EXPLORATIONS™

Activity 8

Objective

- Determine the probability of independent, compound events
- Design simulations and collect data to explore streaking behavior

Gambler's Fallacy: Lucky Streaks and Slumps

Introduction

The expression, "I'm on a roll!" is often heard when a person feels lucky. Many people think that there are such things as lucky streaks and that they will continue to win at a game because their luck is good. Sometimes people talk about their luck running out, thinking that if they have already won many times, they will certainly lose the next time as their luck is bound to run out. Still others who have been in a slump rationalize that they should keep on playing because, surely, their luck is due.

Is there such a thing as a lucky streak? Are some people likely to have their luck run out, while others are more likely to have their luck continue?

Problem

A fair coin has been tossed, and has landed on the same side three consecutive times. Is the next toss more or less likely to be the same?

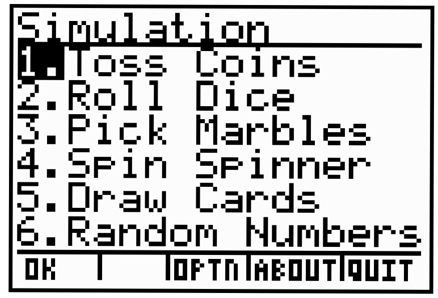
Exploration

Experimental Streaks

Respond to # 1 – 4 on the Student Worksheet

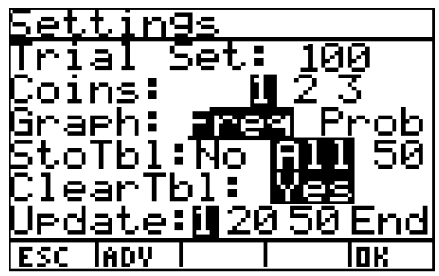
This simulation explores questions # 1 and # 3 on the Student Worksheet. Every time a streak of three heads or three tails occurs, you will examine the behavior of the coin on the next toss..

1. Open the Probability Simulation application and select **Toss Coins**.

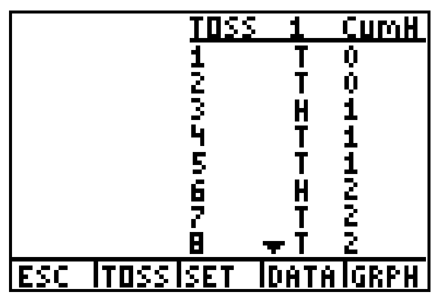


2. Select **SET** and change the settings as shown.

3. Select **OK** to return to the simulation screen, and then select **TABL**.



4. Select **TOSS**.
This simulates tossing a coin 100 times. After the simulation, move the cursor up to Toss 1 in the table.



5. Look for either three Hs or three Ts in a row in the Toss 1 table. Each time you see a streak of three, put a tally mark in the STREAKS OF 3 column in the table in # 5 on the Student Worksheet.

6. Look at the next toss. If the outcome is the same as that of the previous three tosses, put a tally mark in the CONTINUE column.

If a streak continues, other sets of three in a row may be embedded in the streak and must be tallied also. Look at the example:

H H H T T H T T T T T H H H

STREAKS OF 3	CONTINUE
IIII	II

- Three Hs are followed by a T, so put a tally mark in the STREAK OF 3 column.
- The next streak starts with three Ts, followed by a T, so put a tally mark in both columns.
- There are another three Ts (starting with the second T of the first streak) followed by a T, so place another tally mark in both columns.
- The last streak of three Ts (starting with the third T in the tails streak) is followed by an H, so place another tally mark in the STREAKS OF 3 column only. Do not count the last streak of three Hs since you cannot determine what will happen on the next toss.

Respond to # 5 – 6 on the Student Worksheet.

In 100 tosses you may have had a certain number of streaks of three. From the Law of Large Numbers, you know that the greater the number of trials, the closer the experimental probability comes to the theoretical probability.

Pool your results with the rest of the class.

Respond to # 7 – 15 on the Student Worksheet.

Student Worksheet

Name ________________________

Date ________________________

Gambler's Fallacy: Lucky Streaks and Slumps

1. A fair coin has landed on the same side three times in a row. How likely do you think it is that it will continue to land on the same side on the next toss? Explain.

2. A fair coin is tossed four times. How likely do you think it is that the coin will land on the same side every time? Explain. What is the difference between this question and the first question?

3. Predict how many streaks of 3 you think will occur in 100 tosses.

   Note: A toss sequence of TTTTH has two streaks of three: one that starts on the first toss and another that starts on the second toss.

4. For each of the scenarios in # 1– 3, design a simulation to test your hypothesis. Each simulation must identify what one trial is, what a successful outcome is, and how many trials should be conducted.

5. Complete the table based on the coin toss simulation.

	STREAKS OF 3	**CONTINUE**
Tallies		
Total		
Relative Frequency		
Percent Likelihood		

6. How long is your longest streak? Was it a streak of heads or tails?

7. Record the class results of the number of streaks of three in 100 tosses in the table.

# of Streaks of Three	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Class Totals																	

8. Use your calculator to make a box and whiskers plot of the results. Sketch and label your plot.

9. Does the simulation support the conjecture you made in # 3? Explain.

10. Record the class results of the longest streak in 100 tosses in the table.

# of Tosses in Longest Streak	5	6	7	8	9	10	11	12	13	14	15
Class Totals											

11. Use your calculator to make a histogram of the results. What window values did you use? Sketch and label your histogram.

12. Record the class results of streaks of three continuing or halting..

	TOSSES	STREAKS OF 3	CONTINUE
Total			
Relative Frequency			
Percent Likelihood			

13. Does the simulation support the conjecture you made in # 1 above? Explain.

14. Perhaps the results you obtained were expected for a streak of 3. After all, getting three heads or tails in a row is not a very long streak. Do you think that if tails landed up 10 times in a row, it is more likely that the eleventh toss would be heads? Why or why not? Describe a simulation to test your conjecture.

15. Give an example of what is meant by the term *gamblers fallacy*. You may want to research this on the Internet.

Extensions

1. Use the Sample Space for Seven Coin Tosses chart from activity 7 to analyze the theoretical probability of a streak of four continuing in seven tosses of a coin. List all combinations that contain streaks of four and note how many continue.

Note: *Do not count any streaks of four that occur on the last four tosses since you are unable to determine if it would continue or halt.*

2. Why is it unrealistic to analyze a sample space chart to determine the theoretical probability of a streak of three continuing in one hundred tosses?

3. Using the class data and the method used in Activity 3, plot the relative frequency of cumulative halts versus comulative streaks of three as the number of streaks occuring increases. Estimate the theoretical probability from this plot.How does the estimate of the theoretical probability determined with this method differ from the estimate made in # 12?

Teacher Notes

Activity 8

Gambler's Fallacy: Lucky Streaks and Slumps

Objective

- Determine the probability of independent, compound events
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Materials

- TI-84 Plus/TI-83 Plus

Teaching Time

- 60 minutes

Preparation

Much has been written about the gambler's fallacy. A player has such a strong desire to win that he will manipulate his own beliefs to support this desire. Thus if he has been on a winning streak, he reasons that he should keep on playing since luck is with him. On the other hand, if he has been losing, he also reasons that he should keep playing, thinking that his luck is sure to change.

Ironically, these two psychological dispositions that fall under the guise of the gambler's fallacy lead to opposite predictions. Subconsciously, one imagines that a supposedly fair coin that lands heads five times in a row (a winning streak) must really be weighted, even knowing that such rare events do indeed occur naturally. However, if five tails were tossed (a losing streak), that same subconscious mind will now reason that since a fair coin must land head s up 50% of the time, the coin is now more likely to land heads up the next five times so it can catch up.

What is not well understood is that a coin has no memory. The next five tosses of the coin are as likely to be five more tails as they are to be five heads, which is the same likelihood that it will land heads, tails, heads, tails, heads. Even those who have an understanding of the independence of each coin toss or spin of the roulette wheel often fall prey to the gambler's fallacy in the heat of the moment. Jean le Rond d'Alembert, a great 18th century French mathematician, persisted in his belief that after a long run of heads, a tail is more likely. False intuition is hard to conquer.

The purpose of the first two questions on the Student Worksheet is to distinguish between the subtle difference between the wording of each. Aside from the gambler's fallacy, part of the difficulty students have in answering questions about probability is that they misinterpret what is being asked. Discuss student responses to the first four questions before beginning this simulation.

Answers to Student Worksheet

1. A fair coin is 50% likely to land on the same side on the next toss. The outcome of one coin toss is independent of previous outcomes.

2. A fair coin is 12.5% likely to land on the same side when tossed four times in a row. The 16 combinations of four tosses of a coin can be illustrated with a tree diagram. Two of these outcomes (either four heads or four tails) would satisfy the condition. Though this question and the first are both considering situations of four tosses of a coin that always lands on the same side, the first question is only concerned with what happens on the last toss. This question is concerned about what happens on all four tosses.

3. Answers will vary.

4. Answers will vary. Possible simulations include:

    - One trial consists of four tosses of a coin, but only those trials where the first three tosses are all the same (all heads or all tails) are considered when determining the likelihood of the event. A successful outcome is one where the fourth toss is the same as the first three tosses. One hundred trials where the first three tosses are the same could be conducted; this means that many more than 100 sequences of four tosses will need to be simulated in order to come up with 100 sequences of three tosses that are the same.
    - One trial consists of four tosses of a coin, and all trials are considered in determining the likelihood of the event. A successful outcome is one where all four tosses are heads or all four tosses are tails. One hundred trials of four tosses could be conducted.
    - One trial consists of 100 tosses of a coin. This question is about a count, not a likelihood, thus there is no successful outcome; however, a count would consist of every time a sequence of three tails or three heads occur.To verify a prediction, one could conduct 100 trials of 100 tosses.

5. Answers will vary. To determine the relative frequency of a streak of three, calculate the ratio of the streaks of three to 97, because there are only 97 possible streaks of three in 100 tosses. To determine the relative frequency of a streak of three continuing, calculate the ratio of the total "continue" count to the total "streaks of three" count.

6. Answers will vary.

7. Answers will vary but will be the same within a class.

8. Answers will vary but will be the same within a class.

9. Answers will vary.

10. Answers will vary but will be the same within a class.

11. Answers will vary.

12. Answers will vary but will be the same within a class.

13. Answers will vary.

14. The eleventh toss is independent from the first ten, thus there is still a 50% chance that the streak will continue or halt. The simulation would be similar to the one conducted for this investigation with the exception that only streaks of ten would be considered. In order to toss a coin to create enough streaks of ten to see what happens on the eleventh toss, the coin would have to be tossed several thousand times.

15. Many people think that if they have been on a losing streak in a game of chance, their luck is due to change, so they should keep on playing. Conversely, other people think that if they have been winning, they must be on a winning streak, so they should keep on playing.

Extensions

1. The theoretical probability of a streak of four continug in seven tosses of a coin is 50% since 24 of the 48 streaks of four continue. THe combinations in each column that have streaks of four are:

HHHHHHH	HHHHHOH	HHHHHHO	HHHHHOO
OHHHHHH	OHHHHOH	OHHHHHO	OHHHHOO
HOHHHHH	OOOOHOH	HOHHHHO	OOOOHOO
OOHHHHH	HHHHOOH	OOHHHHO	HHHHOOO
OOOOHHH	HHOOOOH	OOOOHHO	HHOOOOO
HHHHOHH	OHOOOOH	HHHHOHO	OHOOOOO
HOOOOHH	HOOOOOH	HOOOOHO	HOOOOOO
OOOOOHH	OOOOOOH	OOOOOHO	OOOOOOO

2. The number of combinations to analyze for 100 tosses is 2^{100} – an extremely formidable task.

3. Answers will vary.