# CALCULUS WITH THE TI-89

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To explore the concepts of the limit of a function f($x$) as $x$ approaches an arbitrary constant $c$ ( $\lim\limits_{x \to c}$ f(x) ) and the limit of a function f($x$) as $x$ approaches infinity ( $\lim\limits_{x \to \infty}$ f(x) ), we investigate the vertical and horizontal asymptotes of a rational function, $f(x) = \dfrac{x^2 - 7x + 12}{x^2 - 4}$ , graphically, numerically and symbolically.  The graphical analysis is depicted in Figures1 and 2.
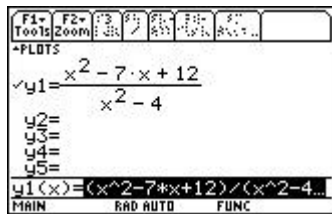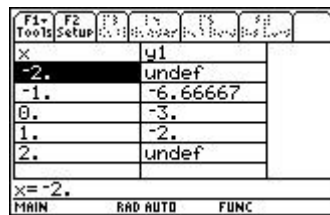

**Figure 1**
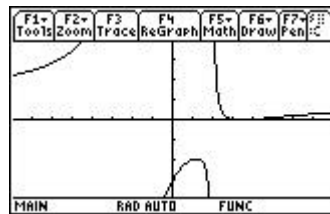

**Figure 2**

The numerical information from tables such as that in Figure 4 suggests the limit as x approaches 2 and -2.


**Figure 3**


**Figure 4**

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The symbolic analysis emphasizes that the right hand limit and left hand limit are not equal and that the limit as x approaches 2 does not exist. The computer algebra system performs the limit operation as portrayed in Figure 5.


**Figure 5**

The numerical and symbolic investigations for the horizontal asymptotes of *f(x)* are illustrated in Figures 6-8.
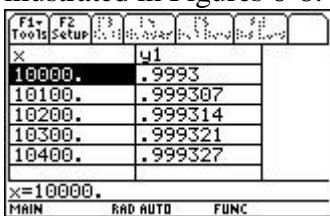


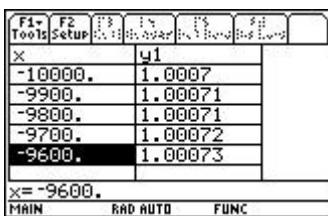**Figure 6**                      **Figure 7**                      **Figure 8**

After the students have experimented with these three points of view, challenge them to consider the limit of $\dfrac{\sin x}{x}$ as *x* approaches zero, the limit of $(1+\dfrac{1}{x})^{x}$ as *x* approaches infinity, and other important limits.

Using either the CBL with a motion detector or a CBR (Calculator Based Ranger with its built-in motion detector), students can interact with and develop a deeper understanding of the concepts of distance traveled, displacement, velocity, speed, acceleration and the connections between these ideas. The first two of the following programs (hiker and ranger) are published by Texas Instruments and are available at http://www.ti.com/calc/docs/89.htm. The programs can be downloaded to your computer and transmitted to your TI-89 with *TI Graph Link* Software and a link cable attached to your computer. The third program (cblm) available for the TI-92 was modified for the TI-89.

In Activity 1 (Figure 9) students explore distance-time graphs. Using the hiker program volunteers try to create distance-time graphs with some of the characteristics listed below. The students verbalize how the walker should move to create the required graph. Students simulate being a particle moving back and forth along a line. This activity should help them to see that their back and forth linear motion will be represented in the position-time graph. Students can experiment with various conditions such as positive constant

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velocity, followed by zero velocity, followed by negative velocity; or variable velocity with zero, positive, or negative displacement.

In Activity 2 (Figure 10) students match distance-time graphs and velocity-time graphs produced with the ranger program. A volunteer tries to match the distance-time graph or velocity-time graphs produced by the TI-89. The class sometimes coaches the volunteer on what actions are necessary in terms of velocity and acceleration.

In Activity 3 (Figure 11) students discuss the relationship between the distance and velocity graphs generated with the cblm program. A volunteer walks back and forth in front of the motion detector connected to a CBL and a TI-89. Students discuss the relationships between the functions.
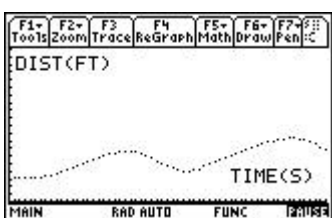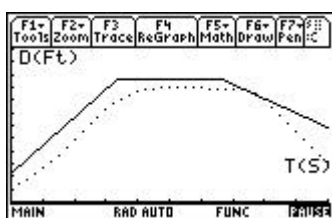
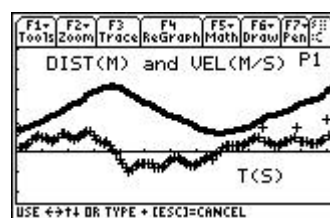| | | |
|---|---|---|
| **Figure 9** **Activity 1** | **Figure 10** **Activity 2** | **Figure 11** **Activity 3** |

The computer algebra system can be used to demonstrate the definition of the derivative using the difference quotient. For example, students can estimate the derivative of $y = \cos(x)$ at $x = \pi/6$ using the average rate of change for increasingly small values of $h$ and then use the definition of the derivative to verify their observations.
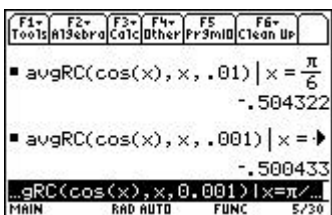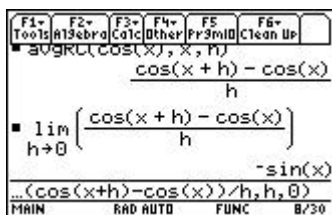
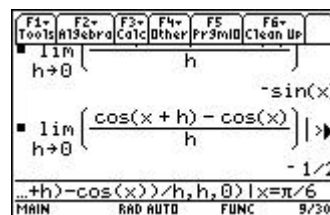| | | |
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| **Figure 12** | **Figure 13** | **Figure 14** |

To observe the graphical representation, students draw the tangent line at $x = p/6$. Noticing the equation in the lower left corner of the screen, they see that the slope of the tangent line is the derivative of the cosine function evaluated at $x = p/6$.
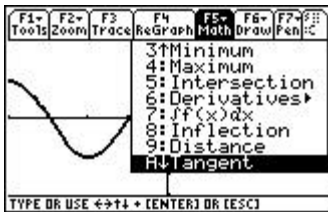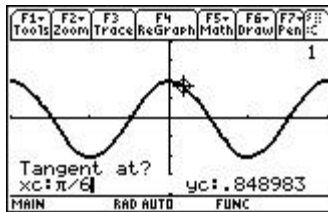
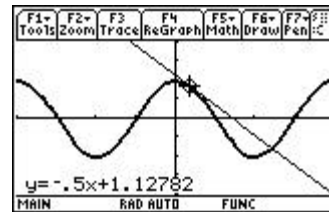| | | |
|---|---|---|
| **Figure 15** | **Figure 16** | **Figure 17** |

Other ways to obtain the result are on the Graph screen using the Derivatives option (Figures 18-20) or on the Home screen (Figure 21). The derivative of cos ($x$) at $p/6$ appears in the lower left corner of the screen in Figure 20.
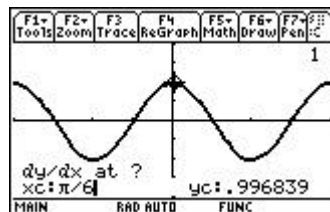
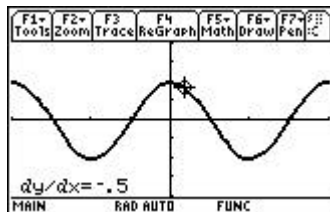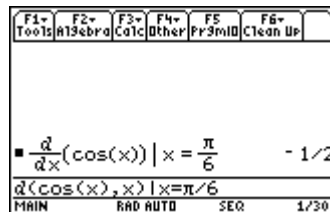| | |
|---|---|
| **Figure 18** | **Figure 19** |

| | |
|---|---|
| **Figure 20** | **Figure 21** |

Programs for visualizing the area under a curve using Riemann Sums are effective in presenting the concept of the integral. In addition to calculating an integral on the Home screen, doing it on the Graph screen provides a visual aid.
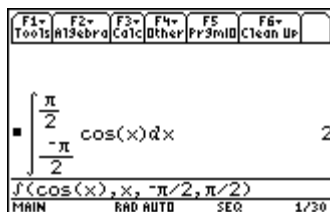
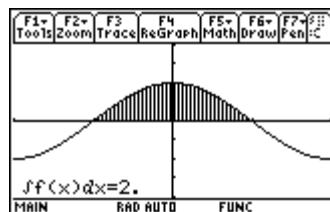| | |
|---|---|
| **Figure 22** | **Figure 23** |

The logistic equation is a staple in a differential equations or modeling course. For example, to model the spread of a disease in a confined population of 4000 people, consider the equation $\dfrac{dy}{dx} = .00025\,y(4000 - y)$ (Figure 24) with initial condition y(0) = 2.

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The TI-89 produces a slopefield and an approximation of the solution using the Runge Kutta or Euler method (Figure 25).
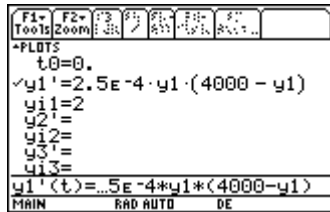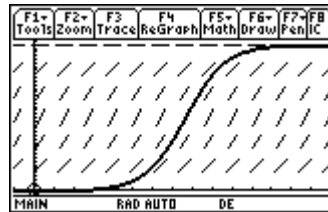
**Figure 24**

**Figure 25**

The deSolve command will obtain an analytical solution on the Home screen (Figures 26-27). The graph of the analytical solution can be drawn in function mode (Figures 28-29) and compared to the approximation using the Runge Kutta method on the slope field graphed in differential equation mode (Figures 30-31).
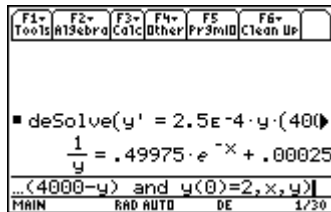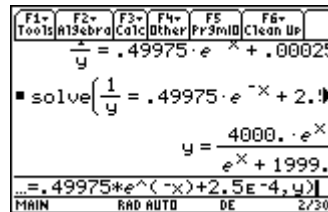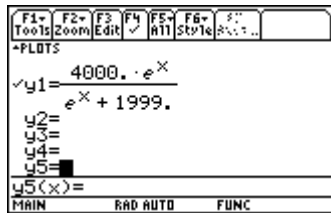
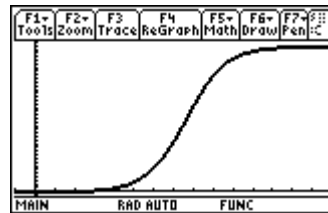**Figure 26**

**Figure 27**
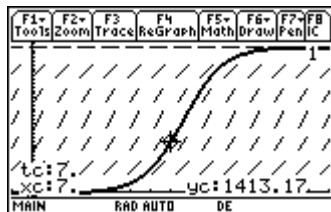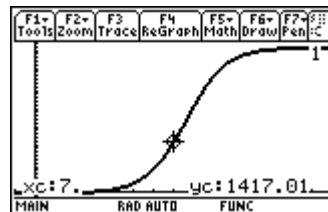
**Figure 28**

**Figure 29**

**Figure 30**

**Figure 31**

These are but a few topics that can be presented and understood more easily from different points of view with the aid of technology. Others include visualization of Taylor series, 3-D graphing with rotation of axes, contour plots, implicit plots and phase portraits.

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# References

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