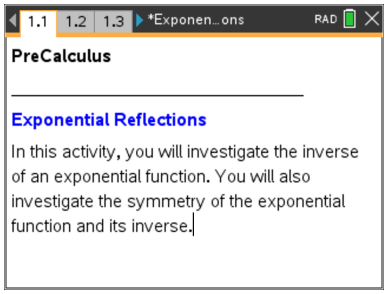
Exponential Reflections

TI-Nspire™ CX/CX II

Teacher Notes

Math Objectives

- Students will determine if a function is invertible.
- Students will find the inverse of an exponential function.
- Students will determine the symmetry of an exponential function and its corresponding logarithmic function.



Vocabulary

- line reflections
- symmetry
- logarithmic function
- exponential function
- inverse functions

About the Lesson

In this activity, students will investigate the inverse of an exponential function by observing a scatterplot. Students will determine that the inverse of an exponential function is a logarithmic function. As a result, students will:

- Analyze the function $f(x) = 2^x$, its corresponding inverse function $g(x) = log_2 x$, and their reflection about the line $y = x$.
- Analyze the function $f(x) = e^x$, its corresponding inverse function $g(x) = lnx$, and their reflection about the line $y = x$.
- Graph the function $f(x) = 10^x$ and its corresponding inverse function $g(x) = logx$.

TI-Nspire™ Navigator™

- Send the TI-Nspire document to students.
- Use Class Capture to view and discuss the graphs.
- Use Quick Poll questions to adjust the pace of the lesson according to student understanding.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

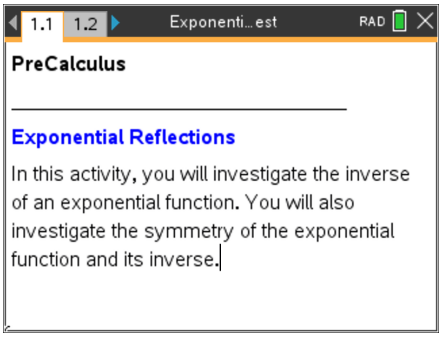
- Exponential_Reflections_Student.pdf
- Exponential_Reflections_Student.doc

TI-Nspire document

- Exponential_Reflections.tns

Open the TI-Nspire document *Exponential_Reflections.tns*

In this activity, you will investigate the inverse of an exponential function. You will also investigate the symmetry of the exponential function and its inverse.



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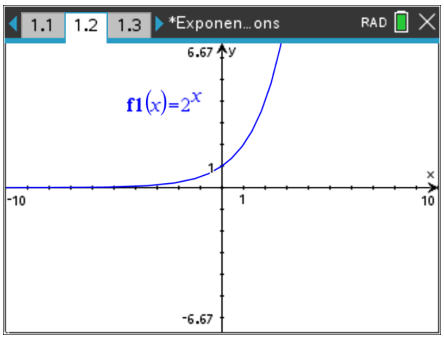
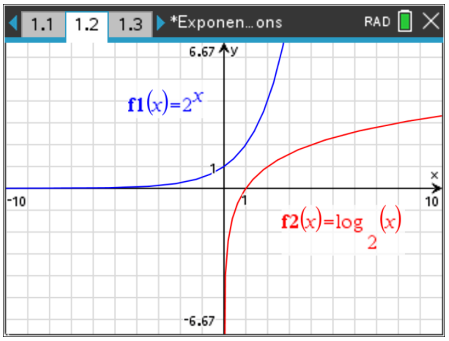
Problem 1 – Reflecting an Exponential Function

1. The exponential function $f(x) = 2^x$ is displayed.

   A function is invertible if each output value is mapped from a unique input value. Is the function $f(x) = 2^x$ invertible? What would the inverse of this graph look like? Sketch the function $y = 2^x$ and its inverse on the grid to the right.

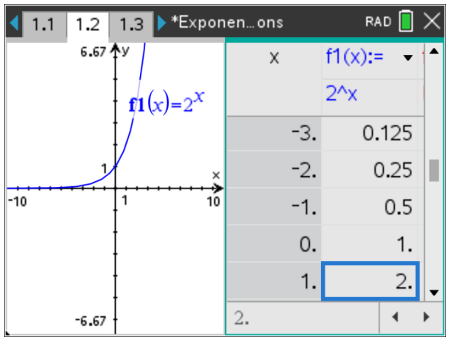




   Answers: Yes, the function $f(x) = 2^x$ invertible. Since the graph of the function $f(x) = 2^x$ is increasing, concave up and has a horizontal asymptote of $y = 0$, the inverse graph would be increasing, concave down, and have a vertical asymptote of $x = 0$. Students may also notice that the inverse passes through (1, 0) and has a domain of $(0,\infty)$ and a range of $(-\infty\infty)$. The sketch drawn by the students should be similar to the calculator screen shot to the right.

Tech Tip: For gridlines press menu and select 2 View, 6 Grid, then 3 Lined Grid.

2. Press ctrl T to access a table of values for your function.



Record the *y*-values under the original *y*-value column in the table below. Recall that if the function $f(x) = 2^x$ consists of input-output pairs (a, b), then the inverse function consists of input-output pairs (b, a).

Next record the inverses of each point by switching the *x*- and *y*-values and recording the results in the inverse columns in the table below.

Press ctrl T again to return to a full screen of the graph.

Answers:

Original *x*-value	Original *y*-value	Inverse *x*-value	Inverse *y*-value
–2	0.25	0.25	–2
–1	0.5	0.5	–1
0	1	1	0
1	2	2	1
2	4	4	2
3	8	8	3

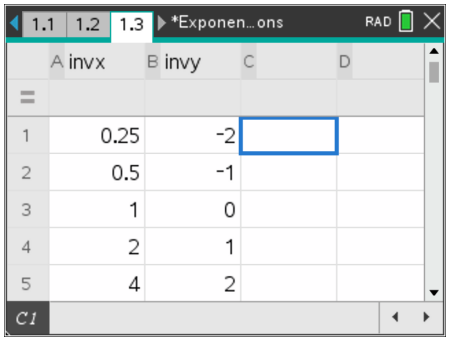
Move to page 1.3.

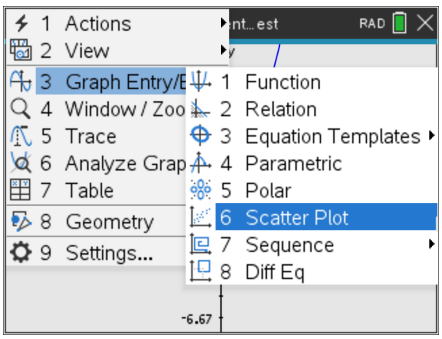
3. Enter the inverse values in **invx** and **invy**. Move back to page 1.2.

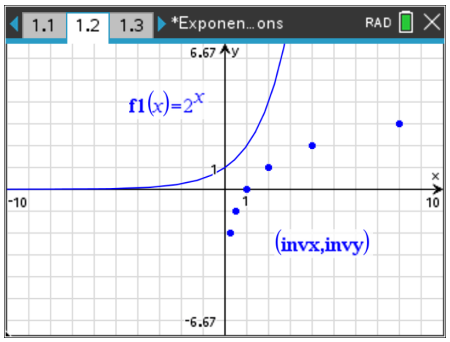
   To set up the scatter plot of the two lists, press menu and select 3 Graph Entry/Edit and then 6 Scatter Plot. For the x, press var and select invx. For the x, press var and select invy. Press enter.

   Do your plotted points appear to be on the graph of the inverse function that you sketched in Question 1?

   Answers: The plotted points should appear to be on the graph sketched by the students in Question 1.



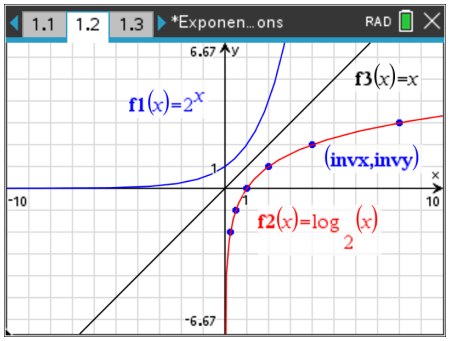




4. The inverse of a general exponential function $f(x) = b^x$ is a logarithmic function of the form $g(x) = log_b x$. Write the inverse of $f(x) = 2^x$.

   Answer: $g(x) = log_2 x$ or $f^{-1}(x) = log_2 x$

5. Check your result by graphing this function in $f2(x)$ to see if it passes through all the plotted points. Also graph the identity function $f3(x) = x$. Are the two graphs symmetric with respect to the line $y = x$?



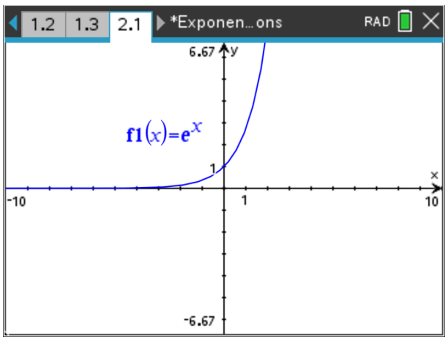
Note: To return to graphing a function, press menu and select 3 Graph Entry/Edit and then 1 Function. The $log_b x$ is found by pressing ctrl log.

Answer: Yes, the graphs are symmetric with respect to the line $y = x$. The graphs appear to be reflected across the line $y = x$.

Move to page 2.1.

Problem 2 – The inverse of $f(x) = e^x$. This function has a natural base of ***e***.

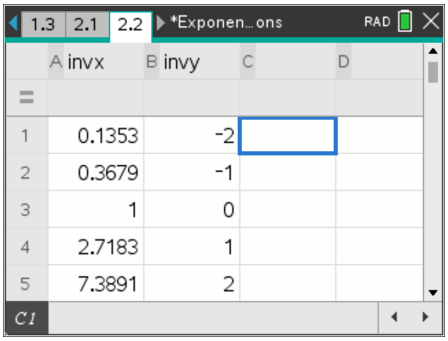
6. Graph $f1(x) = e^x$. Repeat the steps of **Problem 1** using $f(x) = e^x$.

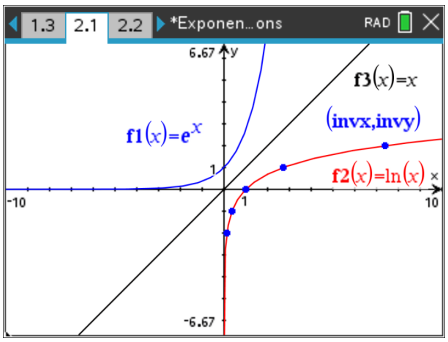


What is the inverse of $f(x) = e^x$.

Note: The inverse of $f(x) = e^x$ is called a Natural Logarithmic function.

Answer: $g(x) = lnx$ or $f^{-1}(x) = lnx$





Teacher Tip: Students will likely write $g(x) = log_e x$ and may use this notation to graph the logarithmic function. This is a good time to have students notice the relationship on the keypad of the e^x key and ctrl $\ln$ and that $g(x) = log_e x$ should be written as $g(x) = lnx$.

Teacher Note: These are the table values if the students make a table while completing **Problem 2.**

Original *x*-value	Original *y*-value	Inverse *x*-value	Inverse *y*-value
–2	0.1353	0.1353	–2
–1	0.3679	0.3679	–1
0	1	1	0
1	2.7183	2.7183	1
2	7.3891	7.3891	2
3	20.086	20.086	3

Move to page 3.1.

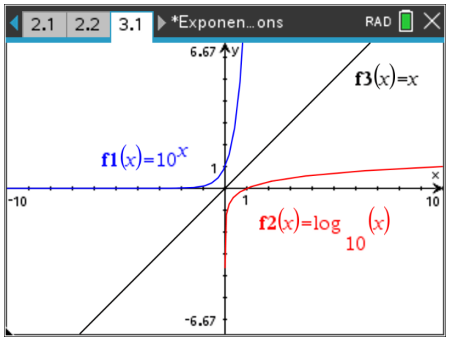
Problem 2 – The inverse of $f(x) = 10^x$.

7. Graph $f1(x) = 10^x$.

   Find the inverse of $f(x) = 10^x$. Check the symmetry of the function and its inverse by graphing.

   Note: The inverse of $f(x) = 10^x$ is called a Common Logarithmic function.

   Answer: $g(x) = logx$ or $f^{-1}(x) = logx$



Teacher Tip: Students will likely write $g(x) = log_{10} x$ and may use this notation to graph the logarithmic function. This is a good time to have students notice the relationship on the keypad of the 10^x key and ctrl log and that $g(x) = log_{10} x$ should be written as $g(x) = logx$. By default, if no base is entered when using ctrl log the base is 10.

Teacher Tip: Students may notice that the graph of the common logarithmic function appears to stop as the graph approaches the *y*-axis (as *x* approaches 0 from the right.). This is a great opportunity to investigate a table of values to convince the students that the graph does not stop. See **Optional Notes**.

Optional Notes: Press ctrl +page. Select 4 Add Lists & Spreadsheet. Press ctrl T to switch to a table of values for your logarithmic function. Make sure you select the correct function from the list. Press menu, 2 Table, and 5 Edit Table Settings. Change Independent to Ask and explore values that approach 0 from the right.

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