Going Back To Your Roots

PrecalcWeek16_BackToRoots.tns

Name ________________________

Class ________________________

Problem 1 - The Fundamental Theorem of Algebra

Every polynomial equation of degree greater than 1, with complex coefficients has at least one complex root.

Consider the polynomial $f(x) = x^3 + x^2$.

- Factor this polynomial so that the degree of each factor is 1.

On page 1.4, graph the function and determine the zeros.

- Which zero matches which factor?
- Read page 1.5. What do you notice about the multiplicities of the roots and the degree of the polynomial?
- Rewrite the roots as complex numbers.

Check For Understanding

1. How many complex roots does $f(x) = x^5 – 7x^2 + 3x +2$ have?
2. What is the multiplicity of the root $x = 3$ in $f(x) = (x – 1)^4(x – 3)^2(x + 3)$?
3. What is the total number of complex roots of $f(x) = (x – 1)^4(x – 3)^2(x + 3)$?

Problem 2- Beyond Real

Graph the polynomial $f(x) = x^2 + 9$ on page 2.2.

- How many complex roots does the polynomial have?
- What type(s) of roots does the polynomial have?
- How did the graph enable you to determine the root type?

Use the **cZeros** command on page 2.6 to determine the values of the roots.

Problem 3 – The Mixed Case

View the graph of $f(x) = x^3 + x^2 + 4x + 4$ on page 3.1

- How many complex roots or zeros does the polynomial have?
- How many of the roots or zeros are real?
- What are the real zeros?
- Rewrite it as a complex number.
- Use pages 1.4 to 1.6 to determine the complete factorization of the polynomial.
- Identify all complex roots of the polynomial.

Extension – Even and Odd Multiplicity

- What is meant by the term *multiplicity*?

Observe the graphs on the pages 4.2 and 4.3. Then change the value of ***a*** on page 4.4.

- Describe how the graph at the x-value of the roots differs for roots with even multiplicities and for roots with odd multiplicities.

Application & Practice

The work for the first polynomial has been started. Find the two remaining factors, roots, and multiplicities for this polynomial. Complete the chart for the remaining polynomials.

Polynomial	Factor(s)	Roots	Multiplicity
$f(x) = x^4 - 9x^3 + 27x^2 - 31x + 12$	$x - 4$	4	1
$f(x) = x^3 - 7x^2 + 11x - 5$			
$f(x) = x^5 + 9x^4 + 31x^3 + 63x^2 + 108x + 108$			