



Activity 8

Testing Hypotheses (Means, Proportions, and Standard Deviations)

This activity starts with testing a hypothesis about a population mean with σ known (Topic 40).

 It is important that you read Topic 40 because it explains the the STAT <TESTS> functions for testing hypotheses as well as the general format that will be used with the other topics in this part.

 The topic number of the related estimation topic is given in parenthesis after the hypothesis topic number. For example, (33) after Topic 40 below indicates that Topic 33 is the related estimation topic for Topic 40.

Topic 40 (33)—Testing Hypothesis of a Population Mean μ (σ Known)

A random sample of size 10 from a population of heights that has a normal distribution (with $\sigma = 2.5$ inches) is given below (with the sample mean). The sample was taken to test the claim that the population has a mean height of 67.25 inches.

Store this data in L_1 .

$$\{66.71, 66.27, 62.81, 66.92, 62.91, 71.42, 67.39, 63.79, 65.81, 62.81\} = L_1$$

$$\bar{x} = 65.68 \quad n = 10$$

Test: $H_0: \mu = \mu_0 = 67.25$ $H_a: \mu \neq \mu_0 = 67.25$

There are two possible methods of input, depending on how the data is presented to you in a problem. Sometimes, all the data is given, and it can be stored in a list. Sometimes, only the summary stats are given, such as \bar{x} and n , and you must work with these. This activity shows you how to work with both.

Activity 8, Testing Hypotheses (Means, Proportions, and Standard Deviations) (cont.)

1. Press **[STAT]** **<TESTS>** **1:Z-Test** for one of the two possible screens (see screen 1 or 3), depending on what input (**Inpt:**) is highlighted.
2. Enter the correct values as in screen 1 or 3, highlight **Calculate** or **Draw** in the last row, and then press **[ENTER]**.

If you selected **Calculate**, output screen 2 or 4 is displayed; for **Draw**, a screen like screen 5 is displayed.

With a p-value of about **0.047**, there is good evidence that the null hypothesis should be rejected. Because **z** is negative (**-1.98**), we conclude that the mean of the population is significantly less than 67.25 inches.

The **z** above was calculated as follows.

$$z = (\bar{x} - \mu_0) / (\sigma / \sqrt{n}) = (65.68 - 67.25) / (2.5 / \sqrt{10}) = -1.9859$$

The Meaning of p-value and One- and Two-Tail Test

The above was a two-tail test ($\mu \neq 67.25$). Our sample mean could have been either in the right or the left tail of the distribution of sample means, as shown in screen 5.

If $H_a: \mu < \mu_0 = 67.25$ for the **Stats** input of the **Z-Test**, then the **Draw** output is given in screen 6. Notice that we only consider the left tail. We would never be able to reject the null hypothesis and accept the alternative hypothesis if our sample mean was greater than the hypothesized mean. The p-value for the one-tail case is half of that for the two-tail case.

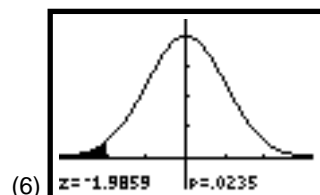
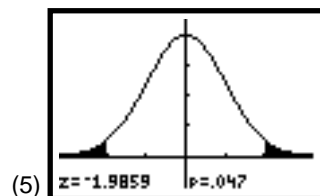
(1)

(2)

(3)

(4)

Note: The slight differences shown in screens 2 and 4 are because the sample mean calculated from the data list had three decimal places ($\bar{x} = 65.684$) while the stats version was rounded to $\bar{x} = 65.68$.



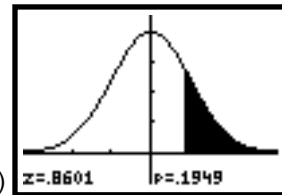
When $H_0: \mu = \mu_0 = 65$ $H_a: \mu > \mu_0 = 65$ and the input of the **Z-Test** is set up as in the screen 7, with the **Draw** output given in screen 8, notice that the sample mean (65.68) is much closer to $\mu_0 = 65$ ($65.68 - 65 = +0.68$ than in the previous case ($65.68 - 67.25 = -1.57$). Thus, we would not reject $\mu_0 = 65$ because it is not unusual to have a sample mean of 65.68 when $\mu_0 = 65$. In fact, this could happen about 19.5 percent of the time. (See the p-value in screen 8.)

The plot resulting from screens 9-12 combines the plots in screens 6 and 8 to make the contrast even clearer. (See screen 12.) We are much more likely to reject the notion that our sample mean (65.68) belongs to the distribution with mean 67.25 than one with a mean of 65.

Note that the sample for this topic (stored in L1) was randomly generated from a normal distribution with a mean of 65 in Topic 33.

```
Z-Test
Inpt:Data  Stats
μ₀:65
σ:2.5
x̄:65.68
n:10
μ:≠μ₀ <μ₀ >μ₀
Calculate Draw
```

(7)



(8)

```
Plot1 Plot2 Plot3
\Y1=normalpdf(X,
65,2.5/√(10))
\Y2=normalpdf(X,
67.25,2.5/√(10))
\Y3=Y1*(Y1≥Y2)+Y
2*(Y2>Y1)
\Y4=
```

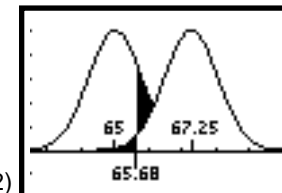
(9)

```
WINDOW
Xmin=62.5
Xmax=70
Xscl=1
Ymin=-.14
Ymax=.56
Yscl=.1
Xres=1
```

(10)

```
Shade(0,Y2,65,65
.68)
Shade(Y2,Y3,65.6
8)
```

(11)



(12)

Activity 8, Testing Hypotheses (Means, Proportions, and Standard Deviations) (cont.)

Topic 41 (34)— Testing Hypothesis of a Population Mean μ (σ Unknown)

A random sample of size 10 from a population of heights that has a normal distribution is given below (with the sample mean and the standard deviation).

Store this data in L_1 .

$$\{66.71, 66.27, 62.81, 66.92, 62.91, 71.42, 67.39, 63.79, 65.81, 62.81\} = L_1$$

$$\bar{x} = 65.68, Sx = 2.72, n = 10$$

Test: $H_0: \mu = \mu_0 = 67.25$ $H_a: \mu \neq \mu_0 = 67.25$

1. Press **[STAT]** **<TESTS>** **2:T-Test** for one of the two possible screens (see screens 13 and 15), depending on what input (**Inpt:**) is highlighted.
2. Enter the correct values as in screen 13 or 15, highlight **Calculate** or **Draw** in the last row, and then press **[ENTER]**.

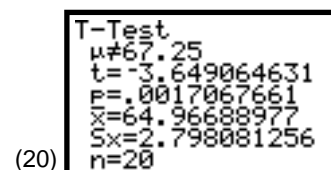
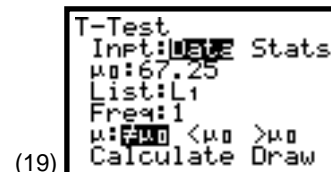
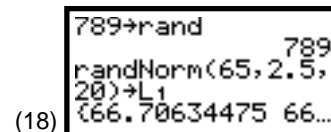
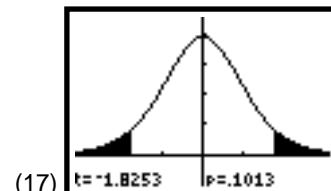
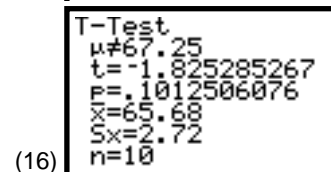
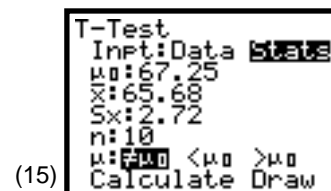
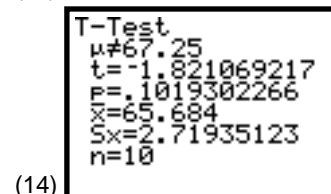
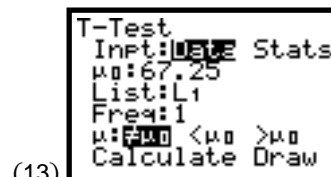
If you selected **Calculate**, output screen 14 or 16 is displayed; for **Draw**, a screen like screen 17 is displayed.

With a p-value of approximately 0.10 and a negative t statistic, there is some evidence that the population mean is less than 67.25. However, at a significance level of $\alpha = 0.05$, we would not reject the null hypothesis. There is insufficient evidence to reject the null hypothesis even if, in fact, it is not equal to 67.25.

The t above was calculated as follows.

$$t = (\bar{x} - \mu_0)/(Sx/\sqrt{n}) = (65.68 - 67.25)/(2.72/\sqrt{10}) = -1.825.$$

Since our sample was randomly generated from a normal population with a mean of 65 and a standard deviation of 2.5 (see Topic 33), we should be able to increase our evidence to reject the null hypothesis that the mean is 67.25 by generating a larger sample (of size 20), as shown in screens 18, 19, and 20, with a p-value of 0.0017.



Topic 42 (35)—Testing Hypothesis of a Population Proportion

When a public policy was introduced several years ago, 67 percent of the people voted for it. Test the claim that a larger percentage of voters would be in favor of the policy today. A random sample of 265 voters has 73.2 percent agreeing with the policy.

Test: $H_0: p = p_0 = 0.67$ $H_a: p > p_0 = 0.67$

Because $n * p = 265 * 0.732 = 194 > 10$ and $n * (1 - p) = 265 - 194 = 71 > 10$, we can use a normal distribution to approximate the binomial.

1. Press **[STAT]** **<TESTS>** **5: 1-PropZTest** for screen 21.
2. Enter the correct values as in screen 21, highlight **Calculate** or **Draw** in the last row, and then press **[ENTER]**.

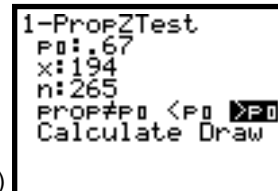
If you selected **Calculate**, output screen 22 is displayed; for **Draw**, screen 23 is displayed.

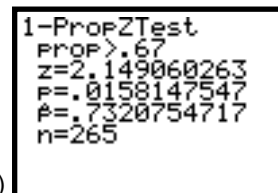
A p-value as small as **0.0158** is strong evidence that the new proportion is probably larger than 67 percent, so we reject the null hypothesis.

The **z** above was calculated as follows.

$$z = (0.732 - 0.67) / \sqrt{(0.67 * (1 - .67) / 265)} = 2.15.$$

Note that in Topic 35 (screen 35), this sample was randomly generated from a population with $p = 0.70$ using **randBin(265, 0.7)/265 = 0.732**.

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(22) 



Activity 8, Testing Hypotheses (Means, Proportions, and Standard Deviations) (cont.)

Topic 43 (36)—Testing Hypothesis of a Normal Population Standard Deviation σ

A battery recharger gives a charge with a mean of 50 hours and a standard deviation of 10 hours. An adjustment reduced the variability of the charge. To test it, the following random sample was taken.

$$\{54.54, 43.44, 54.11, 46.88\} = L_1$$

Test: $H_0: \sigma = 10$ $H_a: \sigma < 10$

The length of the charge is normally distributed, so the sampling distribution of $(n - 1)s^2/\sigma^2$ is chi-square distributed. Note that if s^2 is close to σ^2 , these values will be close to the degrees of freedom $n - 1$ ($4 - 1 = 3$ in this case).

Using 2nd [LIST] <MATH> 7:stdDev(L1 ENTER), or STAT <CALC>1: 1-Var Stats L1 ENTER gives $Sx = 5.48$.

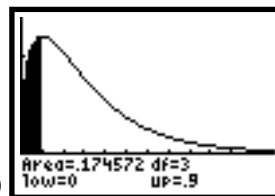
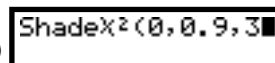
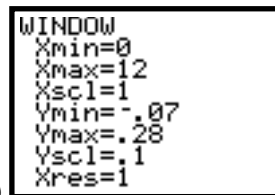
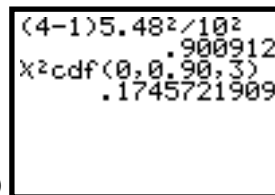
$$\chi^2 = (4 - 1) * 5.48^2/10^2 = 0.90$$

Press 2nd [DISTR] 7: χ^2 cdf(0 , 0 . 90 , 3) ENTER to get the left-tail probability of the χ^2 distribution with three degrees of freedom, or the p-value of **0.17457**, as shown in screen 24.

We do not reject the null hypothesis. There is insufficient evidence from our sample of size 4 to conclude that the variability has, in fact, decreased.

Another method of finding the p value is with 2nd [DISTR] <DRAW> 3:Shade χ^2 , as shown in screen 26, with results in screen 27 (with the WINDOW setup as in screen 25).

The above sample was, in fact, randomly generated from a normal distribution with mean 50 and standard deviation 10 (see Topic 39). Therefore, even with larger samples, it is highly possible that we would not be able to reject the null hypothesis.



Topic 44 (37)—Testing Hypothesis of Two Population Means

Testing Independent Samples

Test the claim that teaching Method A results in higher test scores than Method B based on the following scores from random samples of students taught with the two methods.

| | | | | | | mean | Sx | n |
|---------------|----|----|----|----|----|------|-------|---|
| Method A (L1) | 40 | 38 | 39 | 32 | 35 | 38 | 2.966 | 6 |
| Method B (L2) | 34 | 36 | 31 | 37 | 29 | 33.4 | 3.362 | 5 |

Store Method A data in L1 and Method B data in L2, and test:

$H_0: \mu_1 = \mu_2$, or $(\mu_1 - \mu_2 = 0)$ $H_a: \mu_1 > \mu_2$ or $(\mu_1 - \mu_2 > 0)$

Two Sample T-Test (σ_1 and σ_2 Unknown)

Assume the populations of test scores are normally distributed.


1. Press **[STAT]** <TESTS> 4:2-SampTTest for screens 28, 30, or 32.

Screens 28 and 30 do not pool the sample variances, but screen 32 does. (See “To Pool or Not To Pool” in Activity 7, Topic 37.) Screen 28 uses the **Data List**. However, for screens 30 and 32, you must enter the summary **Stats** of means, standard deviations, and sample sizes.

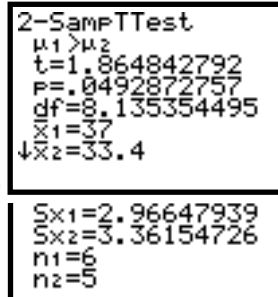
2. Enter the correct values as shown in the screens, highlight **Calculate** or **Draw** and press **[ENTER]**.

In the examples shown, screen 29 is the **Calculate** output for **Data List** input of screen 28. Screen 31 is the **Draw** output from the **Stats** input of screen 30. (The **Calculate** output from screen 30 would be very close to the output in screen 29). Screen 33 is the **Calculate** output of the pooled **Stats** input of screen 32.)

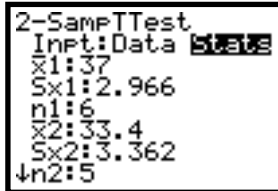
With a p-value of less than 0.05, there is sufficient evidence to reject the null hypothesis and conclude that Method A has significantly higher mean scores than Method B.

(28) 

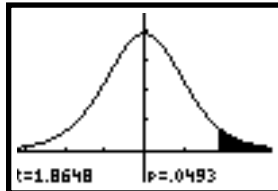
28↓ **Calculate Draw**

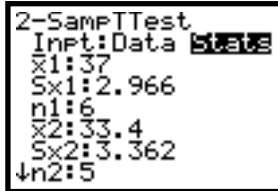
(29) 

29↓ **Calculate Draw**

(30) 

30↓ **Calculate Draw**

(31) 

(32) 

32↓ **Calculate Draw**

Note: The **Calculate** output (screen 33) is at the top of the next page.

Activity 8, Testing Hypotheses (Means, Proportions, and Standard Deviations) (cont.)

Two-Sample Z-Test (σ_1 and σ_2 Known)

Assume the above data is from normal populations but that $\sigma_1 = 2.966$ and $\sigma_2 = 3.362$. (Some texts use this procedure for large samples when σ_1 and σ_2 can be approximated by Sx_1 and Sx_2 .)

1. Press **[STAT]** **<TESTS>** **3:2-SampZTest** for screen 34. The **Data Lists** input is also possible, but we show only the **Stats** version.
2. Make the entries as shown in screen 34, highlight **Draw**, and then press **[ENTER]** for screen 35.

The z statistic (**1.8648**) in screen 35 is the same as the t statistic in screen 29, but the p value is different because of the difference in a z and a t distribution .

We draw the same conclusion as above because a p value of **0.0311** is stronger evidence to reject the null hypothesis.

When the variance is not pooled, the calculation of the z statistic and the t statistic is the same with $Sx_1 = \sigma_1$ and $Sx_2 = \sigma_2$.

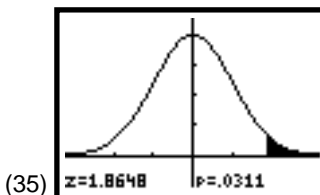
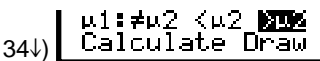
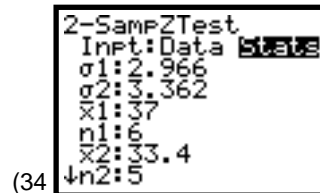
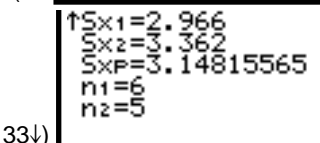
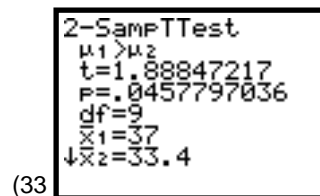
$$(37 - 33.4)/\sqrt{(2.966^2/6 + 3.362^2/5)} = 1.8648.$$

The degrees of freedom for the t case are the same as that calculated in Topic 37, where the same data was used (**df = 8.135**).

When the variance is pooled, $Sxp = 3.148$ (the pooled standard deviation) was calculated in Topic 37 and stored as **S**.

$$t = (37 - 33.4)/(3.148 * \sqrt{(1/6 + 1/5)}) = 1.89.$$

$$\text{Degrees of freedom} = n_1 + n_2 - 2 = 6 + 5 - 2 = 9$$



Testing Dependent Samples

To test the claim that a blood pressure medication reduces the diastolic blood pressure, a random sample of 10 people with high blood pressure had their pressures recorded. After a few weeks on the medication, their pressures were recorded again. See the data in the table below. The mean and the standard deviation of the differences (stored in L_3) were calculated with **1-Var Stats** as shown in screens 36 and 37.

| Subject | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-------------------|----|----|-----|----|-----|----|-----|----|----|----|
| Before (L_1): | 94 | 87 | 105 | 92 | 102 | 85 | 110 | 95 | 92 | 93 |
| After (L_2): | 87 | 88 | 93 | 87 | 92 | 88 | 96 | 87 | 92 | 86 |
| $L_1 - L_2 = L_3$ | 7 | -1 | 12 | 5 | 10 | -3 | 14 | 8 | 0 | 7 |

Assuming the diastolic blood pressure readings are normally distributed, use the following procedure to test the claim with $H_0: \mu_d = 0$ $H_a: \mu_d > 0$.


Note that the mean of the differences is **5.9**, and the standard deviation of the differences is **5.6657** as in screen 37.

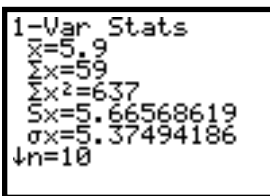
1. Press $\boxed{\text{STAT}}$ \langle TESTS \rangle **2:TTest** and proceed with the differences in L_3 , just as you did for one sample in Topic 41, to get one of the two possible screens (see screen 38 or 40), depending on what input (**Inpt:**) is highlighted.
2. Enter the correct values as shown in screen 38 or 40, highlight **Calculate** or **Draw**, and then press $\boxed{\text{ENTER}}$.

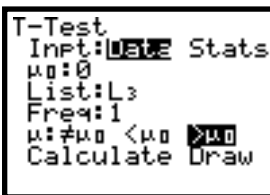
Screen 39 is the **Calculate** output of screen 38. Screen 41 is the **Draw** output of screen 40.

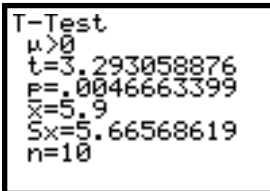
With such a small **p** value of 0.0047 (off the scale to the right in the plot in screen 41), we reject the null hypothesis. Because the mean difference (and t statistic) is positive and the differences were obtained by subtracting the *after* from the *before*, on an average, the *before* must have been larger. We conclude that there is a significant reduction in blood pressure.

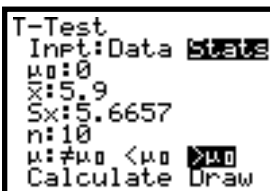
Home screen calculations are similar to those described in Topic 41.

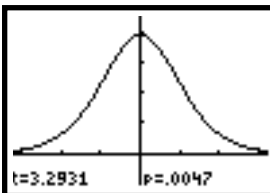
(36) 

(37) 

(38) 

(39) 

(40) 

(41) 

Activity 8, Testing Hypotheses (Means, Proportions, and Standard Deviations) (cont.)

Topic 45 (38)—Testing Hypothesis of Two Population Proportions

Test whether there is a difference in the proportion of men and women in a population who agree with a certain public policy. Base your conclusion on the simple random sample below where x is the number out of the sample of size n that agree.

Women: $x_1 = 694$ $n_1 = 936$
Men: $x_2 = 645$ $n_2 = 941$

Test $H_0: p_1 = p_2$, or $(p_1 - p_2 = 0)$ $H_a: p_1 \neq p_2$ or $(p_1 - p_2 \neq 0)$

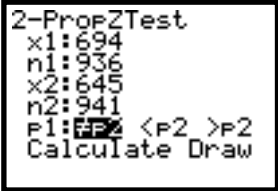
1. Press **[STAT]** **<TESTS>** **6:2-PropZTest** for screen 42.
2. Enter the correct values as in screen 42, highlight **Calculate** in the last row, and then press **[ENTER]** for output screens 43.

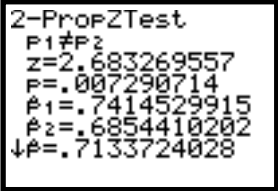
With a p-value of **0.0073**, there is very strong evidence that a larger percentage of women than men agree with the public policy (note that the sample proportions are 74.1 to 68.5 percent).

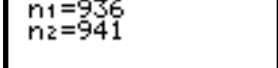
The pooled proportion is $(694 + 645)/(936 + 941) = 0.713$.

$$z = (0.741 - 0.685) / \sqrt{(0.713 * (1 - 0.713) * (1/936 + 1/941))} = 2.68.$$

Note: $n_1 = 936 = 694 + 242$ (both values greater than 5). $n_2 = 941 = 645 + 296$ (both values greater than 5). Therefore, the normal approximation to the binomial can be used.

(42) 

(43) 

43↓ 

Topic 46—Testing Hypotheses of Standard Deviations of Two Normal Populations

To test if there is any difference in the variability of the diastolic blood pressure of men or women, a random sample of each was taken and recorded below.

Male (LMBP): 76 74 70 80 68 90 70 72 76 68 96

Female (LFBP): 70 82 90 68 60 62 80 74 62

| | mean | Sx | n |
|--------|-------|-------|----|
| Male | 76.36 | 9.11 | 11 |
| Female | 72 | 10.34 | 9 |

Assuming that the populations of diastolic blood pressure are normally distributed, test:

H₀: $\sigma_1 = \sigma_2$ or $(\sigma_1^2/\sigma_2^2 = 1)$ H_a: $\sigma_1 \neq \sigma_2$ or $(\sigma_1^2/\sigma_2^2 \neq 1)$


1. Store the male data stored in list **LMBP** and the female data in list **LFBP**.
2. Press **[STAT]** <TESTS> **D:2-SampFTest** for one of the two possible screens (see screens 44 and 46), depending on what input (**Inpt:**) is highlighted.
3. Enter the correct values as in screen 44 or 46, highlight **Calculate** or **Draw**, and then press **[ENTER]**.

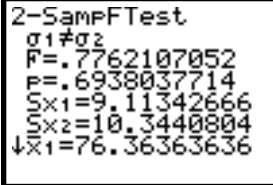
Screen 45 is the **Calculate** output for the **Data List** input of screen 44. Screen 47 is the **Draw** output for the **Stats** input of screen 46.

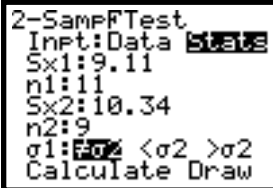
With a p-value of **0.6938**, we cannot reject the null hypothesis. The ratio of variances is not significantly different from 1, so there is no significant difference in the variability of male and female diastolic blood pressure.

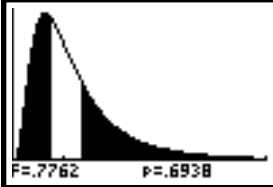
The **F** above was calculated with $9.11^2/10.34^2 = 0.7762$.

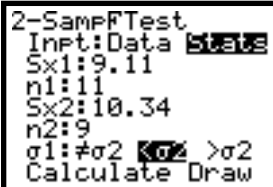
Note that if there were a one-tailed test ($\sigma_1 < \sigma_2$) (as in screen 48) with **F** = $9.11^2/10.34^2 = 0.7762$, we would see that the **p** value is half the value shown in screen 47, or that the p-value = **0.3469**, as in screen 49.

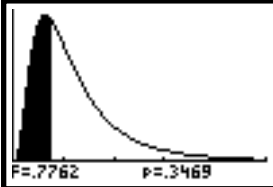
(44) 

(45) 

(46) 

(47) 

(48) 

(49) 

Activity 8, Testing Hypotheses (Means, Proportions, and Standard Deviations) (cont.)

Before the existence of the TI-83, textbooks suggested that the largest sample variance (or standard deviation) be put in the numerator to force us to the right tail so it would be easier to use tables in back of these texts. You will see in screens 50 and 51 that the **p** value and the conclusion (from the textbooks' suggestions) would be the same as above; thus, it is no longer necessary to force the larger value into the numerator. If the larger value is in the numerator, the **F** ratio will be larger than 1; if it is not, the ratio will be less than 1.

Note that $1/1.2883$ from screen 51 equals **0.7762** of screen 49.

