MATH AND SCIENCE @ WORK

AP* CALCULUS Educator Edition

ROBONAUT 2: FIRST HUMANOID ROBOT IN SPACE

Instructional Objectives

Students will

- approximate a rate of change from a table of values;
- predict the graph of the derivative of *f(t)*; and
- use numerical methods to construct the graph of a first and second derivative.

Degree of Difficulty

For the average student in AP Calculus, this problem is at a moderate difficulty level.

Class Time Required

This problem requires 45–50 minutes.

- Introduction: 5–10 minutes
  - Read and discuss the background section with the class before students work on the problem.
- Student Work Time: 30 minutes
- Post Discussion: 10 minutes

Background

This problem is part of a series of problems that apply Math and Science @ Work in NASA's research facilities.

NASA uses robots in many ways to help with space exploration. When it's possible for robots to perform tasks, rather than people, there are some obvious advantages. Robots do not have to eat, drink, breathe, or sleep. They can perform tasks over and over in exactly the same way without getting bored. And they can also perform tasks that are too dangerous or physically impossible for humans.

A previous example of NASA's robotics was the space shuttle's robotic arm—which held astronauts during spacewalks and helped move objects in and out of the shuttle. Mars rovers are also NASA robots and are being used to take photos and collect and analyze samples from the surface of Mars.

In 1996, NASA began developing and using robots that looked and functioned more like humans. Robonaut 1 (R1) was a human-like robot

Grade Level
11–12

Key Topic
Numerical and graphical representations of derivatives

Degree of Difficulty
Moderate

Teacher Prep Time
10 minutes

Class Time Required
45–50 minutes

Technology
- TI-Nspire ™ Learning Handhelds
- TI-Nspire document: *Robonaut2.tns*
- Projector
- Movie player

AP Course Topics
Derivatives:
- Derivative at a point
- Derivative as a function
- Applications of derivatives

NCTM Math Standards
Process:
- Problem solving

prototype that could perform maintenance tasks or be mounted on a set of wheels to explore the surface of the moon or Mars. Through 2006, R1 performed in numerous experiments in a variety of laboratory and field test environments, proving that the concept of a robotic assistant was valid.

Currently, NASA is collaborating with General Motors (GM) on the development of a next-generation Robonaut. Robonaut 2 (R2), a dexterous humanoid robot, was launched to the International Space Station (ISS) in February of 2011. Initially, R2's primary role on the ISS is experimental; performing tasks and operations similar to those that it has already performed on Earth. This will allow engineers to observe how a dexterous robot will behave in space and make any needed adjustments.

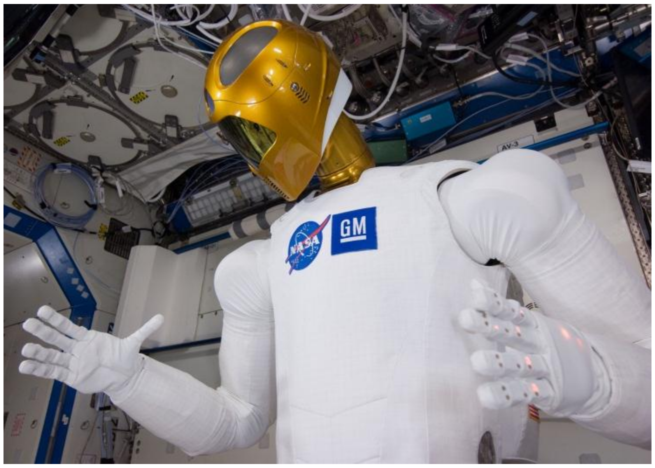


Figure 1: R2 in the Destiny Laboratory of the ISS

Figure 2: R2 holds an instrument to measure air velocity during a system check-out on the ISS

The dexterity of R2's arm joints and fingers was developed to be similar to that of human movement. One advantage of this design is to allow R2 to assist astronauts with (or even take over) simple, repetitive, or dangerous tasks. R2 does not require specialized tools because its dexterous capabilities allow it to grip and maneuver the same tools used by astronauts.

R2 continues to demonstrate capabilities of performing routine maintenance tasks, such as monitoring air velocity and cleaning handrails. Planned upgrades to R2 will allow it to function in the vacuum of space, where it can perform repairs on the exterior of the ISS or simply help astronauts as they perform spacewalks. Engineers observe how the dexterous robot behaves in space, and make adjustments as needed, so R2 can one day take over tasks and responsibilities.

The next step for NASA robots (like R2) could be the exploration of near-Earth objects, such as asteroids and comets, and eventually Mars and its moons. The robots could serve as scouts, providing advanced maps and soil samples, and beginning work on the infrastructure of a base for future missions. The astronaut crew to follow would then be better prepared for the exploration ahead. Humans and robots working together to explore space will provide greater results than either could achieve alone, enabling an exciting future of new discoveries.

For an introduction to Robonaut 2, watch the video, Robonaut 2: Introduction (4:30 min), accessible at the following link: http://youtu.be/Wf8E1Iyelu4.

AP Course Topics

Derivatives

- Derivative at a point
    - Approximate a rate of change from a table of values

- Derivative as a function
  - Corresponding characteristics of graphs of a function and its derivative
- Applications of derivatives
  - Interpretation of the derivative as a rate of change in varied applied contexts

NCTM Standards

Process

- Problem solving
  - Solve problems that arise in mathematics and in other contexts

Problem and Solution Key (One Approach)

Students are given the following problem information within the TI-Nspire document, Robonaut2.tns, which should be distributed to their TI-Nspire handhelds.

Robonaut 2, or R2, moves by performing a series of programmed commands called a script. Engineers and scientists must use knowledge of angles and calculus in order to write scripts for the robots to perform each task as well as keep the robot's motions safe for nearby humans. Watch the short video, "Robonaut 2: Force Control for Working Around People", to see R2 perform a script showing how robots can work with humans in a safe manner. This video shows the R2's basic range of motion and its normal operating speeds. Notice how the angle of R2's arms at the elbow joints are changing as it performs the script (see Figures 3 and 4). As R2 performs the script, the velocity is constantly being calculated for each of its joints, and then is checked against a predefined limit to make sure that it is operating at safe speeds. If the speeds are too fast, R2 executes an emergency stop, setting brakes and turning off motor power.

Play the video, Robonaut 2: Force Control for Working Around People (0:53 sec), accessible at the following link: http://youtu.be/MoDHhq0FiuU.

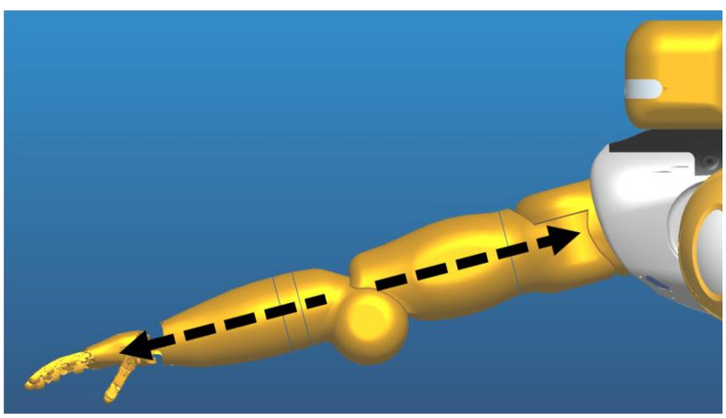


Figure 3: Elbow joint pitch approximately 0°

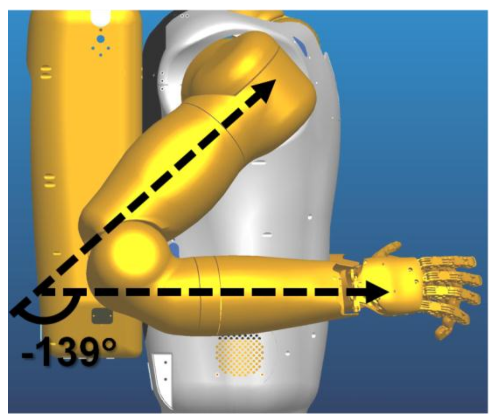
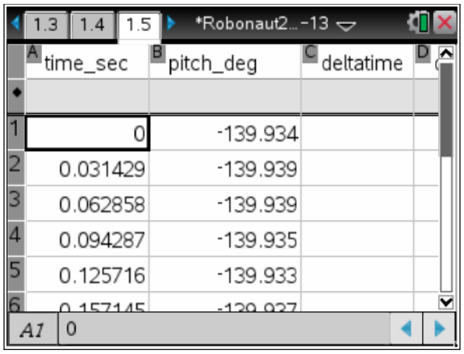


Figure 4: Elbow joint pitch approximately -139°

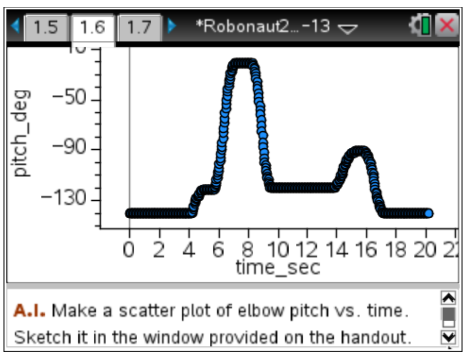
On TI-Nspire page 1.5, arm joint data captured during the first 20 seconds of the script in the video is shown. The first column is time in seconds and the second column is the pitch of the right elbow joint measured in degrees.



A. Analyze the data.

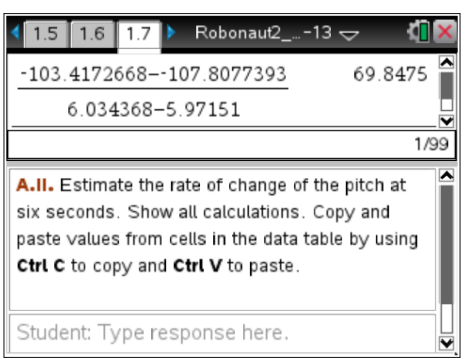
I. On page 1.6, make a scatter plot of elbow pitch vs. time and sketch it in the window below.

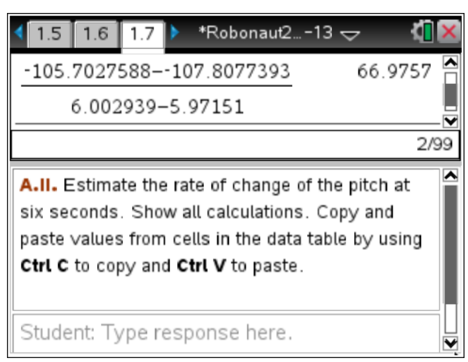
The screenshot below shows the scatterplot on the TI-Nspire. Students sketches should be similar.



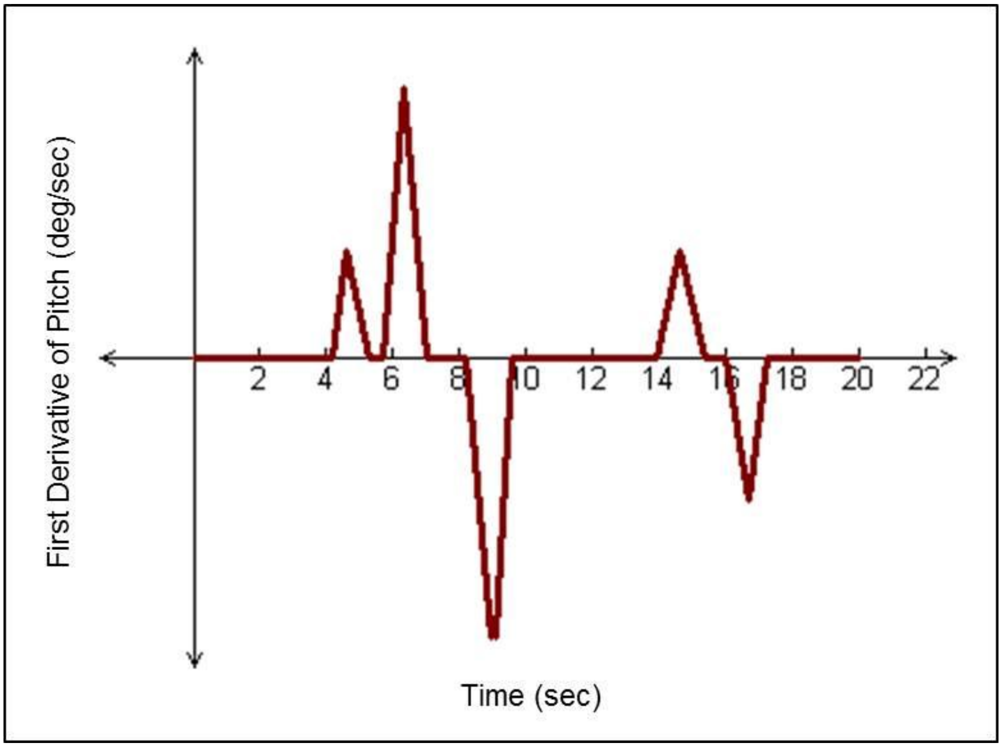
II. On page 1.7, estimate the angular velocity (or rate of change of the pitch) of R2's elbow at 6 seconds. Show all calculations. Copy and paste values from cells in the data table by using **Ctrl C** to copy and **Ctrl V** to paste.

Answers may vary depending on which difference quotients are used. The screenshot on the left shows the symmetric difference quotient which equals 69.8475 deg/sec. The screenshot on the right gives the results if students use the two points closest to 6 to obtain the difference quotient which equals 66.9757 deg/sec.





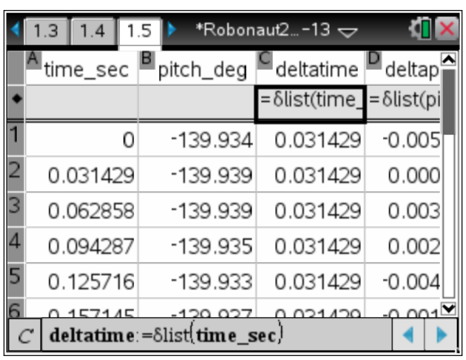
III. Predict the graph of the derivative of pitch with respect to time. Sketch your prediction in the window provided below.



B. Use numerical methods to approximate a derivative.

I. Go back to page 1.5 and enter the following command in the formula cell for the column labeled deltatime: **deltatime:=deltalist(time_sec)**. What does this command do?

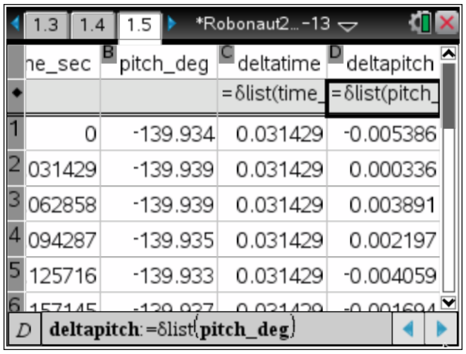
This command calculates the differences between consecutive times.



	A time_sec	B pitch_deg	C deltatime	D deltap
◆			=δlist(time_	=δlist(pi
1	0	-139.934	0.031429	-0.005
2	0.031429	-139.939	0.031429	0.000
3	0.062858	-139.939	0.031429	0.003
4	0.094287	-139.935	0.031429	0.002
5	0.125716	-139.933	0.031429	-0.004

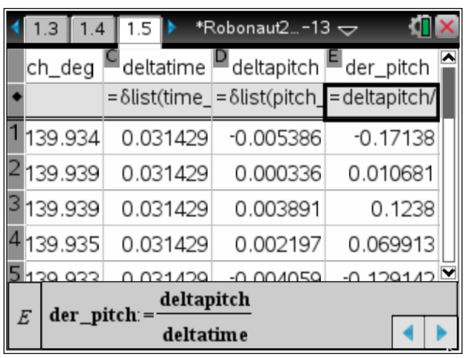
II. Now go back to page 1.5 and enter the following command in the formula cell for the column labeled deltapitch: **deltapitch:=deltalist(pitch_deg)**. What does this command do?

This command calculates the differences between consecutive pitches.



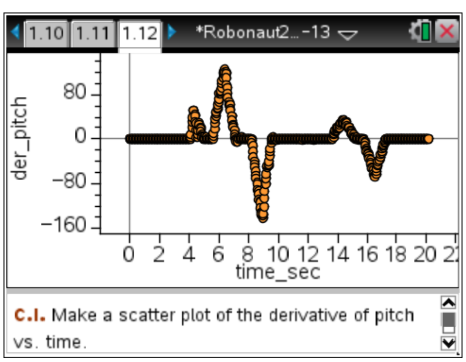
III. Return to page 1.5 and enter the following command in the formula cell for the column labeled der_pitch: **der_pitch:=((deltapitch)/(deltatime))**. What does the resulting list approximate?

The list approximates the derivative of pitch with respect to time.



C. Estimate a rate of change.

I. On page 1.12, make a scatter plot of the derivative of pitch vs. time.



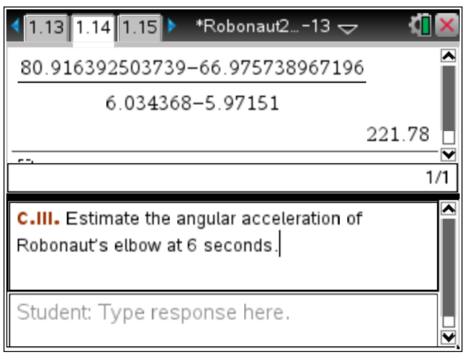
II. Based on the graph in part I, estimate the maximum angular speed of R2's elbow. Explain how you arrived at your answer.

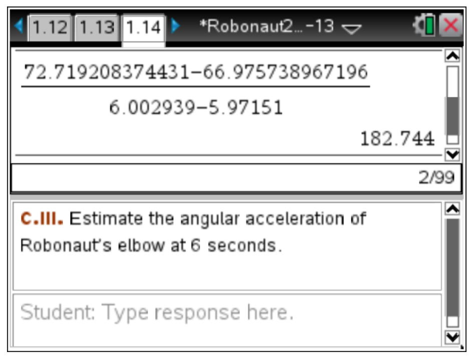
Speed is the magnitude or absolute value of velocity. Thus the maximum angular speed is the greatest distance from 0, or |-142 deg/sec| = 142 deg/sec. (Answers may vary, but should be within 20 deg/sec of this answer.)

Note: This maximum speed can be seen on the graph at about 9 seconds. The value is found by either hovering over the point or by looking at the table on page 1.5.

III. Estimate the angular acceleration of R2's elbow at 6 seconds.

Again, answers may vary depending on which difference quotients are used. The screenshot on the left shows the symmetric difference quotient which equals 221.78 deg/sec^2. The screenshot on the right gives the results if students use the two points closest to 6 to obtain the difference quotient which equals 182.744 deg/sec^2.





IV. Based on the estimated values of angular velocity and angular acceleration of R2's elbow at 6 seconds, explain how R2's arm is moving at that time.

Since the angular velocity is positive, R2's pitch is increasing. This means its arm is moving outward, or straightening out from a bent position. Since the angular acceleration is also positive, its arm is speeding up at that time.

Scoring Guide

Suggested 9 points total to be given.

Question		Distribution of points
A	*4 points*	1 point for correct scatter plot 1 point for an appropriate difference quotient 1 point for correct answer with units 1 point for graph of the derivative with correct shape
B	*2 points*	1 point for entering correct calculation commands 1 point for correct explanations of commands
C	*3 points*	1 point for correct scatter plot 1 point for correct maximum speed 1 point for correct explanation of R2's arm movement at 6 seconds

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school AP Calculus instructors.

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