10, using the regression model?

_____

_____

Teacher Notes

Activity 3

Watching Your Weight

Objectives

- To find the *y* value of a function, given the *x* value
- To use technology to find a best fit line
- To use technology to plot a set of ordered pairs

Materials

- TI-83 Plus
- Bathroom scale, kitchen scale, or small scale
- Block(s) equal in height to the height of the scale
- Wooden plank 2 – 4 cm thick by 25 – 30 cm wide and 120 – 140 cm long (or meter stick or ruler), one per group
- Textbooks that weigh at least 10 – 14 kilograms or 25 – 30 pounds (or a bathroom size paper cup and at least 50 pennies), one set of weights per group
- Meter stick or tape measure, one per group

Preparation

- The wooden plank can be obtained from a lumberyard or a home improvement store. You can also use a bookshelf.
- A bathroom scale works well. You can also get a small scale from the science department in your school or use a kitchen scale.
- Make sure that students place the wooden plank at the middle of the scale.
- If you are using a bathroom scale with textbooks, make sure that the books weigh at least 10 – 14 kilograms or 25 - 30 pounds.
- If you are using small scales, use a bathroom cup with at least 50 pennies. You can also use marbles, metal washers, or any other small object with weight.

Answers to Data Collection and Analysis questions

Collecting the data

- Sample data for a bathroom scale with textbooks.

Distance from scale (in)	Weight shown on scale (lbs)
0	26
10	23
20	21
30	17
40	14
50	12
60	9
70	7
80	5

Distance from scale (cm)	Weight shown on scale (kg)
0	11.7936
25.4	10.4328
50.8	9.5256
76.2	7.7112
101.6	6.3504
127	5.4432
152.4	4.0824
177.8	3.1752
203.2	2.268

- Sample data for a small scale with pennies.

Distance from scale (in)	Weight shown on scale (oz)
0	4.25
3	4.00
6	3.50
9	3.00
12	2.75
15	2.25

Distance from scale (cm)	Weight shown on scale (g)
0	120
8	115
16	100
24	90
32	80
60	70

Analyzing the data

Use your equation from number 9 in the **Analyzing the data: Finding a trend line** section to answer questions 1 through 6.

1. The *slope* of the line is _____ .

Answers may vary.
For the sample data in Table 1 the slope is approximately -0.12096.
For the sample data in Table 2 the slope is approximately -1.666666....

2. Explain what the *slope* represents.

The slope represents the decrease in the number of pounds per inch increase in distance from the scale.

3. The *y*-intercept of the line is _____ .

The y-intercept for the sample data in Table 1 is approximately 11.7936.
The y-intercept for the sample data in Table 2 is approximately 115.

4. Explain what the *y*-intercept represents.

The y-intercept represents the weight at which the distance from the scale is zero.

5. Find the weight if the objects that you are using are placed at a distance of 25 centimeters from the scale. (If you used a small scale, find the weight of the objects at a distance of 10 centimeters from the scale.)

The weight is 8.7696 kilograms for the data in Table 1.
The weight is 93.333… grams for the data in Table 2.

6. At what distance are the objects if the weight shown on the scale is one-half the original weight of the objects?

The distance is 48.75 centimeters for the data in Table 1.
The distance is 34.5 centimeters for the data in Table 2.

Use the regression equation that you found in number 4 in the **Analyzing the data: Finding a best fit line** section to answer questions 7 through 10.

7. The *slope* of the regression line is _____ .

For the sample data in Table 1 the slope is approximately -.121716.
For the sample data in Table 2 the slope is approximately -1.666666....

8. The *y*-intercept of the regression line is _____ .

The y-intercept for the sample data in Table 1 is approximately 11.62224.
The y-intercept for the sample data in Table 2 is approximately 115.

9. Find the weight if the objects that you are using are placed at a distance of 25 centimeters from the scale. (If you used a small scale, find the weight of the objects at a distance of 13 centimeters from the scale.)

The weight is 8.57934 kilograms for the data in Table 1.
The weight is 93.3333… grams for the data in Table 2.

10. At what distance are the objects if the weight shown on the scale is one-half the original weight of the objects?

The distance is 47.039337 centimeters for the data in Table 1.
The distance is 34.5 centimeters for the data in Table 2.

11. How do the values that you found in questions 1, 3, 5, and 6, using your model, compare with the values that you found in questions 7 through 10, using the regression model?

The values should be close.