Activity 15

Objectives

- To investigate the relationships that exist in similar triangles
- To investigate and explain the various ratios that exist in similar triangles

Cabri® Jr. Tools

Similar Triangles and Proportions

Introduction

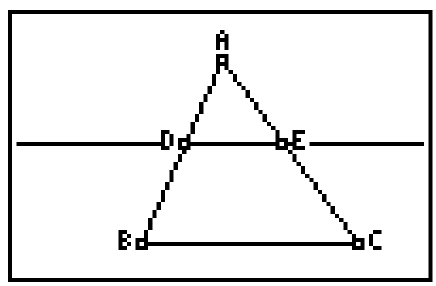
In this activity, you will investigate the relationships among similar triangles formed when a segment is constructed parallel to one side of a triangle. You can do this exploration in conjunction with Activity 16, Midsegments of a Triangle.

Construction

Construct a triangle and a line intersecting the triangle that is parallel to one of its sides.

Draw $\triangle ABC$ near the center of the screen.

Construct $\overleftrightarrow{DE}$ parallel to $\overline{BC}$ that intersects $\overline{AB}$ at point D and $\overline{AC}$ at point E.



Exploration

Measure the lengths of sides $\overline{AD}$, $\overline{AB}$, $\overline{AE}$ and $\overline{AC}$. Drag the vertices of $\triangle ABC$ and observe any relationships among the measured quantities.

Calculate and label the ratios $\dfrac{AB}{AD}$ and $\dfrac{AC}{AE}$. Drag the vertices again and observe any relationships among the calculated ratios.

Calculate the ratio $\dfrac{AD}{DB}$. Investigate other ratios that are equal to this ratio. Be sure to test your conjecture by dragging the vertices of $\triangle ABC$.

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Questions and Conjectures

1. Make a conjecture about the ratios $\frac{AB}{AD}$ and $\frac{AC}{AE}$. Do you think your conjecture holds true for any triangle? Explain your reasoning.
2. Are there any other ratios that are equal to $\frac{AB}{AD}$? If so, what are they?
3. Make a conjecture about the relationship between $\triangle ADE$ and $\triangle ABC$. Support your conjecture.
4. Are there any other ratios that are equal to $\frac{AD}{DB}$? If so, what are they?

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Teacher Notes

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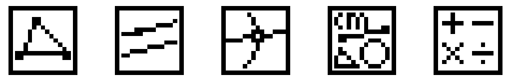


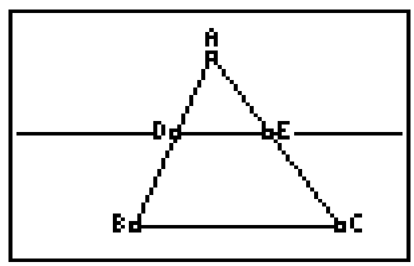
Additional Information

Because of the accuracy issue (see A Specific Cabri Jr. Issue on page vi), when measuring segment length using the Cabri Jr. application, you can obtain the best results by looking at ratios that are greater than one.

Answers to Questions and Conjectures

1. Make a conjecture about the ratios $\frac{AB}{AD}$ and $\frac{AC}{AE}$. Do you think your conjecture holds true for any triangle? Explain your reasoning.

   The ratios are equal and hold true for any triangle. Make sure students understand that the ratios hold for any triangle.



2. Are there any other ratios that are equal to $\frac{AB}{AD}$? If so, what are they?

   Another ratio equal to $\frac{AB}{AD}$ is $\frac{BC}{DE}$.

3. Make a conjecture about the relationship between ΔADE and ΔABC. Support your conjecture.

   ΔADE is similar to ΔABC. Parallel lines cut by a transversal form congruent corresponding angles, so $\angle ADE \cong \angle ABC$ and $\angle AED \cong \angle ACB$. The triangles are similar by AA similarity.

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4. Are there any other ratios that are equal to $\frac{AD}{DB}$? If so, what are they?

$\frac{AD}{DB} = \frac{AE}{EC}$. Here is an algebraic proof verifying this relationship:
If $\frac{AB}{AD} = \frac{AC}{AE}$ then $AD \times AC = AB \times AE$. But, since $AD + DB$ = AB and $AE + EC = AC$, by substitution, $AD \times (AE + EC) = AE \times (AD + DB)$. By expanding and using the distributive property, $AD \times AE + AD \times EC = AE \times AD + AE \times DB$. Since the term $AD \times AE$ is common on both sides of the equation, subtract it away to get $AD \times EC = AE \times BD$. Dividing both sides of the equation by DB and EC gives the desired result, $\frac{AD}{DB} = \frac{AE}{EC}$.

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